HYDRAULICS

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BY Blue

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PREFACE TO THE SECOND EDITION

In this edition it has been possible to add new material which the author has found necessary in his present courses. This consists principally of graphical methods of solving certain practical problems, the determination of the economic size of pipe, and problems of flow through compound pipes, branching pipes, pipes with laterals, and through rotating channels. The treatment of various topics has also been extended in many cases, and certain other portions have been rewritten where experience has indicated that this was desirable for greater effectiveness.

The author has been aided in this revision by the helpful suggestions of many individuals and in particular of Professor F. G. Switzer of Cornell University. R. L. D.

Troy, N. Y.,
May, 1919.

PREFACE TO FIRST EDITION

This book has been prepared as a text for students who are required to cover a wide field in hydraulics in a limited amount of time. Therefore the treatment has been made as brief and concise as is consistent with clearness. Attention has been given mostly to matters which are of fundamental importance and but little space has been devoted to those things which are of small practical value, except where necessary to illustrate basic principles. As a step in saving the student's time a liberal use has been made of diagrams, curves, and half-tones. These not only save words but often give a clearer idea at a glance than can be obtained in any other way.

The treatment throughout has been made as consistent as is possible. The solution of all problems involving the flow of water is made to depend upon applications of Bernoulli's theorem, which is the key to a rational treatment of the subject. The student is not told in the very beginning that $V = \sqrt{2gh}$ and then compelled to unlearn it later. Experience in the class room has shown that many students will persistently apply that formula whether it fits the case or not. By deriving it at a later time by an application of Bernoulli's theorem, they will more readily see that it is a very special case and thus realize more fully its limitations.

An effort has been made to avoid special cases so far as is possible. The treatment in the text and the equations are for the most part perfectly general. Special cases are given only when necessary to illustrate the application of some general principle, or where a special case makes some proposition clearer, and when the general treatment is too complex. But the attention of the reader is called to the fact that the equations there given are not universally applicable.

Class-room experience has shown that very few students obtain a true physical conception of the subject of hydraulics. To most of them, even some of the best, it is very largely an abstract subject. This is partly due to the fact that, with their limited experience and observation, they have actually seen but few of

the things with which the book deals and hence they can form no adequate mental picture of the physical facts. In order to overcome this, so far as possible, a large number of illustrations from photographs have been employed. As a further step in implanting a true physical idea in the mind of the student, a great deal of care has been exercised in the arrangement and presentation of the subject and a constant attempt has been made to connect one part with another. In many cases the problems have been taken from actual practice and have also been arranged so as to be instructive in themselves.

In considering turbines and centrifugal pumps the first essential is to convey a fair idea as to the general appearance, construction, and arrangement of such machines and possibly some simple features of their operation, since it is useless to plunge directly into a mass of equations which are no more than mathematical gymnastics to most students. The second step should be the presentation of the principles of operation together with a general idea as to actual characteristics. These facts could then be explained by as much theory as one had time to go into. In this text but very little theory has been given and that of the simplest kind, though it is believed that what is given is both general and rational. By the aid of this theory the nature of the characteristics of these machines can be accounted for. After that one is ready to take up certain very useful and practical commercial factors by the aid of which one can classify turbines or pumps, can compare one type with another, and can make an intelligent selection of the best type for certain conditions.

The simple theory of hydraulic machinery that has been given here covers about all that is really useful in a text of this scope. The design of turbines and pumps is too empirical, and requires too much judgment and experience backed up by a good supply of test data, to be expressed by a few equations. Any brief treatment of this phase of the subject would be false and misleading, hence it has been omitted. For any more extended treatment of these subjects the reader is referred to other publications of the author.

The main idea underlying the entire text has been to present fundamental principles. After this ground has once been covered, those who desire to specialize in hydraulics are prepared to

study certain topics more intensively. The devotion of considerable space to an account of experiments and test data is unwarranted here, though the student should not lose sight of the fact that the study of such is desirable when important work is undertaken. However, a sufficient amount of information on experimental coefficients and empirical factors has been given so that a correct idea may be formed both as to the range of values and the considerations that enter into the choice of a suitable value for a given case.

Very naturally some very important topics in practical hydraulics have been omitted altogether or treated very briefly and superficially because they did not involve fundamental principles and hence were not within the scope of this text, or else were of such a nature as to belong to advanced treatises. The final apology which the author makes for this work is that it has been prepared primarily to meet the needs of his own classes.

The author wishes to acknowledge his indebtedness to the various parties whose names are attached to certain of the illustrations for their kindness in furnishing the same. He is also indebted to E. H. Wood, Professor of Mechanics of Engineering in Sibley College, and to D. R. Francis, Instructor in Sibley College, for valuable assistance in the criticism of the manuscript and the reading of the proof. R. L. D.

Ithaca, N. Y.,
April, 1916.

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NOTATION

A = angle between V and u (Fig. 142)
a = angle between v and u (Fig. 142)
c = coefficient of discharge or coefficient of flow
c_c = coefficient of contraction
c_v = coefficient of velocity
D = diameter of turbine runner or pump impeller in inches
d = diameter of pipe in feet
d'' = diameter of pipe in inches
e = efficiency
e_h = hydraulic efficiency
e_m = mechanical efficiency
e_v = volumetric efficiency
F = area in square feet; in turbines and pumps it is the total area of the streams measured normal to the absolute velocity of the water
f = friction factor in pipes
= area in square feet in turbines or pumps measured normal to the relative velocity of the water
G = any weight in pounds
g = acceleration of gravity in feet per second per second
H = total effective head in feet, $= p + z + V^2/2g$
H' = any loss of head in feet
h = head in feet
I = moment of inertia
k = any coefficient of loss
l = any length in feet
m = hydraulic mean depth (or hydraulic radius) in feet
N = revolutions per minute
N_s = specific speed, $= N \times \sqrt{\text{B.hp.}}/\text{h}^{5/4}$
n = factor in Kutter's formula
= any abstract number
P = total pressure or force in pounds
p = intensity of pressure in feet of water
p' = intensity of pressure in pounds per square foot
p'' = intensity of pressure in pounds per square inch
q = rate of discharge in cubic feet per second
r = radius to any point in feet
s = slope of hydraulic gradient, $= H'/l$
= tangential component of absolute velocity, $= V \cos A$
T = torque or moment of a force in foot pounds
u = linear velocity of a point in feet per second
V = absolute velocity of water (or relative to earth) in feet per second

v = velocity of water relative to some moving point in feet per second
W = pounds of water per second, $= wq$
w = density of water in pounds per cubic foot
z = any vertical distance in feet; in measuring "head" it is a vertical elevation *above* any arbitrary datum plane
ϕ = ratio of peripheral speed of turbine runner or pump impeller to $\sqrt{2gh}$
ϕ_e = value of ϕ for which the maximum efficiency is obtained
ω = angular velocity in radians per second, $= 2\pi N/60 = u/r$

Values of quantities at specific points will be indicated by subscripts. In the use of subscripts (1) and (2) the water is always assumed to flow from (1) to (2).

ABBREVIATIONS

G.P.M. = gallons per minute
Sec. ft. = cubic feet per second
R.p.m. = revolutions per minute
Hp. = horsepower
B.Hp. = brake horsepower = D.Hp.
W.Hp. = water horsepower

HYDRAULICS

CHAPTER I

INTRODUCTION

1. Definition of Subject.—*Hydromechanics* is the science of the mechanics of fluids. It may be subdivided into three branches: *Hydrostatics* is the study of the mechanics of fluids at rest, *hydrokinetics* deals with the flow of fluids, while *hydrodynamics* is concerned with the forces exerted by or upon fluids in motion.

Hydraulics is practical hydromechanics, that is, it is the study of the applications of hydromechanics to engineering problems.[1] While it might deal with any fluid it is generally restricted to liquids and especially to water.

By idealizing conditions and ignoring phenomena that are known to exist, it is possible to study hydromechanics as a subject in pure mathematics. But naturally the results of such studies, though interesting, are often of little practical value. The determination of actual results by rigorous mathematics is often impossible because of the fact that the exact nature of certain hydraulic phenomena are either unknown or if known are so complex that it is not feasible to express them as mathematical functions. We must, therefore, resort to a combination of rigid mathematics, empirical expressions, and experimental coefficients. The science that results, based partly upon pure reasoning and partly upon experimental evidence, is called hydraulics.

It is seen that hydraulics is not an exact science. In its actual applications much depends upon the judgment and the experience of the engineer. In many cases it is necessary to compute or estimate results for which satisfactory experimental data is lacking. And in applying any experimental factors or empirical formulas it is well to have some familiarity with the

[1] The derivation of the word "hydraulics" means "flow of water in a pipe" but usage has given the word a much broader significance.

work upon which they were based in order to judge as to their application to the case in hand.

2. Distinction between a Solid and a Fluid.—The distinction between a solid and a fluid is ordinarily quite clear but there are plastic solids which flow under the proper circumstances and even metals may flow under high pressures. On the other hand, there are certain very viscous liquids which do not flow readily and it is easy to confuse them with the plastic solids. The definition of a fluid as a substance which flows must be extended therefore. The distinction is that any fluid, no matter how viscous, will yield in time to the slightest stress. But a solid, no matter how plastic, requires a certain magnitude of stress to be exerted before it will flow.

Also when the shape of a solid is altered by external forces the tangential stresses between adjacent particles tend to restore the body to its original figure. With a fluid these tangential stresses, which are proportional to the viscosity, can act only while the change is taking place. When motion ceases the tangential stresses disappear and the fluid does not tend to regain its original shape.

3. Distinction between a Gas and a Liquid.—A fluid may be either a gas or a liquid. A gas is quite compressible and when all external pressure is removed it tends to expand indefinitely. A gas is, therefore, in equilibrium only when it is completely enclosed. A liquid, on the other hand, is relatively incompressible and if all pressure, except that of its own vapor, be removed the cohesion between adjacent particles holds them together so that the liquid does not expand indefinitely. Therefore, a liquid may have a free surface, that is, a surface from which all pressure is removed, except that of its own vapor.

The volume of a gas is greatly affected by changes in either pressure or temperature or both. It is usually necessary, therefore, to take account of changes in volume and temperature when dealing with gases. Since the mechanics of gases is largely one of heat phenomena it is called thermodynamics.

The volume of a liquid is affected to a very small extent by changes in pressure or temperature and for most purposes the changes in volume or temperature may be ignored.

4. Compressibility of Water.—Water is usually said to be incompressible and as compared with gases it is relatively so. But it is much more compressible than many solids such as steel or even wood where the elastic limit is not passed. Its bulk or

volume modulus of elasticity, the ratio of the change of pressure per unit area to the change per unit of volume, is

$$E_v = 294{,}000 \text{ lb. per sq. in.}$$

This value holds only for pressures below 1,000 lb. per sq. in. and for temperatures near the freezing point. For higher temperatures it increases slightly. Thus at 77°F. it is about 327,000 lb. per sq. in. and at 212°F. it is 360,000 lb. per sq. in. Also for higher pressures than the above the modulus is materially larger. Thus at a pressure of 65,000 lb. per sq. in. Hite found a value of $E_v = 650{,}000$ lb. per sq. in.

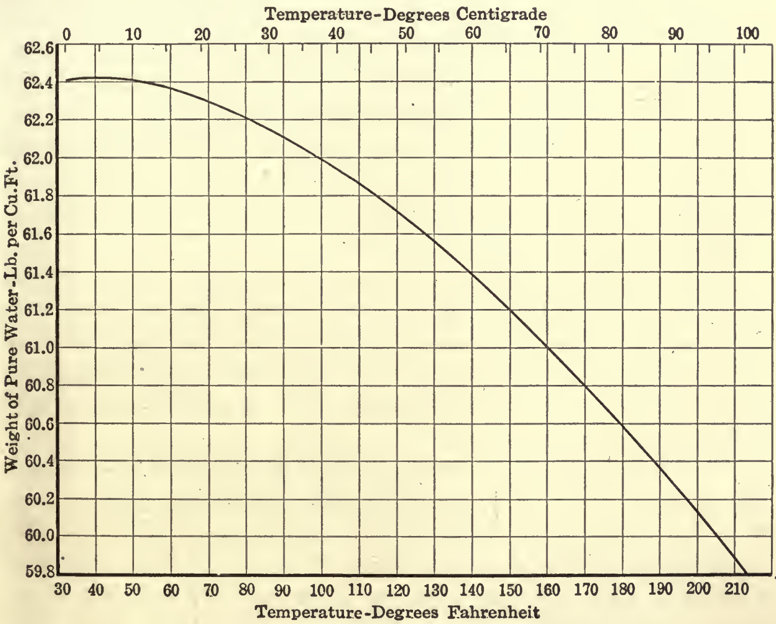


FIG. 1.—Density of pure water.

It may be seen that increasing the pressure from atmospheric to 1,000 lb. per sq. in. will reduce the volume of a body of water by 985 ÷ 294,000 or about ⅓ of 1.0 per cent. Therefore, it is seen that the usual assumption regarding water as incompressible is justified.

5. Density of Water.—The density of water varies somewhat with the temperature as well as with the pressure. In Fig. 1 can be seen the values of the density of pure water at atmospheric

pressure for the range of temperature from freezing to boiling. The presence of impurities increases these values somewhat. Thus ocean water may ordinarily be taken as weighing 64.0 lb. per cu. ft. In the computations in this book it will be sufficient to take

$$w = 62.4 \text{ lb. per cu. ft.}$$

6. Accuracy of Computations.—No computed result can be more accurate than the data upon which it is based and it is therefore not only useless but also misleading to carry out results to more significant figures than the data warrant. It should be noted that the number of significant figures has no relation to the location of the decimal point. Thus 347,000, 34.7, 0.0000347 are all values given to three significant figures and are of the same degree of accuracy. It is incorrect in such a figure as the first to preserve any more figures such as 347,129 if three figure work is all that is warranted. And if it is warranted it is likewise incorrect in such a value as the last to abbreviate it to 0.00003 for that is equivalent to saying that its value is 0.0000300 to three significant figures.

There are some quantities that may be known with a high degree of accuracy but in hydraulic work most experimental factors are uncertain in the third significant figure and there are some coefficients or values which are uncertain even in the second significant figure. Thus slide rule work is all that is usually justified.

Suppose for example that the product is desired of two quantities whose values are 34.7 and 125. Multiplying these two numbers together we obtain 4,337.5 but the answer that should be given is 4,340. If our values were known to be 34.700 and 125.00 then the exact product would be permissible. But if our values are experimental they may range for example from 34.6 to 34.8 and 124 to 126 respectively. The products of the minimum and maximum values in each case are 4,290 and 4,380, thus showing that our result of 4,340 is uncertain in the third significant figure as we should expect when the given data are not correct in the third figure.

7. Notation.—The use of a systematic and consistent notation is highly desirable and familiarity with the notation will save time and trouble. A table of the notation employed in this book is given on page xiii.

So far as possible an attempt has been made to employ the same notation that the majority of other writers use in this and in related subjects. This will result in a few cases of the same letter being used for different quantities but in such instances the quantities are so unlike that it is believed no real confusion can result. Unfortunately, the necessity of avoiding real conflicts in notation prevents the use of certain letters that most naturally suggest themselves for several different quantities, the quantities not being sufficiently removed from each other to permit the duplication.

8. Units.—The standard system of units employed in this book is based upon the foot, pound, and second. With few exceptions all formulas are to be used with such units. There are some few exceptions that commercial practice makes necessary or desirable. For instance the diameter of a pipe is customarily given in inches rather than in feet. Any exceptions to the general rule will be clearly indicated.

It should be noted that the units of the answer in any computation can be determined from the units involved in the separate items. Thus the product of velocity and area is the product of *(ft./sec.)* $\times$ *sq. ft.* = *cu. ft./sec.* The familiar quantity $v^2/2g$ is *(ft./sec.)*^{2}/*(ft./sec.*^{2}*)* = *ft.* The product of the depth of water by the density of water is *ft.* $\times$ *lb. per cu. ft.* = *lb. per sq. ft.*, etc.

It will frequently be necessary to use the value of g, the acceleration of gravity. Its units are *feet per second per second,* often written *ft./sec.*^{2} The value of g varies with latitude and elevation. Its value for any locality may be computed by the following formula according to Pierce,

$$g = 32.0894\ (1 + 0.0052375 \sin^2 l)(1 - 0.0000000957\ e),$$

where l is the latitude in degree and e is the altitude in feet. For ordinary purposes g may be taken as 32.2 ft. per sec.^{2}

9. PROBLEMS

1. If a body of water is subjected to a pressure of 65,000 lb. per sq. in. how much less will its volume be than in a perfect vacuum?

Ans. 10 per cent.

2. What pressure will be required to reduce the volume of a body of water by 0.1 of 1.0 per cent. if the temperature is 32°F. and the initial pressure 10 lb. per sq. in.

Ans. 304 lb. per sq. in.

3. If the temperature is 77°F. what would be the result in problem 2? *Ans.* 337 lb. per sq. in.

4. A cubic foot of ocean water at the surface and at ordinary temperature weighs 64.0 lb. At the surface it is under a pressure of 14.7 lb. per sq. in. What will be the weight of a cubic foot at a depth such that the pressure is 2,000 lb. per sq. in.? Assume $E_v = 310{,}000$ lb. per sq. in. (Density is inversely proportional to volume.)

5. The radiator of an automobile holds 2.0 cu. ft. of water. It is filled with water at 50°F. After the engine has been running the temperature of the water is 180°F. Assuming no loss by evaporation or otherwise and neglecting expansion of radiator, how much water will have run out the overflow?

6. If we multiply *cubic feet* of water by the density of water in *pounds per cubic foot* and by *feet*, in what units will the answer be?

7. If we multiply *torque*, which is the product of a force in pounds and a distance in feet, by *angular velocity* in radians per second, what units will be involved in the answer?

8. If we multiply *pounds* per second by *feet per second* and divide by g, in what units is the answer?

9. If we multiply a force in *pounds* by velocity in *feet per second* in what units is the answer?

CHAPTER II

INTENSITY OF PRESSURE

10. Definition of Intensity of Pressure.—By intensity of pressure is meant pressure per unit area. It may be expressed in various units such as pounds per square inch, pounds per square foot, or, as will be seen later, in feet of water, inches of mercury, etc.

If P represents the total pressure on some finite area, F, while dP represents the total pressure on an infinitesimal portion of area, dF, the intensity of pressure is

$$p' = \frac{dP}{dF} \tag{1}$$

If the pressure is uniformly distributed over the area in question the intensity of pressure would then be $p' = P/F$. If the pressure is not uniformly distributed the latter expression will give the average value only.

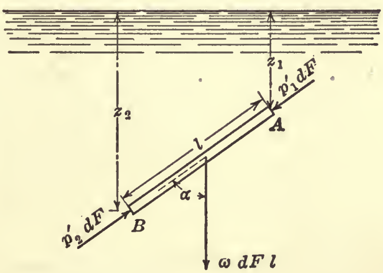


FIG. 2.

The word "pressure" is usually used for "intensity of pressure" though the latter term should be employed where there is any possibility of misunderstanding. The word "pressure" is also used to designate the resultant force exerted on an area. In order to clearly distinguish this usage from intensity it would be well to employ the term "resultant pressure" or "total pressure."

11. Variation of Pressure in a Liquid.—Let us consider a slender prism of the liquid in Fig. 2 as a free body in equilibrium. The forces acting upon it will be the pressures on its various faces and the pull of gravity. If the intensity of pressure at A be denoted by p'_1, the total pressure on the end at A will be p'_1dF, where dF is the cross-section area. In similar manner the total pressure on the end at B will be p'_2dF. The weight of the volume

of liquid is evidently $wdFl$. Since the prism of water is in equilibrium the algebraic sum of the components in any direction of all the forces acting on it will be zero. If the forces be resolved along the axis AB the three forces mentioned will be the only ones that will appear since the forces acting on the sides of the prism are all normal to the axis. Hence we may write

$$p'_1\,dF - p'_2\,dF + wdFl\cos\alpha = 0.$$

Since $l\cos\alpha = z_2 - z_1$ it follows that

$$p'_2 - p'_1 = w(z_2 - z_1) \tag{2}$$

This equation shows that the difference in the intensity of pressure at two different points varies directly as the difference in the depths of the two points. Also if point A be taken at the level where p'_1 is zero and if z be the elevation of such level above any other point, then in general

$$p' = wz \tag{3}$$

From this equation it can be seen that the intensity of pressure varies directly as the depth of the point in question below the level where p' is zero.

The results of equations (2) and (3) are strictly true only for an incompressible fluid in which the density, w, is constant at all depths. For practical purposes water is an incompressible fluid and hence these equations may be applied. But, owing to the high degree of compressibility of gases, they should not be used for a gas except where there are relatively small differences in pressure.

12. Surface of Equal Pressure.—It may be seen from equation (3) that all points in a connected body of water at rest are under the same intensity of pressure if they are at the same depth. This indicates that a surface of equal pressure is a horizontal plane. Strictly speaking it is a surface everywhere normal to the direction of gravity and it is, therefore, approximately a spherical surface concentric with the earth. But a limited portion of such a spherical surface is practically a plane area.

A *free surface* is strictly one on which there is no pressure. Usually however the surface of a liquid exposed only to the pressure of the atmosphere is said to be a free surface.

13. Pressure the Same in all Directions.—In a solid, owing to the existence of tangential stresses between adjacent particles,

the stresses at a given point may be different in different directions. But in a fluid at rest no tangential stresses can exist and the only forces exerted between adjacent surfaces are normal to the surfaces. Therefore, the intensity of pressure at a given point is the same in every direction.

This can be proven by reference to Fig. 3 where we have a small triangular element of volume whose thickness perpendicular to the plane of the paper is constant and equal to dz. Let α be any angle, p' the intensity of pressure in any direction, and p'_x the intensity of pressure on a vertical plane. The following forces act upon this volume of fluid: The pressure on the vertical face is $p'_x\, dydz$, the pressure on the slanting face is $p'dldz$, then there are the pressures on the horizontal face and on the two faces parallel to the plane of the paper, and the weight of the volume. Their values are not required. Since this volume is a fluid body at rest there are no other forces besides these normal to the surfaces, and, since it is a condition of equilibrium, the sum of the components in any direction is equal to zero. Writing such an equation for components in a horizontal direction we have only

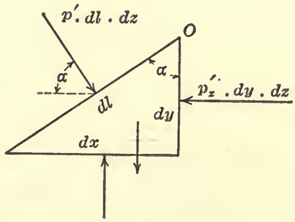


FIG. 3.

$$p'dldz \cos \alpha - p'_x\, dydz = 0.$$

Since $dy = dl \cos \alpha$, it is seen that

$$p' = p'_x$$

This result is independent of the angle α, and, therefore, it follows that the intensity of pressure is the same upon any plane passing through O.

14. Pressure Expressed in Height of Liquid.—In Fig. 4 imagine a body of liquid upon whose surface there is no pressure. Then by equation (3) the intensity of pressure at any depth z is $p' = wz$. For a given liquid w is constant and thus there is a definite relation between p' and z. That is any pressure per unit area is equivalent to a corresponding height of liquid. In hydraulic work it is often more convenient to express intensity of pressure in terms of height of a column of water rather than in pressure per unit area.

Even if the surface of the liquid in Fig. 4 is under some pressure

the relation stated is still true. For this pressure on the surface could be expressed in terms of height of the liquid and such value added to z. The resulting value of p' would thus be increased by the amount of this surface pressure.

The intensity of pressure expressed in terms of the height of a column of liquid will be denoted by the letter p. It will thus be seen that we have the relation

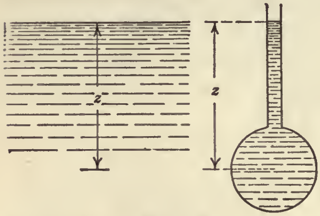


FIG. 4.

$$p' = wp \tag{4}$$

This equation is true for any consistent system of units. Thus if w is density in pounds per cubic foot, p must be expressed in feet, and p' will then be in pounds per square foot. For pure water at ordinary temperatures we have the relation $p' = 62.4p$. It is quite common to express intensity of pressures in pounds per square inch, but p is rarely found expressed in other units besides feet of water. Since $p' = 144p'' = 62.4p$ we have

$$p'' = 0.4333p \text{ and } p = 2.308p''$$

EXAMPLES

1. Neglecting the pressure of the atmosphere upon the surface, what is the pressure in pounds per square inch at a depth of 3,000 ft. in fresh water? At a depth of 3,000 ft. in the ocean?

2. A certain pump for a hydraulic press delivers water at a pressure of 5,000 lb. per sq. in. To what height of pure water would that be equivalent? To what height of liquid having a density of 100 lb. per cu. ft.?

3. The specific gravity of mercury is 13.57, that is, its density is 13.57 times that of pure water. How many feet of mercury is equivalent to a pressure of 100 lb. per sq. in.?

4. If the specific gravity of mercury is 13.57 how many feet of water is equivalent to a pressure of 10 ft. of mercury?

5. The pressure of the atmosphere is about 14.7 lb. per sq. in. To what height in feet of water is this equivalent? What is the equivalent height in inches of mercury?

15. Barometer.—If a tube such as that in Fig. 5 has its lower end immersed in a liquid and the air is exhausted from the tube the liquid will rise in the latter. If the air is completely exhausted we shall have zero pressure on the surface of the liquid in the tube, and the liquid will have reached its maximum height.

This device is called the barometer and is used for measuring the pressure of the atmosphere.

By Art. 12 it may be seen that the intensities of pressure at o (within the tube) and at a (at the surface of the liquid outside) are the same. That is $p_o = p_a$. And, since the pressure on the surface of the liquid in the tube is zero, the intensity of pressure at o is by equation (3)

$$p'_o = wy.$$

And by equation (4) $p'_o = wp_o$. Thus the pressure of the air in terms of height of the liquid column is

$$p_a = y \tag{5}$$

The liquid employed is usually mercury because of the fact that its density is sufficiently great to enable a reasonably short tube to be used, and because its vapor pressure is negligible at ordinary temperatures. If water were employed the height of tube would be inconvenient and also its vapor pressure at ordinary temperatures is appreciable so that instead of having a perfect vacuum at the top of the tube we should have a space filled with water vapor. The height attained by the liquid would consequently be less than what would otherwise be the case. The diameter of the tube should be at least 0.5 in. in order to eliminate errors due to capillarity.

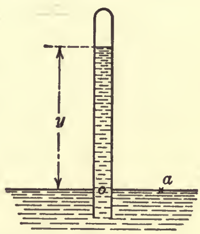


FIG. 5.—Barometer.

The pressure of the atmosphere is different in different localities, depending upon elevation, and at a given point it varies from time to time according to the temperature and other factors.

In round numbers the pressure of the atmosphere may be taken as 14.7 lb. per sq. in., 30 in. of mercury, and 34 ft. of water. (These values are not exact equivalents.)

EXAMPLES

1. If the barometer reads 29.92 in. of mercury, what is the pressure in pounds per square inch?

2. If the pressure of water vapor at 80°F. is 0.505 lb. per sq. in. what would be the height of the water barometer if the atmospheric pressure were 14.7 lb. per sq. in.? (Use correct density of water for this temperature.)

3. Assuming the density of air to be 0.0807 lb. per cu. ft., what would be the height of the air surrounding the earth and producing a pressure of 14.7 lb. per sq. in., if air were incompressible?

16. Vacuum.—Pressures less than that of the atmosphere are usually called vacuums, a perfect vacuum meaning an entire absence of all pressure. Vacuum is usually measured from the pressure of the atmosphere as a base and is commonly, though not necessarily, measured in inches of mercury. If the atmospheric pressure is 30 in. of mercury, a perfect vacuum would then be a vacuum of 30 in. And a vacuum of 10 in. of mercury would mean that there was a real pressure of 20 in. of mercury.

EXAMPLES

1. If the barometer reads 28.5 in. of mercury and the absolute pressure in the condenser for a steam turbine is 1.5 in. of mercury, what is the value of the vacuum?

2. The barometer reads 30 in. of mercury and within a certain vessel there is a vacuum of 22 in. of mercury. What is the real pressure within that vessel in pounds per square inch? What is the excess external pressure on the walls of the vessel in pounds per square inch?

17. Absolute and Relative or Gage Pressures.—If the pressure is measured above absolute zero pressure it is called absolute pressure. If it is measured from the atmospheric pressure as a base it is called relative or gage pressure, since it is only relative pressure that a gage measures. Thus Fig. 6 shows a compound gage which will measure pressures either above or below that of the atmosphere. When the gage is open to the atmosphere the hand points to zero. If the gage is connected to any vessel in which there is a pressure above that of the surrounding air the hand will turn in a clockwise direction from zero. If the pressure is a vacuum the hand will move in the opposite direction. Thus the gage measures only the difference between the pressure on the inside of the gage tube and that of the air surrounding the gage.

FIG. 6.—Compound gage.

In Fig. 7 let O indicate entire absence of all pressure or absolute zero and the ordinate OA represent the pressure of the atmosphere. Then suppose we have any pressure such as at B. The gage will read the value AB and this is the gage pressure. The absolute pressure is OB. Also if we have a vacuum of AC, the gage pressure is $-AC$, the minus sign merely indicating a value below atmospheric just as a plus sign indicates a pressure above that of the atmosphere. But the absolute pressure is OC.

When dealing with absolute pressures all values are positive. In the case of gage pressures only values above that of the atmospheric pressure are positive, but the minus sign for pressures below that of the atmosphere serves only to indicate a vacuum. There may still be a real pressure between adjacent particles of water. A true negative pressure would mean that the water was in a state of tension and as water can sustain only a very slight tensile stress it is impossible to have a pressure below absolute zero. Absolute zero is the point where the stress in the liquid would change from compression to tension.

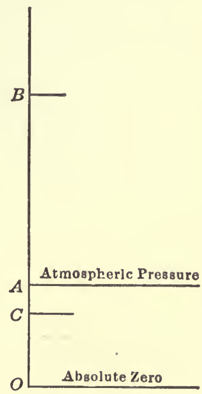


FIG. 7.

In most problems in hydraulics we are not interested in absolute pressures. We are concerned with the differences in pressure inside a vessel and that outside for example and in general that would be the gage pressure. And in many other cases the atmospheric pressure acts alike at all points and balances out.

EXAMPLES

1. A gage reads 20 lb. per sq. in. when the gage itself is surrounded by the atmosphere. If the air surrounding the gage be exhausted to a vacuum of 20 in. of mercury while the real pressure of the fluid on the inside of the gage tube remains the same, what will be the reading of the gage?

2. If the barometer reads 30 in. of mercury and a vacuum gage reads 5 in. of mercury, what is the absolute pressure?

3. A gage pressure of -25 ft. of water is how much less than a gage pressure of 10 ft. of water?

18. Instruments for Measuring Pressure.—*Gage.*—The familiar pressure or vacuum gages have already been referred to and the combination of the two is shown in Fig. 6. In this type of instrument a curved tube is caused to change its curvature by changes of pressure within the interior of the tube. The moving end of the tube then rotates a hand by means of some intermediate links. It is usually assumed that the pressure indicated by the gage is that existing at the center of the gage. Thus the location of the center of the gage should always be taken into consideration. For instance, referring to Fig. 8, the

pressure at A is the gage reading plus the distance z. If the gage reads pounds per square inch, as is customary,

$$p_A = 2.308\, p'' + z.$$

Piezometer Tube.—A piezometer tube is a simple device for measuring moderate pressures. It consists of a tube in which the liquid can freely rise, without overflowing, until equilibrium is established. To prevent error due to capillarity the diameter of the tube should be at least 0.5 in. The height of the surface of liquid in the tube will give the pressure desired directly. It should be noted that if the water, whose pressure is desired, is flowing the true pressure can be obtained only by having the axis of the tube at the point of connection perpendicular to the stream flow and furthermore the interior opening should be smooth and free from all projections. If the end of the pipe projects into the stream, as in the case of the fourth tube in Fig. 94, the pressure read will be too low.

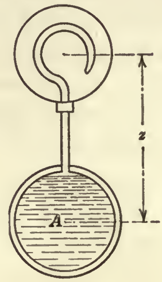


FIG. 8.

Mercury U Tube.—For high pressure the water piezometer is not suited and some modification must be adopted. The mercury

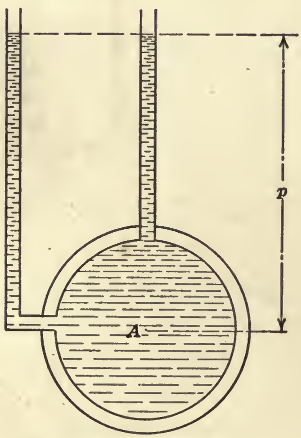


FIG. 9.—Piezometer.

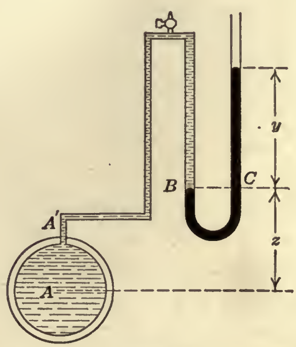


FIG. 10.

U tube shown in Fig. 10 may then be used. If s is the specific gravity of the mercury (or whatever liquid may be employed) the pressure at the point C is sy. This is also the pressure at B but the pressure at A is greater than this by the amount z, if the

tube from A' to B is filled with water. If it were filled with air, then, neglecting the slight weight of the air within the tube, the pressures at B and at the surface of the water at A' would be equal. In practice it would be difficult to insure the absence of air and if the tube were partially filled with air and partially with water it would be troublesome to make correction and accuracy would be impossible unless it were known just what proportion of the tube was filled with water and what with air. It is therefore desirable to provide some means of permitting all the air to escape and its place to be taken by water. If the connecting tube in Fig. 10 is filled with water the pressure at A is

$$p_A = z + sy.$$

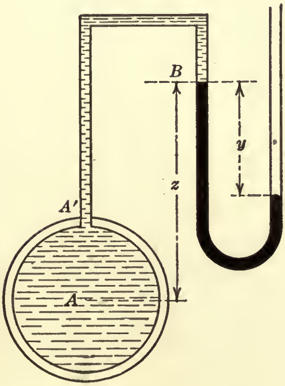


Fig. 11.

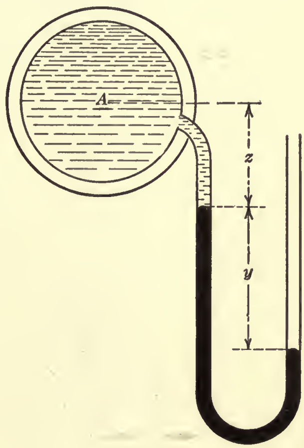


Fig. 12.

In measuring a vacuum we must interpret y as a negative quantity in Fig. 11 so that we have, if the tube is filled with water,

$$p_A = z - sy.$$

If this connecting tube from A' to B were filled with air then the correction for the height above A' would be negligible but it is difficult to insure this being filled with air and error will be introduced if it is not. Thus the arrangement in Fig. 12 is much better as that permits no air to collect in the tube and introduce errors in the readings. In this case z is negative so that

$$p_A = -z - sy.$$

Differential Gage.—The differential gage is used for measuring differences of pressure only. One form of this is shown in Fig. 13. Assuming the entire connecting tubing to be filled with water except that portion of the *U* that is filled with the denser liquid, such as mercury for instance, the pressure at A' will exceed that at B' by the amount sy. That is

$$p_{A'} - p_{B'} = sy.$$

But

$$p_{A'} = p_A - z_A$$

and

$$p_{B'} = p_B - z_B.$$

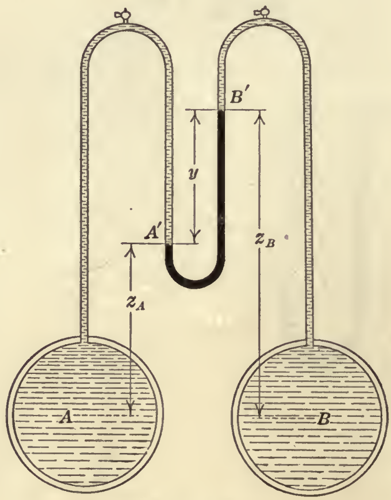


FIG. 13.

Substituting these values we have

$$\begin{aligned} p_A - p_B &= sy + z_A - z_B \\ &= sy - y = (s - 1)y. \end{aligned}$$

In the differential manometer the left-hand column of mercury, or whatever it is, has a column of water of height y resting upon it that is not balanced by a corresponding amount on the right-hand column, hence the pressure difference is not sy alone.

EXAMPLES

1. Two pressure gages are connected to the same vessel containing water under pressure. One of these gages is 10 ft. below the point where the pres-

sure is measured and it reads 40 lb. per sq. in. The other gage is located 15 ft. above the point in question. What will it read? What is the pressure in the vessel? (It is assumed that the connecting pipes are filled with water.)

2. Two vessels are connected to a differential manometer using mercury (specific gravity = 13.57). When the mercury reading is 36 in. what is the difference in pressure in feet of water between the two vessels?

19. The Hydraulic Press.—The most important device operating upon the principle of equal transmission of intensity of pressure in all directions is the hydraulic press. If in Fig. 14 a force P_1 be applied to the small piston whose area is F_1 the intensity of pressure throughout the whole volume of liquid will be increased by the amount $p' = P_1/F_1$. Then the total additional force exerted upon the face of the large piston will be $p'F_2. = (P_1/F_1)F_2 = P_1(F_2/F_1)$. It is thus seen that a small force

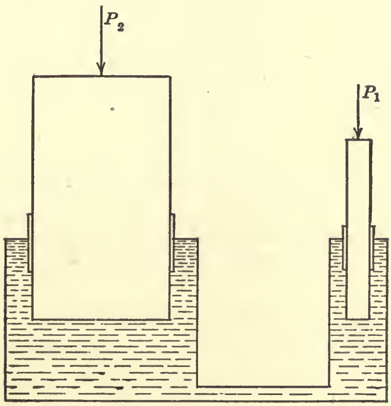


FIG. 14.—Hydraulic press.

exerted on the smaller piston is enabled to oppose a much greater load on the large piston. If G_1 and G_2 denote the weights of the pistons while z is the difference in elevation of their faces, we have for equilibrium

$$\frac{P_1 + G_1}{F_1} = \frac{P_2 + G_2}{F_2} \pm wz.$$

Since the volume of liquid in the vessel must remain constant, it follows that the distance moved by the larger piston must be much less than that moved through by the smaller piston.

EXAMPLES

1. In Fig. 15 the diameter of the small piston is ¾ in. and that of the large one is 20 in. The big plunger weighs 1,000 lb. and sustains an external load

of 6,000 lb. The liquid used is water. What total force P applied to the small piston will secure equilibrium?

Ans. 4.12 lb.

2. When the small piston has descended 20 ft. how far will the plunger have been raised?

20. PROBLEMS

1. In Fig. 16 what are the values of absolute pressure at A, B, C, and D, assuming the liquid to be water? What are the values of gage pressure?

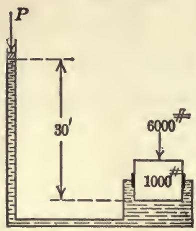


FIG. 15.

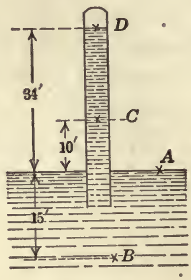


FIG. 16.

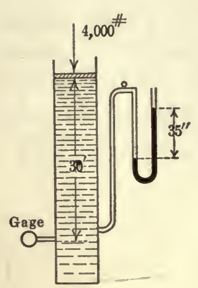


FIG. 17.

What is the value of the vacuum at C? (Give answers in feet of water, pounds per square inch, and inches of mercury in every case.)

2. In Fig. 17 the cylinder is 2 ft. in diameter and the weight of the piston and load is 4,000 lb. What will be the gage reading in pounds per square inch?

3. If the mercury manometer in Fig. 17 reads 35 in., how far is the top of the lower mercury column below the piston? If the manometer remained at

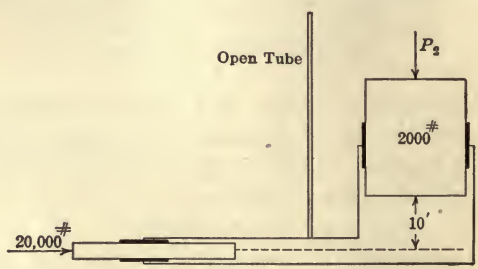


FIG. 18.

this same place but the connection were made to the tank at a different level, would the mercury reading change?

4. The small piston in Fig. 18 has a diameter of 3 in. Neglecting friction, when a force P_1 of 20,000 lb. is applied to it, what will be the force P_2 that can be exerted by the plunger with a diameter of 24 in.? To what height would water rise in the piezometer tube shown?

CHAPTER III

HYDROSTATIC PRESSURE ON AREAS

21. Total Pressure on Plane Area.—Since we are dealing with fluids at rest, no tangential forces can be exerted and hence all pressures are normal to the surfaces in question. If the pressure were uniformly distributed over an area, the total or resultant pressure would be the product of the area and the intensity of pressure and the point of application of the force would be the center of gravity of the area. In general the intensity of pressure is not uniform, hence further analysis is necessary.

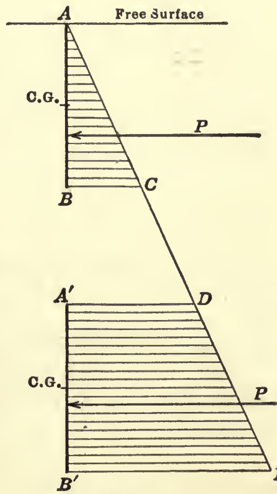


FIG. 19.

In Fig. 19 consider a vertical plane whose upper edge lies in the free surface. Let this plane be perpendicular to the plane of the paper so that AB is merely its trace. The intensity of pressure will vary from zero at A to BC at B. It will thus be seen that the total pressure P will be the summation of the products of the elementary areas and the intensities of pressure upon them. It is also apparent that the resultant of this system of parallel forces must be applied at a point below the center of gravity of the area, since the center of gravity of an area is the point of application of the resultant of a system of uniform parallel forces. If the plane be immersed to $A'B'$ the intensity of pressure varies from $A'D$ at A' to $B'E$ at B'. Since the proportionate change of intensity of pressure from A' to B' is less than before, it is clear that the center of pressure will approach nearer the center of gravity.

In Fig. 20 let AB be the trace of a plane making any angle θ with the horizontal. The view to the right is the projection of this area upon a vertical plane which is also normal to the plane containing AB. Let z be the depth of any point and y be the

distance of any point from OX, the axis of intersection of the plane, produced if necessary, and the free surface. The coordinates of the center of gravity of the area may be denoted by $\bar{z}$ and $\bar{y}$ respectively.

Take an element of area dF such that all portions of said element are at the same depth z. Then the total pressure on dF is

$$dP = p'dF = wzdF;$$

Hence $$P = w\int zdF.$$

But $$\int zdF = \bar{z}F,$$

hence $$P = w\bar{z}F \tag{6}$$

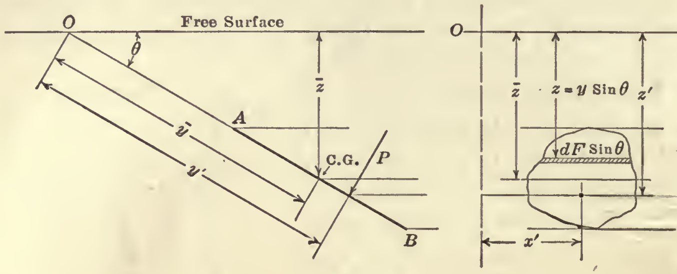


FIG. 20.

22. Depth of the Center of Pressure.—The point of application of the resultant force on an area is called the center of pressure. We usually locate the line of action of a force by taking moments. In this case it is convenient to take OX in Fig. 20 as the axis of moments. On any element of area dF the total pressure is

$$dP = wzdF = wy \sin \theta dF$$

and its moment is

$$ydP = wy^2 \sin \theta dF.$$

If the distance of the center of pressure from OX be denoted by y'

$$y' = \frac{\int ydP}{\int dP} = \frac{w \sin \theta \int y^2dF}{w \sin \theta \int ydF} \tag{7}$$

But $\int y^2dF$ is the moment of inertia of the area F about the axis OX, and $\int ydF$ is the statical moment of the area with respect to the same axis, hence

$$y' = \frac{I_o}{\bar{y}F} \tag{8}$$

This may be put in a more convenient form by noting that, if I_g is the moment of inertia of the plane area about its gravity

axis and k_g is the radius of gyration about the gravity axis, we have

$$y' = \frac{\bar{y}^2F + I_g}{\bar{y}F} \tag{9}$$

$$= \bar{y} + \frac{k_g{}^2}{\bar{y}} \tag{10}$$

From these equations it may be seen that the location of the center of pressure is independent of the angle θ, that is, the plane area may be rotated about the axis OX without affecting the location of the center of pressure. However, this will not hold for $\theta = zero$ since the value of P would also be zero.

From equation (10) it may also be seen that the center of pressure is always below the center of gravity. Also as the depth of immersion is increased for a given value of θ, the distance $\bar{y}$ increases. But as k_g remains constant in value it may be seen that the last term in equation (10) becomes relatively small, hence y' approaches $\bar{y}$ in value. The same thing would be true if the depth of the center of gravity $\bar{z}$ remained constant while the plane was rotated so as to approach a horizontal direction. (This is entirely different from rotation about the axis OX, since $\bar{y}$ no longer remains constant.)

EXAMPLES

1. A rectangular plane area is 5 ft. by 6 ft., the 5-ft. side is horizontal, and the 6-ft. side vertical. Determine the resultant pressure and the location of the center of pressure when: (*a*) the top edge is in the water surface; (*b*) the top edge is 1 ft. below the water surface; (*c*) the top edge is 100 ft. below the water surface.

Ans. (*a*) P = 5,620 lb., y' = 4 ft.; (*b*) P = 7,500 lb., y' = 4.75 ft.; (*c*) P = 193,000 lb., y' = 103.03 ft.

2. Suppose in Fig. 20 that we have a rectangular area 5 ft. by 6 ft., that AB = 6 ft., the 5-ft. edge being normal to the plane of the paper, and that $\bar{y}$ = 4 ft. Find the magnitude of the total pressure and the location of the center of pressure when θ has values of 90°, 60°, 30°, and 10°.

Ans. (*a*) P = 7,488 lb., y' = 4.75 ft.; (*b*) P = 6,490 lb.; (*c*) P = 3,744 lb.; (*d*) P = 1,302 lb.

3. Suppose that in problem (2) $\bar{y}$ was variable but that $\bar{z}$ = 4 ft. Solve with values of θ of 90°, 60°, 30°, and 0°.

Ans. (*a*) P = 7,488 lb., y' = 4.75 ft.; (*b*) y' = 5.265 ft.; (*c*) y' = 8.375 ft.; (*d*) y' = infinity, z' = 4 ft.

4. Find the depth of the center of pressure on a vertical triangular area whose altitude is h and whose base is b if: (*a*) its vertex lies in the water surface and base is horizontal; (*b*) its base lies in the water surface.

Ans. (*a*) $y' = \frac{3}{4}h$; (*b*) $y' = \frac{1}{2}h$.

23. Lateral Location of Center of Pressure.—For most practical problems the depth of the center of pressure is all that requires solution since the areas with which we deal are usually such that a straight line can be drawn through the centers of all horizontal lines. In such cases the center of pressure is seen to lie on this line. But in case this is not so we should have to compute x' as in Fig. 20, x' being measured from any axis parallel to trace AB.

Again we employ moments as in the preceding article. If x is the distance of an element from the axis in question the moment of dP is

$$xdP = wxy \sin\theta \, dF$$

Hence the value of x' is

$$x' = \frac{\int xdP}{\int dP} = \frac{w \sin \theta \int xydF}{w \sin \theta \int ydF}$$

$$= \frac{\int xydF}{\bar{y}F} \tag{11}$$

This equation differs from (7) simply in the fact that we have $\int xydF$ instead of $\int y^2dF$. The latter quantity is more frequently met with, it is given a name, symbolized by the letter I, and values of I for different areas can usually be found in tables. The former expression is called "product of inertia," is symbolized by the letter J, but owing to the infrequent use that is made of it values of J cannot usually be obtained save by integration. Lacking the knowledge of the value of J for any area, we should simply proceed to evaluate $\int xydF$ just as we should evaluate $\int y^2dF$ in case we did not know the value of I for the area in question.

It will be found that reduction formulas can be used here as with moments of inertia. If J indicates the product of inertia with respect to the intersection of any two axes, while a and b are the coordinates of the center of gravity of an area about which the product of inertia is J_g, it will be found that

$$J = J_g + Fab.$$

In using equation (11) it must be noted that y is to be measured as in Fig. 20, while x may be measured from any axis in the plane of the figure and perpendicular to OX.

EXAMPLES

1. Given a right triangle with height, h, and base, b, with its vertex in the water surface and its plane vertical. Find the value of y' and then determine x': (*a*) by inspection; (*b*) by calculus.

Ans. $y' = \frac{3}{4}h$; $x' = \frac{3}{8}b$.

2. Find the center of pressure on an area which is a quadrant of a circle. It is placed in a vertical plane and one edge lies in the water surface.

Ans. $\bar{y} = 4r/3\pi$; $y' = 3\pi r/16$; $x' = 3r/8$.

23a. Graphical Solution for Pressure on Plane Area.—It is not always feasible to apply equations (6), (8) and (11) directly, especially if the plane area in question is irregular in outline so that its center of gravity and moment of inertia cannot be readily determined. The problem may then be solved as follows.

In Fig. 20a is shown a plane area at I which makes any angle θ with the horizontal. As in Art. 21 let us taken an element of area such that every portion is at the same depth below the water surface. Then $dF = xdy$. Note also that $z = y \sin \theta$. The intensity of pressure, $p' = wz$, is shown in II, values of wz being plotted perpendicular to the inclined base line.

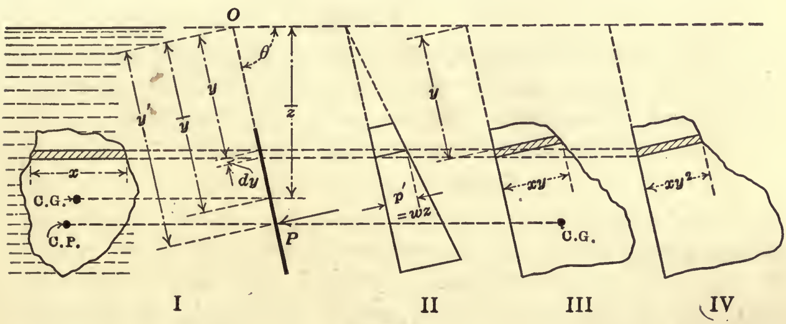


FIG. 20a.

The total pressure on an element of area is $dP = wzdF = wzxdy$. The total pressure on the entire area is

$$P = w\int zxdy = w \sin \theta \int xydy.$$

In III xy is plotted as a function of y. The elementary rectangle there shown is of length xy and width dy. The sum of all such areas is the total area shown in III. But the sum of all values of $xydy$ is the value of the definite integral. Hence to find the total pressure on the area shown at I, it is only necessary to

multiply values of x by the corresponding values of y and plot the product against the values of y. The area then represents to some scale the value of the integral between the limits used, and when multiplied by $w \sin \theta$ will give the value of the total force.[1]

Since the ordinates of III, when multiplied by the proper constant, represent values of forces on elementary areas, the resultant force will act through the center of gravity of III. This then serves to locate the y co-ordinate of the center of pressure of the actual area in I. It will also be perceived that, in general, the centers of gravity of areas I and III do not coincide. Hence the center of gravity of the actual area I and its center of pressure never coincide, except in the special case where the intensity of pressure is uniform. To determine the location of the center of gravity of III or in other words to locate the center of pressure in I we take moments. Thus

$$Py' = w \sin\theta \int xy^2 dy$$

The area shown in IV is seen to represent the value of this integral. Since $y' = Py'/P$,

$$y' = \frac{\int xy^2 dy}{\int xydy}$$

or y' is obtained by dividing the area of IV by the area of III after reducing each to its proper scale value. A similar procedure could be employed for finding the value of x' if desired.

It may be noted that the area of III represents

$$\int xydy = \int ydF = \bar{y}F$$

or is the moment of the actual area in I. And the area represented by IV is

$$\int xy^2 dy = \int y^2 dF = I_o$$

or is the moment of the area III or the moment of inertia of the actual area I.

[1] In general when one has an integral of the form $\int udv$ it may be impossible to integrate it either because the calculus solution cannot be discovered or because u cannot be expressed as a mathematical function of v. In either case a numerical value of the definite integral may be obtained by plotting values of u against values of v and determining the area between the curve and the v axis.

If u is plotted to such a scale that 1 in. = a units and v to such a scale that 1 in. = b units, then the scale for the area is 1 sq. in. = ab units.

If it is not convenient to plot the curve and measure the area with a planimeter, the ordinates can be computed and the area determined without actually plotting it by some method of approximation, such as Simpson's rule.

EXAMPLES

1. A vertical plane area, whose upper edge coincides with the water surface, has the following widths starting with the surface and at 1 ft. intervals below it. 4.90 ft., 4.48, 4.00, 3.46, 2.82, 2.00, and 0. Plot values of zx and z^2x and determine the magnitude of the resultant pressure and the depth of the center of pressure.

Ans. $P = 2930$ lb., $y' = z' = 3.43$ ft.

2. Find the area of the plane and the depth of the center of gravity.

Ans. $F = 19.6$ sq. ft., $\bar{y} = \bar{z} = 2.4$ ft.

3. Solve the above problems by Simpson's rule.

24. Resultant Thrust on Plane Areas.—So far we have dealt with the total pressures on one side of a plane area alone. Of course, when the area is completely immersed in a fluid as shown in some of the previous illustrations, the total pressure on one side is balanced by that on the other and the net effect is zero. But when the two sides are not subjected to the same pressure, there is a resultant thrust whose value we desire.

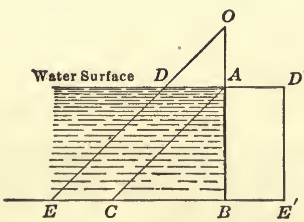


Fig. 21.

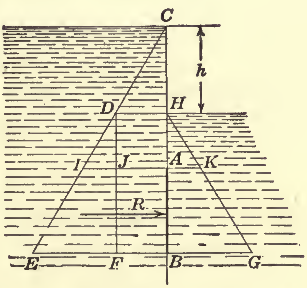


Fig. 22.

So far we have considered the surface of the liquid as being free from all pressure. Thus in Fig. 21 we should consider the intensity of pressure as varying from zero at A to BC at B. But in reality there is some pressure, in general, from the atmosphere acting upon the water surface equivalent to a height of about 34 ft. of water, and thus the true free surface might really be at point, O, the distance AO being equal to the height of the water barometer. The absolute intensity of pressure upon the left-hand side of the plane, AB, therefore varies from AD to BE. But in practical applications we desire the difference between the pressure on the left-hand side and that on the right-hand side. But the pressure on the right-hand side is that due to the atmosphere and its intensity is uniform from A to B being equal to AD'. But $AD' = AD = CE$. Hence atmospheric pressure is added alike to both

sides, and it is useless to consider it. Therefore, we neglect atmospheric pressure altogether and treat the water surface as a true free surface in most calculations.

Suppose we have an area such as AB in Fig. 22 with a fluid pressure on both sides but of different intensities. Of course, we could compute the magnitudes of the total pressures on both sides of the area and the difference would be the resultant desired. But we should also have to find the centers of pressure on both sides and then locate the line of action of the resultant of these two forces. The following analysis will indicate a much easier solution.

At A the intensities of pressure on the two sides are AI and AK. If IJ be laid off equal to AK the net difference in the intensity of pressure will be AJ. In similar manner at B the net intensity of pressure is BF. And it is readily seen that, since CDE and HKG make the same angle with the vertical, the values of HD, AJ, and BF are equal. Thus the resultant intensity of pressure on the area, AB, is uniform and equal to HD in value. But HD is the intensity of pressure at the depth, h. Hence the resultant thrust on any area with both sides completely covered by the same liquid is

$$R = whF \tag{12}$$

where h is the difference in level of the two liquids. And since the net intensity of pressure is uniform, the resultant thrust will act through the center of gravity of the plane area.

EXAMPLES

1. Suppose that a rectangular area is 2 ft. wide by 3 ft. high and that its upper edge lies in a water surface. What twisting moment will be necessary in a shaft through A (Fig. 21), perpendicular to the plane of the paper, to withstand the water pressure? It will be assumed that the gate received no support save what the shaft affords, and that atmospheric pressure acts alike on the water surface and the right-hand side of the gate.

Ans. 1,123 ft.-lb.

2. Suppose that the right-hand side of the gate in problem (1) is under a vacuum of 30 in. of mercury, and that the barometer reading is 30 in. of mercury. What twisting moment would be required?

Ans. 20,200 ft.-lb.

3. Suppose that the barometer reads 30 in. of mercury and that the right-hand side of the gate in problem (1) is under a vacuum of 20 in. of mercury. What twisting moment would be required?

Ans. 13,800 ft.-lb.

4. Suppose the barometer reads 30 in. of mercury, that the right-hand

side of the gate in problem (1) is under atmospheric pressure, while the surface of the water is under a gage pressure of 50 lb. per sq. in. What twisting moment would be required?

Ans. 65,900 ft.-lb.

5. Suppose in Fig. 22 that AB is a circular gate of 3-ft. diameter, that $BC = 10$ ft. and $BH = 4$ ft. Find: (a) magnitude and line of action of total pressure on left-hand side only; (b) magnitude and line of action of total pressure on right-hand side only; (c) resultant thrust on gate.

Ans.(a) 3,747 lb., 1.566 ft. below top of gate; (b) 1,102 lb., 1.725 ft. below top of gate; (c) 2,645 lb., 1.500 ft. below top of gate.

25. Horizontal Pressure on Curved Surface.—On any curved or irregular area in general, such as that whose trace is AB in Fig. 23, the pressures upon different elements are different in direction and an algebraic or calculus summation is impossible.

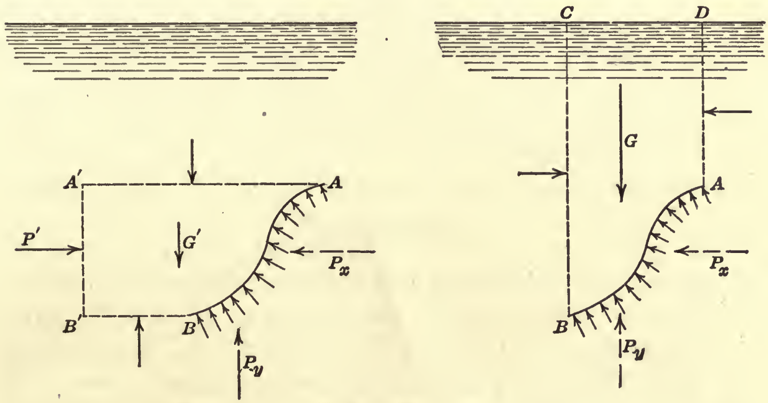


Fig. 23.

Hence equation (6) can be applied only to a plane area. But we may find the component of pressure in certain directions. Thus if we multiplied each dP by $\cos\theta$, θ being a variable angle which each elementary force makes with the horizontal, the total horizontal force would be

$$P_x = \int dP \cos\theta. \tag{13}$$

In general it will be tedious to integrate the latter and often practically impossible. Hence the following procedure may be employed

Project the irregular area in question upon a vertical plane, the trace of the latter being $A'B'$. The projecting elements are AA', BB', etc. It is seen that these projecting elements, which are all horizontal, enclose a volume whose ends are the vertical

plane $A'B'$ and the irregular area whose trace is AB. This volume of liquid is in equilibrium under the action of the following forces. Upon the vertical plane at the left there is a force P', gravity G' acts upon the volume and is vertical, the pressures on the projecting elements are all normal to these elements, hence normal to P'. Then there are the pressures upon the area in question at the right-hand end, the horizontal component of pressure being represented by P_x and the vertical component by P_y. Since we have a condition of equilibrium the sum of all the forces in any direction must be equal to zero. But in a horizontal direction the only forces are P' and P_x.

Hence $$P_x = P'. \tag{14}$$

That is the component, in any given horizontal direction, of the pressure upon any area whatever is equal to the pressure upon the projection of the area upon a vertical plane which is perpendicular to the given horizontal direction. The lines of action must also be the same.

26. Vertical Pressure on Curved Surface.—The vertical component of pressure on an irregular surface can be found by a method similar to that for the horizontal pressure. Thus in Fig. 23 if we take a volume of liquid of which the area in question forms the base and vertical elements such as AD and BC form the sides, we find the following forces are acting. Considering CD a free surface the pressure on the upper face is zero. The pressure on the lower face is composed of the two components P_x and P_y. Gravity, G, is the only other vertical force, the pressures on the sides all being horizontal. Summing up the vertical forces and equating to zero we have

$$P_y = G. \tag{15}$$

Hence the vertical component of pressure on any area whatever is equal to the weight of that volume of liquid which would extend vertically from the area to the free surface.

27. Component of Pressure in any Direction.—In general the component of pressure in any direction aside from horizontal and vertical cannot be found, since the weight of the volume of liquid, such as $AA'B'B$ in Fig. 23 would have to enter the equation. But if the depth of immersion is great so that the pressures on AB and $A'B'$ are great compared with the weight G' the latter may be neglected. Hence in such cases only, the component of

pressure in any direction may be taken as the pressure upon an area projected in that direction upon a plane which is perpendicular to the given direction.

Of course with a plane area the component of pressure in any direction may be found by multiplying P by the proper function of some angle. Or it may be convenient to find it by the methods of Arts. 25 and 26. Also for a plane area, since $P \cos \theta = (w\bar{z}F) \cos \theta$, it may be seen that the component of pressure is the same as the pressure upon an area of value $F \cos \theta$ provided the center of gravity of such area be the same depth as the center of gravity of the given plane.

28. Resultant Pressure on Curved Surface.—In general there is no single resultant pressure on an irregular surface, for a system of non-parallel and non-coplanar forces does not usually reduce to anything simpler than two single forces. Thus in general P_x and P_y are not in the same plane and hence cannot be combined. But in some special cases of symmetrical surfaces, these two components will lie in the same plane and hence can be combined into a single force.

EXAMPLES

1. In Fig. 24 is shown a quadrant of a circular cylinder, AB, whose length perpendicular to the plane of the paper is 4 ft. (*a*) Find the horizontal component of pressure. (*b*) Find the vertical component of pressure. (*c*) Find the magnitude and direction of the resultant water pressure. (*d*) What locates its line of action?

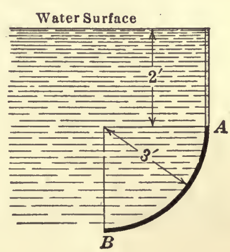


Fig. 24.

29. Pipes under Pressure.—If the internal pressure in a cylindrical pipe is great enough to be considered in determining the thickness of pipe wall necessary, it will be large enough so that the weight of the water may be disregarded. Hence according to Art. 27 we may compute the resultant pressure in any direction. Suppose that in Fig. 25, we pass a plane XY through a diameter of the pipe as shown. The total pressure on one-half of the pipe in any direction, such as that normal to XY, will evidently be $p' \times 2r \times l$, l being any length of pipe. This follows directly from Art. 27 or may be seen from the fact that the thrust of the water on the wall of the pipe normal to XY must be balanced by the thrust of the water on the plane XY. This pressure will tend to rupture the pipe across the plane XY and is resisted by the

tensions in the walls of the pipe, such as T. Evidently $2T = 2p'rl$. If the thickness of the pipe wall be denoted by t, and the stress induced in it by S_h, then $T = S_htl$. Hence

$$S_ht = p'r \tag{16}$$

From (16) the thickness of wall necessary may be computed for any allowable unit tensile stress. However, it is well to note that p' should be the maximum intensity of pressure that may occur and in case of water hammer these intensities are much greater than the static pressures alone. Also it may often be found that (16) gives entirely too thin a wall to stand ordinary handling and to allow for a certain amount of corrosion. In practice p' is, therefore, increased to allow for possible water hammer and the thickness determined by (16) is then increased

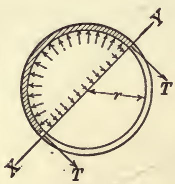


FIG. 25.

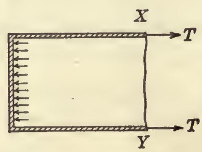


FIG. 26.

to a value necessary for these other reasons. The tension in the case shown is called *hoop tension.*

Referring to Fig. 26 it may be seen that a cylindrical pipe may also be ruptured by forces parallel to the axis. Thus the pressure on the blank end is balanced by the tension in any section such as XY. The total pressure, assuming it to be of uniform intensity, is $p' \times \pi r^2$. And the tension across a section XY is $T = S_l \times 2\pi rt$. Hence, equating these two,

$$2S_lt = p'r \tag{17}$$

This stress is called *longitudinal tension* and it may be seen that it is one-half the hoop tension.

For cylinders with thin walls these formulas will hold, since they assume uniform intensity of stress across the metal. But with thick walls they do not hold. In the case of hoop tension in a cylinder with thick walls it is usually assumed that the intensity of stress is a maximum at the inner face and decreases to zero at the outside of the wall. Also the elasticity of the material enters

into the hypothesis. John Sharp[1] gives the following empirical formula for hoop tension in a cast-iron cylinder with thick walls

$$S \log_e \frac{r_2}{r} = p' \tag{18}$$

where r_2 = external radius and r = internal radius. For wrought iron and steel cylinders he gives the empirical expression

$$S\left[\left(\frac{r_2}{r} - 1\right) + \log_e \frac{r_2}{r}\right] = 2p' \tag{19}$$

Equation (16) with S understood as compressive stress would also hold for external pressure provided the pipe remained truly cylindrical. But actually it may become slightly distorted from the cylindrical form and then there is a possibility of sudden collapse. A large thin tube which can stand a high internal pressure can withstand only a small external pressure. All formulas for determining the strength of pipes against external pressure are purely empirical. So far no satisfactory expression has been deduced, and sufficient data is lacking.

30. Buoyant Force of the Water and Flotation.—Considering the body *EHDK* immersed in a fluid in Fig. 27, we see that it is acted upon by gravity and the pressures from the surrounding fluid at least. In addition there may be other forces applied. On the upper surface of the body the vertical component of the pressure, P_y, will be equal to the weight of the volume of fluid *AEHDC*. In similar manner the vertical component of the pressure on the under surface, P'_y, will be equal to the weight of the volume of fluid *AEKDC*. It is evident that P'_y is greater than P_y and that the total vertical force exerted by the fluid is upward and is equal in magnitude to

$P'_y - P_y$ = weight of volume *AEKDC* – weight of volume *AEHDC*.

But the difference between these two volumes is the volume of the body *EHDK*. Hence for any body immersed in a fluid such as water the buoyant force of the water is equal to the weight of the water displaced.

If the body remains in equilibrium in the position shown in Fig. 27, when no other forces are acting, it is seen that $G = P'_y - P_y$. Hence the body must be of the same density as the fluid in which it is immersed. If it is lighter than the fluid, a downward

[1] "Some Considerations Regarding Cast Iron and Steel Pipe."

force will have to be applied whose value is $B - G$, B being the buoyant force of the fluid. If the body is denser than the fluid, it will have to be supported by a force whose value is $G - B$. But if the body rests on the bottom of a body of fluid (Fig. 28) in such a way that the fluid does not have access to the under side, there will be no buoyant effect for then P'_y = zero. Thus in the case of a ship, for example, sunk in the mud at the bottom of a body of water, the pull T necessary to raise the ship is not only

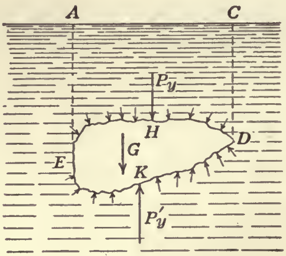


FIG. 27.

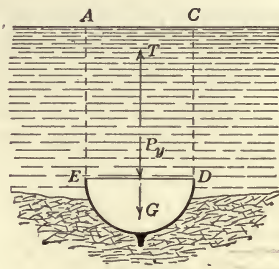


FIG. 28.

the weight of the ship but also the weight of the entire volume of water resting on top of it. Thus in Fig. 28, $T = G + P_y$.

If no external forces are applied to a body which is lighter than the fluid, it will float on the surface, such portion of its volume being immersed as is necessary to displace an amount of fluid equal in weight to the weight of the body.

If the body is slightly heavier than the fluid, it will sink. If it is less compressible than the fluid and there is sufficient depth, it will sink until such a depth is reached that the density of the fluid is equal to its own density. If it is more compressible than the fluid its own density will be increased more rapidly than that of the water and it will sink to the bottom.

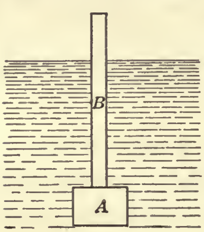


FIG. 29.

EXAMPLES

1. A body whose volume is 2 cu. ft. weighs 200 lb. What will be the force necessary to sustain it when it is immersed in fresh water? In ocean water?

2. In Fig. 29 the cube A is 12 in. along each edge and weighs 100 lb. It is attached to the square prism B which is 6 in. by 6 in. by 8 ft. and weighs 30 lb. per cu. ft. What length of B will project above the water surface?

Ans. 1.76 ft.

3. A balloon weighs 250 lb. and has a volume of 10,000 cu. ft. When it is filled with hydrogen which weighs 0.0056 lb. per cu. ft. what load will it support in air which weighs 0.08 lb. per cu. ft.?

Ans. 494 lb.

4. The specific gravity of a solid is 0.8. What portion of its volume will be above the surface of the water upon which it floats?

5. A body weighs 50 lb. and has a volume of 4 cu. ft. What vertical force is necessary to sink it beneath the surface of the water?

31. Metacenter.—For a body floating on the surface of the water, such as in Fig. 30, there are only the two vertical forces, its weight G and the buoyant force of the water B. The latter acts through the center of gravity of the water displaced. This point is called the center of buoyancy. If the body is in equilibrium, these two forces must be in the same straight line.

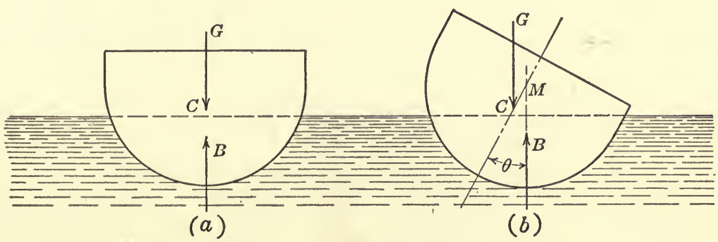


Fig. 30.

Suppose that by some external agency the body is rolled or displaced through some angle θ. The center of gravity is naturally unchanged in its position in the section but the center of buoyancy, in general, will change. Thus G and B constitute a couple. In Fig. 30 (*b*) this is a righting couple since it tends to restore the body to the upright position.

It may be seen that the line of action of B cuts the axis at point M. This point is called a *metacenter*. As the angle θ varies, the amount of this couple will vary and the point M will also change its location. The position which M approaches as θ approaches zero is the *true metacenter*. It may be seen that if the couple is a righting couple the point M must always be above C the center of gravity. It is necessary in ship design to insure that M will be above the center of gravity for all angles of heel. Thus not only is it necessary to locate the true metacenter but also to compute the moment of the righting couple for all values of θ which are likely to be encountered. Further consideration of this topic properly belongs to the subject of ship design.

CHAPTER IV

APPLICATIONS OF HYDROSTATICS

32. The Gravity Dam.—One of the most important of the many applications of hydrostatics is the design of dams, of which there are several types. The gravity dam is one which depends for its stability upon its weight. A typical cross-section of such a dam is shown in Fig. 31. If the face AB is curved it will be necessary to compute the two components of the water pressure, H being equal to the pressure on a plane whose trace is $A'B$ while V is the weight of the volume of water represented by

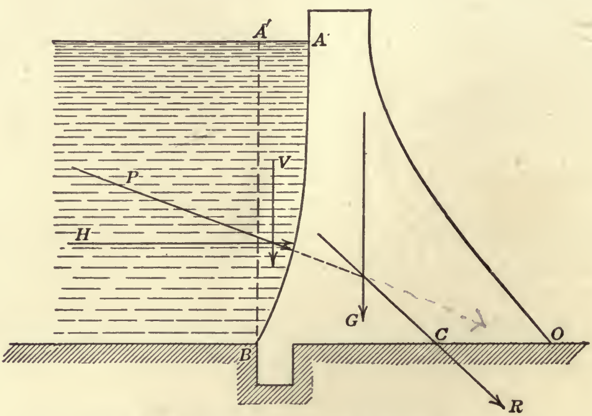


FIG. 31.—Cross-section of gravity dam.

ABA'. In all computations it is customary to consider a length of dam (perpendicular to the plane of the figure) of 1 ft. Evidently the stability of a gravity dam is independent of the total length of the dam.

The total water pressure P combined with the weight of the section G gives a resultant pressure on the base whose value is R. This pressure is distributed all over the base BO but may be considered to have a single point of application C. If R be resolved into two components at the point C, evidently the value of the horizontal component must be equal to H while that

of the vertical component will equal $G + V$. By taking moments of all the forces about O it will be easy to locate the point C.

If the dam rests solidly upon impervious rock and there is no leakage of water along any plane, or if a cutoff wall at B runs down deep enough to stop percolation, and the base of the dam is well drained, the above forces are all that act upon the structure, excepting of course the support of the earth which is equal and opposite to R. But if water does have access to the under side of the dam there will be exerted upon BO a vertical upward pressure due to this. How much this may amount to depends upon conditions. Thus if water saturates the foundation but does not have an opportunity of escaping past O the whole base of the dam will be subjected to a water pressure equal to BA' in intensity. But if the water can escape past O there will be a flow of water under the dam and consequently the pressure must decrease from BA' at B to a very much smaller value at O. It is often reasonable to assume the pressure as zero at O. But in any event the admission of water to the base of the dam tends to decrease the safety of the structure.

It may be seen that the horizontal thrust of the water H is opposed solely by the friction between the dam and the foundation upon which it rests. If the coefficient of friction here be denoted by μ then it is clear that if the dam is safe against sliding the value of H must be less than $\mu(G + V)$. The factor of safety against sliding is the ratio of the latter quantity to H. Any leakage of water under the base of the dam decreases the pressure between the dam and the material upon which it rests and thus tends to decrease the frictional resistance. The frictional resistance can be increased by sinking portions of the dam into trenches such as in the case of the cutoff wall at B.

If it were possible for the dam to act as a rigid body under all circumstances it could then fail by overturning about O as an axis. It is seen that with O as a center of moments, H tends to overturn the dam but is resisted by G and V. If water pressure acted upon the base it would also tend to overturn the dam. The factor of safety against overturning is the ratio of the moment of $G + V$ to the moment of H and the water pressure on the base, if any is allowed for. However, before a masonry dam of any size would overturn, the material along the base near O would be crushed due to the high intensity of pressure it would be under. Thus although the point C might be to the left of O in Fig. 31 so that

the structure is safe against overturning, the base would still not be safe against crushing. Hence, the second consideration of the stability of the dam is not as to whether it will or will not overturn but is concerning the distribution of stresses along the base BO.

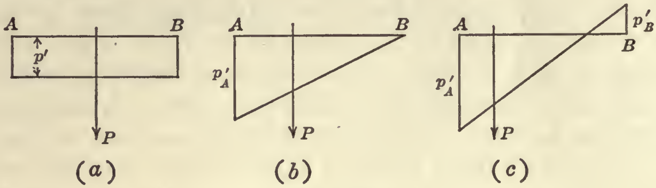


FIG. 32.

Referring to Fig. 32, a uniform intensity of stress p' distributed over an area represented by AB gives a resultant pressure P applied midway between A and B. If, however, the stress varies uniformly from p'_A at A to zero at B, the resultant P will pass through a point one-third the distance from A to B. If the total pressure

From a photograph by the author.

FIG. 33.—Concrete dam at Crystal Springs Lake, California. 145 feet high.

P has the same value in both cases, it is clear that the intensity of pressure at A is greater in the latter case than in the former. And if P is applied at a point less than one-third the distance from A to B, the intensity of stress will be still greater at A and at B the intensity of stress p'_B will be opposite in sign to that at

A. It is thus clear that it is desirable to have the resultant pressure pass as nearly through the midpoint as possible. And if tensile stresses are to be avoided the resultant pressure must be kept within the middle third. As masonry is not supposed to endure tensile stresses, it is customary to so design the dam that the resultant pressure falls within the middle third of any section.

It is not only necessary to undertake such an analysis of the dam as a whole but also to investigate the stability of *all* portions of the dam with respect to *any* horizontal plane. In all such studies the maximum height of water should be assumed. But also the pressures should be determined when the reservoir is empty as the inner face of the dam might then be subjected to excessive vertical stresses.

33. The Framed Dam.—Contrasted with the gravity dam we have the framed dam shown in Fig. 34 which depends for its

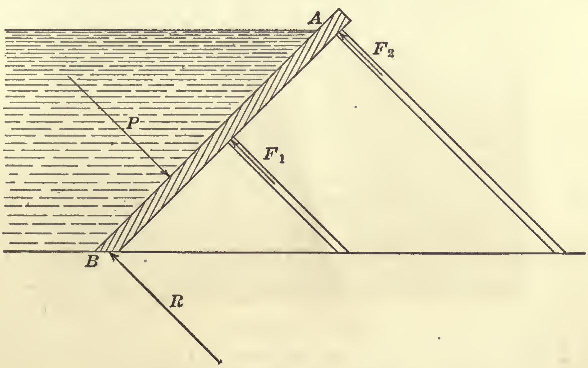
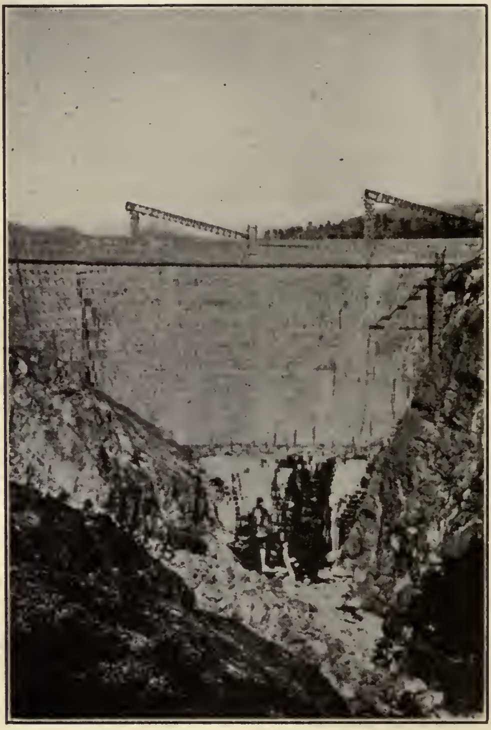


FIG. 34.—Framed dam.

stability upon the strength of its members. It consists of a water-tight deck AB supported by struts, trusswork, or buttresses at certain intervals along the length of the dam (perpendicular to the plane of the figure). The deck is always inclined so that the weight of the water upon it may hold the structure down and increase the factor of safety against sliding.

34. The Arch Dam.—In the case of a short high dam in a situation where firm support can be had from the walls on either side the arch dam is desirable. It is designed to withstand the water pressure by pure arch action and to transmit the pressures to the abutments at either end. The material in an arch dam is usually much less than in a pure gravity dam but any arch dam

acts to some extent as a gravity dam. Its analysis is not within the scope of this text.



From a photograph by the author.

FIG. 35.—Lake Spaulding, Cal., variable radius arch dam. Ultimate height will be 325 ft.

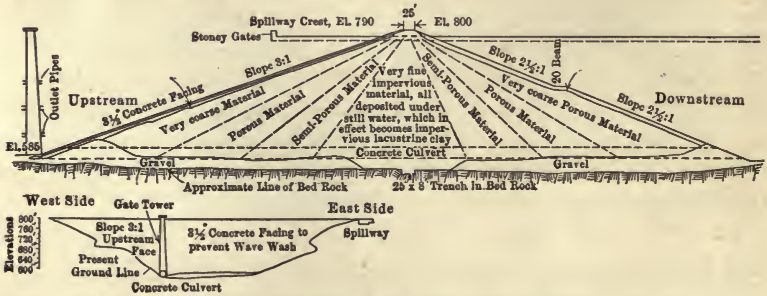


FIG. 36.—Section of Calaveras earth dam.

35. The Earth Dam.—Under favorable circumstances the earth dam is a very economical type. A typical section of such

a dam may be seen in Fig. 36. The slopes on both the upstream and downstream faces are less than the angle of repose of the

From a photograph by the author.

Fig. 37.—Upstream face of San Andreas earth dam. 90 ft. high.

From a photograph by the author.

Fig. 38.—Incompleted Calaveras earth dam. Ultimate crest will be at dotted line making it the highest earth dam in the world.

material used. In order to make such a dam water-tight it is provided with an impermeable core which may be a thin vertical wall of concrete or other material, or, as in Fig. 36, it may be

obtained by depositing fine earth under water. Fig. 38 shows the pool of water in the center of the dam where such a core is being formed. There is little mathematical analysis to be made for such a dam. The main problems are those of construction and careful selection of the materials employed.

36. Additional Notes on Dams.—In most cases there are times when there is an excessive quantity of water that must be disposed of, usually by allowing it to flow over a spillway that is provided for that purpose. The spillway may be located at a different place from the dam so that no water ever overtops the latter as will be the case in Fig. 35. Again the spillway may

From a photograph by the author.

Fig. 39.—Low dam at Ithaca, N. Y.

occupy a portion of the crest of the dam as in Fig. 33 where the spillway can be seen in the middle. In other cases, such as in Figs. 36, 37, and 38, the spillways are located at one end of the dam and consist of rectangular canals through which the flood waters are discharged. But in Fig. 39 it may be seen that the entire crest of the dam is used for a spillway. This dam also shows the curved face that is provided to minimize the scouring effect of the waterfall upon the bed of the stream at the toe of the dam. For it must be recognized that water in falling over a dam acquires kinetic energy that must be expended in some way and unless suitable provision is made for this it may be expended in undermining the dam itself.

37. Flashboards.—In storing water by means of a dam it is desirable to keep the water level as high as possible without flooding any lands upstream. If, therefore, the crest of the dam were located at the elevation allowable under normal conditions it would be excessively high in times of flood. In order to overcome this difficulty movable devices are employed called flashboards, movable crests, and various other names (Fig. 40). These are all schemes for increasing the height of the dam by equipment which can be removed when necessary. In some cases they work automatically, being either washed away when the water reaches a certain stage or caused to drop to a horizontal position. Other types require removal by hand in such emergencies. After the flood is past and the drier season comes on

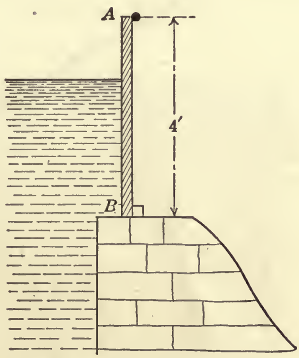


Fig. 40.—Flashboard.

they may be replaced again. Some of these are entirely automatic in their action as in the case of the Stickney automatic crest outlined in Fig. 41. We have here two planes AB and BC rigidly connected and rotating about B. The water pressure on AB together with the weight of the shutters and the additional weight added at C tend to rotate the device in one direction but that is opposed by the pressure of the water on BC. By a suitable adjustment of area and weights it is possible to keep this crest in the position shown until the water reaches the level of A. Then the pressure on AB may be sufficient to cause it to drop to the position $A'BC'$. Hence the crest of the dam will then be reduced to the height of B, and the flood water will pour over the shutter BA' and hold it down. But when the excess waters have passed and the water level drops to B, or thereabouts, the pres-

sure on BC', no longer opposed by that on BA', will raise the crest to the initial position.

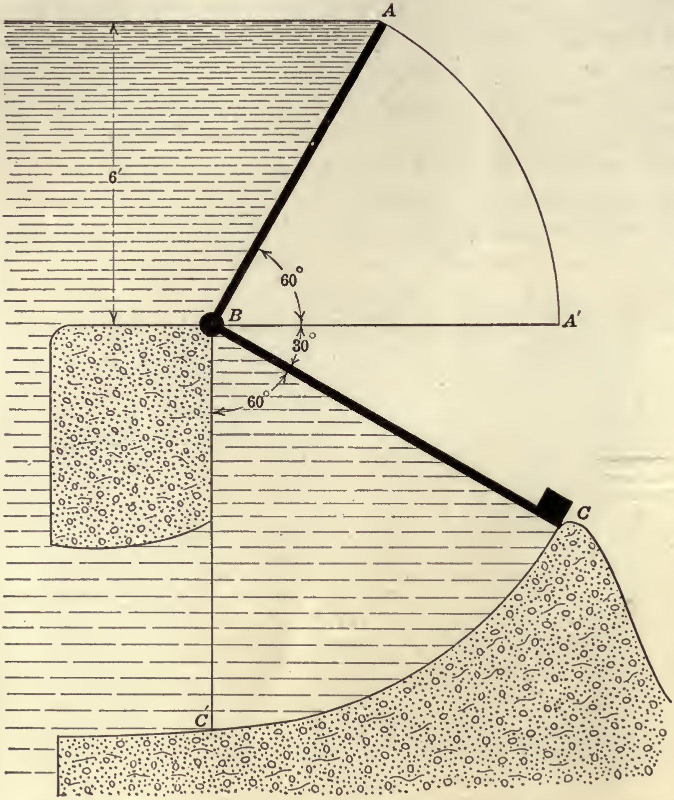


Fig. 41.—Automatic dam crest.

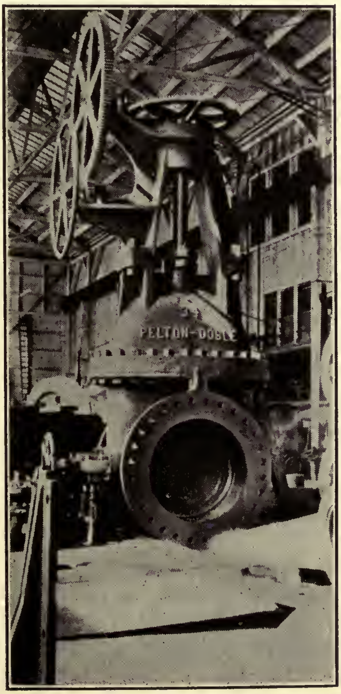
38. PROBLEMS

1. The intake tower in Fig. 42 will be surrounded by water when the reservoir is filled and the outflow of water will take place through the openings provided in the tower. Assume one of these gates to be 3 ft. wide and 4 ft. high and to weigh 1,000 lb. When the inside of the tower is subjected to the pressure of the air only, what vertical pull on a gate rod will be necessary to open the gate when the water stands 10 ft. above its top, if the coefficient of friction between the gate and its guides is 0.3?

2. The valve in Fig. 43 is 34 in. in diameter. If it is closed and under a pressure of 1,000 ft. of water on one side and atmospheric pressure on the other, what pull will have to be exerted on the valve stem to open it if the coefficient of friction is 0.4?

From a photograph by F. H. Fowler.

FIG. 42.—Intake tower at Elizabeth Lake Reservoir on Los Angeles Aqueduct.



From a photograph by the author.

FIG. 43.—A 34-in. high-pressure gate valve in the shop of the Pelton Water Wheel Co.

3. Find the magnitude and point of application of the resultant pressure on the 2-ft. circular gate shown in Fig. 44.

4. The gate *AB* in Fig. 45 rotates about an axis through *B*. If the width is 4 ft., what torque applied to the shaft through *B* is required to keep the gate shut?

5. What value of *b* in Fig. 46 is necessary to keep the masonry wall from sliding? Masonry weighs 150 lb. per cu. ft. and the coefficient of friction equals 0.4. Will it also be safe from overturning? If it has a factor of safety against sliding of 2, where will the resultant of the water pressure and its weight cut the base?

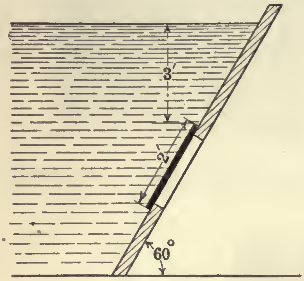


Fig. 44.

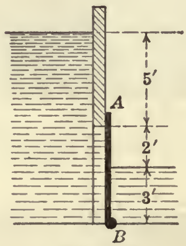


Fig. 45.

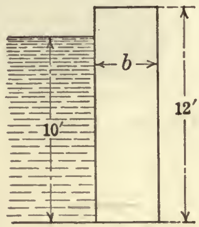


Fig. 46.

6. In the framed dam shown in Fig. 47, the struts *CD* are placed 5 ft. apart along the dam (perpendicular to the plane of the figure). What will be the load on each strut? What will be the value of the reaction at *A*? If the length *BE* is 4 ft. and the depth of the water flowing over the crest at *E* is 3 ft. what will be the load on the strut?

7. Assume the weight of the dam in Fig. 48 to be 150 lb. per cu. ft., that there is no seepage of water under its base, and that the coefficient of friction between the dam and the material upon which it rests is 0.6. For 1 ft.

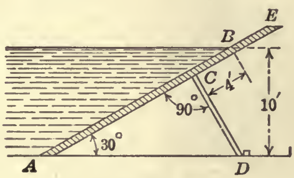


Fig. 47.

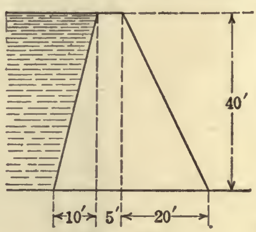


Fig. 48.

length compute: (*a*) Horizontal component of water pressure. (*b*) Vertical component of water pressure. (*c*) Weight of dam. (*d*) Is it safe against sliding? (*e*) Is it safe against overturning? (*f*) Where does the resultant of the water pressure and the weight of the dam cut the base?

8. In Fig. 40 the flashboard *AB* rests against a solid block at *B* but there is a pin at either end at *A* which is breakable. If the length of a section of flashboard is 6 ft., what must be the shearing strength of the pins if they give way when the water level reaches *A*?

9. In Fig. 41 what weight must be added at C per foot of length in order that the crest may drop when the water level reaches A? Neglect the weight of the rest of the movable crest, and assume $BC = 7.5$ ft.

10. Figure 49 shows a cylindrical tank. What is the total pressure on the bottom? What is the total pressure on the annular surface A-A? Find the maximum intensity of longitudinal tensile stress in side walls B-B: (*a*) If the tank is suspended from the top. (*b*) If it is supported on the bottom.

Ans. 392 lb., 147 lb., (*a*) 20.8 lb. per sq. in., (*b*) 7.8 lb. per sq. in.

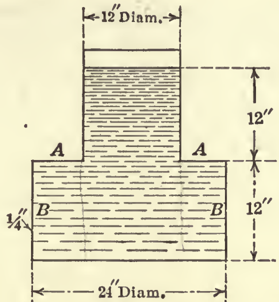


FIG. 49.

11. A pipe line 3 ft. in diameter is to carry water under a pressure of 1,000 ft. If the allowable tensile stress is 20,000 lb. per sq. in., what should be the thickness of steel used?

12. With the thickness of metal computed in the preceding problem what would be the tensile stress across a circumferential section if a valve was closed, the pressure on the other side of it being atmospheric?

CHAPTER V

HYDROKINETICS

39. Actual and Ideal Conditions.—From the standpoint of pure mechanics the subject of hydrokinetics is rather unsatisfactory. This is due to the fact that so many assumptions are necessary, many of which are known not to be true. Thus in the greater portion of the work all particles of water in any cross-section of a flowing stream are assumed to move in parallel paths and with equal velocities. This is shown in Fig. 50, a

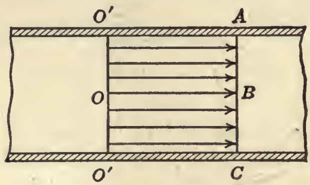


FIG. 50.

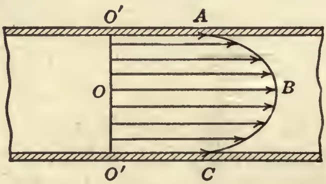


FIG. 51.

particle of water at point O moving along the axis of a pipe with a velocity OB. But every other particle of water across section $O'OO'$ is assumed to move with the same velocity giving us the velocity curve ABC, in this case a straight line. But it is well known that in a pipe the actual velocity curve is similar to ABC in Fig. 51, the velocity of a particle of water at O in the center of the pipe being OB while that of a particle near the wall of the pipe is $O'A$. Experiment shows that in general OB is about twice the value of $O'A$ and that the mean velocity of all the particles is about 0.84 OB. It is this mean velocity that is actually used in our computations. Hence our results are based upon a velocity which is possessed by only a few of the particles of water, the greater portion of them moving with either higher or lower velocities. We usually do not attempt to deal with the actual velocity curve ABC of Fig. 51 because we

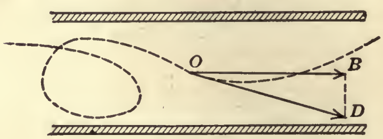


FIG. 52.

have no assurance as to its exact nature in every case and, if we did, our equations would be too complicated for practical use.

But in Fig. 51 all particles of water have been assumed to be moving in straight lines parallel to the axis of the pipe, which we know is very seldom the case. In fact the path of a given particle is very irregular as is shown in Fig. 52 and at the instant

From a photograph by the author.

Fig. 53. Showing vortices on surface of canal.

in question a particle at point O may be moving with some velocity OD. But in most practical problems we are concerned with OB which is the axial component of the true velocity. Thus not only do our equations ordinarily deal with a mean velocity, but they deal with a component of the true velocity. Instead of water flowing in parallel threads the true phenomena has been

very aptly compared to the motion of a cloud of feathers blown along by the wind. Water tends to travel in vortices as may often be observed upon the surface of an open stream such as the canal shown in Fig. 53. In this particular scene the water was flowing with a moderate velocity (about 3 miles an hour) over a reasonably smooth bed but the surface was covered with little vortices.

Since actual conditions depart so widely from the ideal conditions assumed by our imperfect theory we can expect our theory to provide little more than a framework upon which may be hung the results of experimental investigation.

The mean velocity at any section (strictly the mean axial component of velocity) is obtained by dividing the total rate of discharge by the total area of the section. That is $V = q/F$.

EXAMPLES

1. Experiment indicates that the velocity curve ABC of Fig. 51 is approximately a semi-ellipse and that OB is about twice $O'A$. Assuming this to be so, find the ratio between the mean velocity and the maximum velocity. (The total rate of discharge is $\int VdF$ and the value of this integral is the volume of the solid $O'ABCO'$. Dividing the solid by the area of the base, πr^2, we should have the mean ordinate or in this case the mean velocity. The volume of an ellipsoid is two-thirds that of the circumscribing cylinder.)

Ans. 0.833.

2. A stream is divided into five equal areas and the mean velocity of each portion is found by some method. These velocities are 3, 3, 4, 4, and 5 ft. per sec. What is the mean velocity of the entire stream?

Ans. 3.80 ft. per sec.

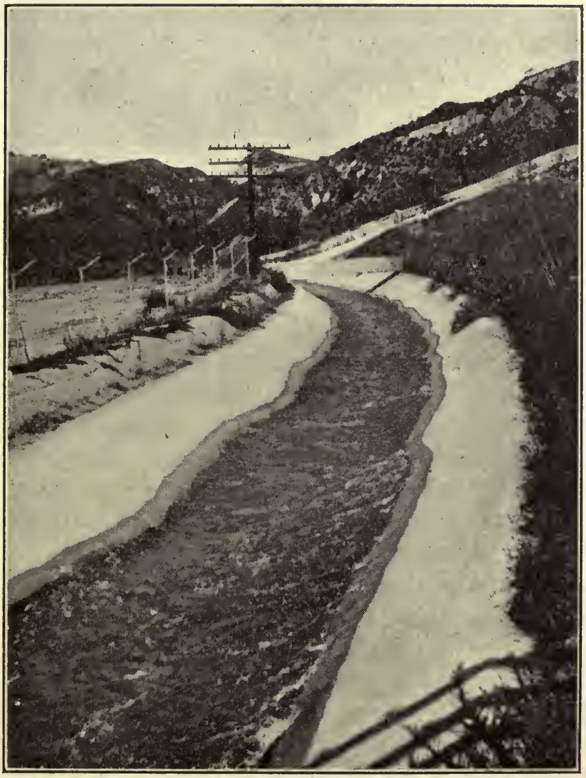
3. Suppose that the areas are not equal but have values of 2.5, 2.5, 2.0, 2.0, and 1.0 sq. ft. while the velocities are 3, 3, 4, 4, and 5 ft. per sec., respectively. What is the total rate of discharge? What is the mean velocity?

Ans. 36 cu. ft. per sec.; 3.60 ft. per sec.

40. Critical Velocity.—The path followed or assumed to be followed by a single particle of fluid is called a stream line. It has been found that for very low velocities the stream lines are straight parallel lines as shown in Figs. 50 and 51, but that as soon as a certain velocity is exceeded the flow becomes turbulent or sinuous as in Fig. 52. The velocity at which the change occurs is called the critical velocity. The value of the critical velocity is affected by the temperature and also the size of the tube or pipe; the larger the latter the lower the critical velocity. For ordinary size pipes with which the engineer has to deal the critical velocity is so low that its value is of no interest.

41. Steady Flow.—By steady flow is meant that at any point in a stream all conditions remain constant with respect to time. This does not mean that the conditions at any one point are necessarily like those at some other point.

Unsteady flow is met with in cases where change is taking place. Thus suppose a pipe line is flowing full of water and a



From a photograph by the author.
FIG. 54. The Los Angeles Aqueduct.

valve is closed suddenly at its lower end. The velocity of the water would be brought to zero and in so doing there would be certain pulsations of pressure, which if violent enough would be recognized as water hammer. While such changes are in progress we should have unsteady flow. Again suppose that a gate is opened so as to admit water into an open canal originally empty. As the canal filled with water the level at any point would stead-

ily rise and also the velocity would in general be changing at all points. While such changes were under way the flow would be unsteady. But when equilibrium is finally established, the water level at any point and the velocity of flow across any section no longer vary from time to time and we then have steady flow.

In the strictest sense of the word steady flow is seldom met with in ordinary engineering work as it would be found only with velocities below the critical velocity. For with all velocities above the critical we have continual fluctuations of flow at any point due to the irregular motion of the individual particles. It is for this reason that manometers or pressure gages attached to pipes, in which water is flowing, continually pulsate. Another evidence may be seen in Fig. 54, the dark band on either side of the water being where the latter has wet the concrete by wave action.

For all practical purposes we disregard these slight fluctuations at individual points. If the average conditions over the entire section are reasonably constant with respect to time, we consider the flow as steady. While problems of unsteady flow are often problems of great practical value, especially in connection with the speed regulation of water power plants, they are rather difficult of mathematical treatment. Fortunately they are not as common as the more simple problems of steady flow. For the most part this text will be devoted to the latter.

42. Rate of Discharge.—The volume of water flowing across any section per unit time is called the *rate of discharge*. It must not be confused with velocity, since it is the product of the cross-section area of the stream and the velocity of flow across the section. It may be expressed in various units such as cubic feet per minute, gallons per day, etc., depending upon the custom in that particular class of work. In the foot-pound second system of units such as are employed in this text it would naturally be in cubic feet per second. This is often called "second foot" for brevity and written as "sec. ft."[1]

43. Equation of Continuity.—In Fig. 56 it is apparent that the volume of water between any two sections such as (1) and (2) must remain constant if the flow is steady. Hence it follows that the rate at which water flows in at (1) must be equal to the rate at which it flows out at (2), otherwise there would be a change

[1] In irrigation work in India the term "cusec" has gained acceptance for this rate of discharge.

in the volume contained between the two sections. Thus we may say that for steady flow, $q_1 = q_2$.

If the flow is unsteady this is not necessarily so. For suppose that the closure of a gate above (1) shut off the flow of water at (1), we would still find water flowing for a time past (2) though at the expense of the volume stored between the two sections. Hence in case of unsteady flow, where the volume in any distance is changing, the equation of continuity no longer applies.

The equation of continuity states that for steady flow

$$q = F_1V_1 = F_2V_2 = \ldots\ldots = FV = \text{constant} \qquad (20)$$

This equation justifies the use of the term "rate of discharge" for the rate of flow across any section even though it be in the middle of a length of pipe or at some point in a river. For at some ultimate point the pipe or stream actually discharges in the usual sense of the word. And the rate of discharge at this point is equal, if the flow be steady, to the rate of volume flow at all sections throughout the stream.

EXAMPLES

1. In Fig. 55 the portion of pipe between A and C is the frustum of a right circular cone with vertex at O. If the rate of discharge is 10 cu. ft. per sec., what are the velocities at A, B, and C? Between A and C the velocity will vary as what function of the distance from O? What shape of tube should be between A and C in order that the velocity may decrease uniformly with respect to distance? (Take origin at point where velocity would become zero.)

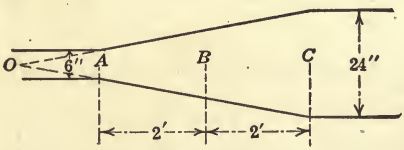


FIG. 55.

2. The canal shown in Fig. 53 is 14.5 ft. wide and 4.2 ft. deep. If the velocity of the water is 3.5 miles per hour, what will be the rate of discharge in cubic feet per second?

3. The water in the canal of problem (2) finally flows down a steel penstock (Fig. 111) which is 52 in. in diameter. What is the velocity of flow?

4. At the end of the pipe line in problem (3) the water is discharged through four nozzles the jets from which are approximately 7 in. in diameter. What is the jet velocity?

44. General Equation for Steady Flow.—In the case of steady flow we may derive a very useful equation commonly known as

Bernoulli's theorem in honor of Daniel Bernoulli who proposed it in 1738. We shall make use of the principle of *work and kinetic energy*, and the following conditions will be assumed:

(*a*) Flow is steady.
(*b*) Fluid is incompressible.
(*c*) Velocity across any cross-section is uniform.

In Fig. 56 let A and B be any two cross-sections of a filament of a stream in steady flow. Suppose that during an infinitesimal time interval particles passing A and B move to A' and B' respectively. The pressure, elevation, velocity, and cross-section area between A and A' will be denoted by p_1, z_1, V_1, and F_1 respectively, while between sections B and B' these will be p_2, z_2, V_2 and F_2 respectively. Since the flow is steady and the fluid is incompressible, the volumes of water passing A and B during any time interval must be equal so that $F_1ds_1 = F_2ds_2$.

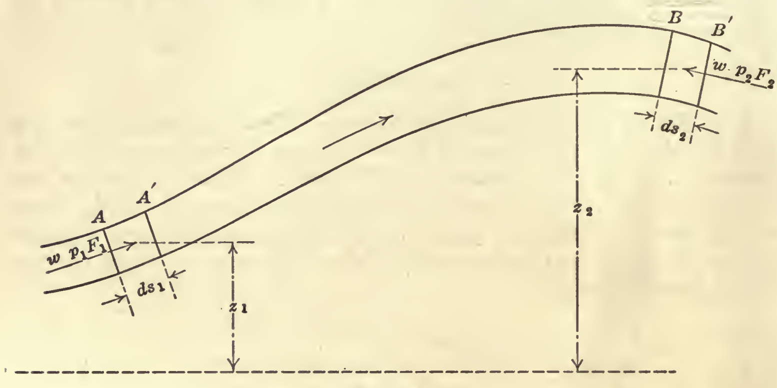


FIG. 56.

From the principle of work and kinetic energy, the net work done on the volume between A and B while it moves to the position between A' and B' is equal to the corresponding change in its kinetic energy. The net work done on the volume under consideration is the sum of three parts: (1) The work done by pressures normal to the external surface of the filament; (2) the work done by gravity; (3) the work done by frictional forces.

In the first item it is necessary to consider only the work done by the pressures on the end cross-sections, since the side pressures do no work. These forces at A and B are wp_1F_1 and wp_2F_2 re-

spectively and the displacements of their application points in the directions in which they act are ds_1 and $-ds_2$ respectively. Hence the net work done by these forces is $wp_1F_1ds_1 - wp_2F_2ds_2$.

Since the location of the center of gravity of the portion of the filament between A' and B remains unchanged, the net work done by gravity during the time interval is equal to that due to the change of elevation of the volume of water F_1ds_1 from z_1 to z_2. The net work done by gravity is thus $wF_1ds_1(z_1 - z_2)$.

The work done by friction will be neglected for the present.

Since the kinetic energy of the portion between A' and B remains unchanged if the flow is steady, the whole change of kinetic energy is the difference between the kinetic energies of the parts between B and B' and between A and A'; that is

$$wF_2ds_2\frac{V_2^2}{2g} - wF_1ds_1\frac{V_1^2}{2g}.$$

Combining all the work and energy terms in an equation and noting that $F_1ds_1 = F_2ds_2$

$$wF_1ds_1(p_1 - p_2 + z_1 - z_2) = wF_1ds_1\frac{V_2^2 - V_1^2}{2g}$$

Dividing both terms by wF_1ds_1 and rearranging, we have

$$p_1 + z_1 + \frac{V_1^2}{2g} = p_2 + z_2 + \frac{V_2^2}{2g}.$$

This is Bernoulli's theorem, but, since all real fluids are viscous, it is impossible for flow to take place without fluid friction and hence a term should always be included to account for the energy converted into heat and hence lost.

Experiment indicates that the friction loss is some function of the velocity and for the present we shall represent it as $H' = kV^n/2g$, realizing that this is purely an empirical expression. Hence we may write the general equation as

$$p_1 + z_1 + \frac{V_1^2}{2g} - k\frac{V^n}{2g} = p_2 + z_2 + \frac{V_2^2}{2g}. \tag{21}$$

In case the velocity varies between (1) and (2) the V for the friction term might be taken as the average velocity, or by using

a suitable value of k it may be written as V_1 or V_2. In practical work the difficulty of using equation (21) lies largely in estimating proper values of k and n, and it is necessary to rely entirely upon experimental evidence.

45. Use of the Word "Head."—Examining each term of equation (21) in detail we find: The term p indicates intensity of pressure expressed in feet of water, hence it is a linear quantity and indicates the height of a column of water necessary to produce the given pressure. There may be no such real height of water in the problem, as in the case of a small volume of water enclosed within a cylinder and subjected to pressure by a piston. The quantity p is called *pressure head.*

The elevation of a point *above* any arbitrary datum plane is indicated by z. It is a linear quantity and in our system of units it should be expressed in feet. It is called *elevation head* or *potential head.*

The third term $V^2/2g$ may also be seen to reduce to a linear quantity when we analyze the units involved in V and g. The linear quantity equivalent to $V^2/2g$ is the height through which a body might fall in a vacuum from rest and acquire the velocity V. In many cases it is a purely artificial quantity in that there is no actual height in the figure illustrating the problem that gives any indication of its value. It may be called *velocity head.*

Since all the other quantities in equation (21) are in linear dimensions, or feet in our system of units, it follows that $kV^n/2g$ must also be in feet. It may be called the *lost head*, and is represented by the letter H'.

The sum of the pressure, elevation, and velocity heads at any section is called the *total* or the *effective head* at that section. However, the effective head, or any of the individual terms composing it, may be called "head" without any qualifying adjective.

It is often convenient to let a single letter stand for the effective head, hence we may write

$$H = p + z + \frac{V^2}{2g}. \tag{22}$$

Using this brief notation we may rewrite equation (21) as

$$H_1 - H' = H_2. \tag{23}$$

We see that the effective head must decrease in the direction of flow by an amount H'. Hence, although either pressure, eleva-

tion, or velocity may increase in the direction of flow, the sum of all three of them must continually decrease. Therefore an increase in one of these items must always be accompanied by a corresponding decrease in one or both of the other heads.

EXAMPLES

1. Assuming a body of water at rest in Fig. 57, so that there is no loss of head, what are the values of the pressure head at A, B, C, and D? What are the values of the elevation head? What are the values of the effective head at these four points?

2. In Fig. 58 the point A is 30 ft. higher than B. Assuming the pipe to be of uniform diameter, in which direction will the water be flowing if the pressure at A is 20 lb. per sq. in. and that at B is 40 lb. per sq. in.? What is the head lost between the two points? What would the pressure be at A if the flow were to be in the opposite direction, the rate of discharge remaining the same?

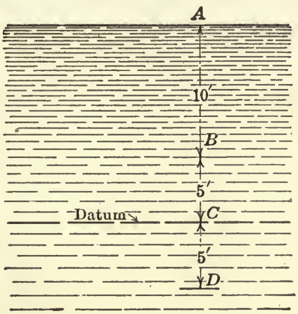


FIG. 57.

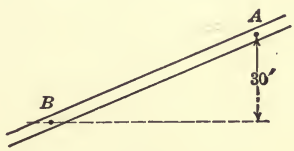


FIG. 58.

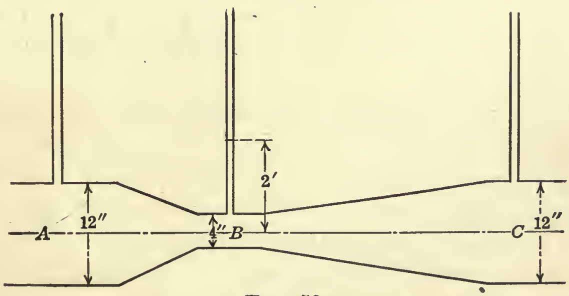


FIG. 59.

3. In Fig. 59 suppose 8 cu. ft. of water per sec. to be flowing from A to C. Assume a loss of head from A to B to be equal to $0.001 V_B^2$ and an equal loss to occur between B and C. If the pressure head at B is 2 ft., how high will the water stand in piezometer tubes at A and C?

4. Neglecting all loss of head in Fig. 55, what kind of a curve would express the variation of pressure from A to C?

46. Energy and Power Meaning of Head.—Suppose we multiply the elevation head z by the weight G of a definite volume of water. The product Gz being pounds times feet represents foot-pounds and we recognize it as potential energy. That is the body of weight G possesses Gz foot-pounds of potential energy by reason of its elevation z.[1] In like manner if we multiply the velocity head by G we have $GV^2/2g$, which represents the kinetic energy of G pounds of water due to its mass and velocity. By analogy we might expect that if we multiplied the pressure head p by G we should also have Gp foot-pounds of energy which we could call pressure energy.

But we here face the difficulty that we recognize energy in only two fundamental forms which we call potential energy or kinetic energy as the case might be. All other forms of energy may be reduced to one of these two. It is not clear that pressure energy can be reduced to either of these and so we have to seek further for an explanation of this term. Energy is ability to do work and we feel that water under pressure is capable of doing work. But if a particle of water should in some manner suddenly be disconnected from its fellows it would still have its initial elevation and velocity; these are qualities that it possesses in itself. But its pressure would be lost, since that is derived from contact with other particles. Thus water can do work due to pressure only so long as it is still connected with other particles. Hence we might conclude that pressure energy, if the term is permissible, is not something that a particle possesses but is merely energy that is transmitted from one to the other by virtue of the pressure and the motion.

A good analogy to the way that energy may be transmitted past any point is offered by a belt connecting two pulleys. The belt possesses kinetic energy due to its mass and velocity and this energy is carried past any stationary reference point. But in addition to this the belt is under tension and is in motion and hence transmits energy from one pulley to the other that it does not in itself possess. In like manner in the case of a flowing stream of water, the water carries across any transverse section a certain amount of energy which it possesses in either the potential or the kinetic form or both. In addition to this it

[1] Strictly speaking this energy is possessed by the system consisting of the earth and the water.

may transmit energy across the section due to its pressure and motion.

Suppose we consider a particle of water flowing from (1) to (2) in Fig. 56 and assume that there is no loss of head so that $H = p + z + V^2/2g =$ constant. In the case of a freely falling body acted upon by no other forces save gravity we should find that the loss of potential energy was compensated for by an equal increase in kinetic energy so that its energy remains constant. But if we assume that the stream of water is confined in a channel of uniform area the velocity is constant according to the equation of continuity, hence the kinetic energy cannot change. If the particle loses potential energy without any increase in its kinetic energy, it follows that its total energy must decrease. This is true for the particle. But the total energy in the system does not change for we are assuming no loss. The reason the velocity of the particle of water cannot increase is that negative work is being done upon it by the pressure $dp'dF$ (Fig. 56). But this negative work, although it reduces the energy of the particle, is not lost from the system. If conditions permit the particle of water to again ascend to a higher elevation or to increase in velocity the pressure acting on it will then do positive work and restore the potential or kinetic energy to it.

The discussion may be concluded by stating that head represents energy per unit weight of water. In the case of lost head H' represents the energy lost and dissipated in the form of heat per unit weight of water. Thus head may represent foot-pounds per pound of water.

If we multiply the effective head H by the weight of water W flowing across any section per unit time, the product of the two will be energy per unit time. But energy per unit time, or the rate at which energy is transmitted, is power. In our system of units the product WH will be in foot-pounds per second. Thus head is equal to the *energy* per unit weight of water or it is equal to the *power* per unit rate of discharge.

EXAMPLES

1. The surface of a lake is 500 ft. above a certain arbitrary datum plane with respect to which energy is to be measured. (*a*) What is the energy per pound of water? (*b*) If the lake is capable of furnishing 200 cu. ft. of water per sec. what power is available at the datum plane?

2. In a pipe line which is 24 in. in diameter we have water flowing with a velocity of 15 ft. per sec. under a pressure of 10 lb. per sq. in. What power

is being transmitted through the pipe due to pressure? What is the total power delivered?

Ans. 123.2 hp.; 141.8 hp.

3. A jet of water free from all pressure is 7 in. in diameter and has a velocity of 250 ft. per sec. What is the horsepower?

Ans. 7,390 hp.

4. A pipe line draws water from a lake and delivers it to a power house at a point 500 ft. below the level of the surface of the lake. The water is delivered at a velocity of 170 ft. per sec. by a jet 6 in. in diameter, and free from all pressure (save that of the atmosphere). What horsepower has been lost in the pipe line?

47. Correct and Incorrect Applications of Bernoulli's Theorem.—Bernoulli's theorem states that along any stream line the effective head remains constant. But in a real fluid which is viscous there can be no flow whatever without some loss due to friction. Hence the correct statement is that along any stream line the effective head always decreases in the direction of flow.

It should be emphasized that the general equation should be written only between two points in the *same stream line* so that a particle of water may be assumed to flow from one point to the other. If there were no loss of energy it would follow that the effective head is constant at all points throughout a connected body of fluid and in that event only we might apply the equation to any two points whatever. But in reality this would lead us to incorrect conclusions as the following will show.

Suppose that we have no loss due to friction; it would then follow that we should have all particles of water at a section moving with equal velocities in parallel paths as shown in Fig. 50. All particles of water would have the same amount of kinetic energy and it is clear that all particles of water through the section would have the same amount of energy. But in reality the velocity across any section of a circular pipe, for example, is like that in Fig. 51. This is due to the fact that the greater frictional resistance near the walls of the pipe has retarded the water near them. Certain persons have incorrectly applied Bernoulli's theorem between a point near the wall and a point at the center of the pipe and reasoning that there could be no loss between two such points come to the conclusion that the pressure is less at the center of the pipe than it is near the wall because the velocity head is higher. This would lead to an excessive pressure difference if true. This reasoning has even been bol-

stered up by claims of experimental evidence, but in reality the data were inaccurately determined.

It is no more permissible to apply the general equation between two points in adjacent stream lines than between two separate streams in different channels. It is a mistake to assume that the effective head is constant across any section. Correct experimental evidence shows that the pressure head across any section varies only according to the depth, the same as in the case of water at rest. Hence if the sum of pressure head and elevation head is constant across any section while the velocity head varies, it follows that the total head varies at different points in the section. This is in harmony with a correct application of Bernoulli's theorem along the different assumed stream lines. If all particles started with the same store of energy it is clear that those near the pipe wall would have lost more by friction than those near the center. Hence the energy of those particles near the pipe wall should be less than that of those in the center.

In practical application of the general equation we do not deal with stream lines but with entire streams. Hence we have for H the average head across the section and for V the average velocity across the section. But we do equate the average head at some section of a stream to the average head at some other section of the *same* stream.

In considering an entire stream, rather than a single stream line, we assume the kinetic energy per unit time to be $WV^2/2g$, where V is the average velocity. This is not strictly true for, if the velocity varies from point to point over the section, the kinetic energy is the sum of the kinetic energies of all the individual particles. Considering an elementary area dF, the flow through it will be $wV'dF$, where V' is the actual velocity at the point in question. The kinetic energy of the elementary stream would be $wV'^3dF/2g$. Hence the total kinetic energy for the entire stream is

$$\frac{w}{2g}\int V'^3 dF. \tag{24}$$

If the velocity is constant this will become $wV^3F/2g = WV^2/2g$ since the true velocity at every point is the average velocity for the section. But in reality the velocity does vary to some extent over the section and hence (24) gives the true kinetic

energy. If the law of variation of V' throughout the section is known this integral can be evaluated, but in any event it can be shown that the kinetic energy so obtained is greater than that computed by using the average velocity.[1] Thus making the assumption that the velocity in the center of a circular pipe is twice that near the walls and that the velocity curve is a semi-ellipse it will be found that the true kinetic energy is 1.06 times that based upon the mean velocity. Fortunately the difference is not great in important cases met with in practice. Thus in the case of a jet of water from a good nozzle, where there is little variation in velocity, the difference may be a matter of about 1 per cent. only.[2] A correct application of equation (21) would require us to insert some factor before the velocity head, based upon the average velocity, to give a correct value. But if the velocity curves at sections (1) and (2) are similar and the velocities nearly the same in value the error in one may nearly balance that in the other. Hence it is not customary to allow for this discrepancy between the true kinetic energy and that computed by using the average velocity.

EXAMPLE

1. Assume that in a rectangular stream the velocity of the water is uniform from side to side at any depth but that it varies from the top to the bottom inversely as the depth. If the velocity at the top is twice that at the bottom find the ratio between the true kinetic energy passing a section per unit time and that based upon the mean velocity.

Ans. 1.11.

48. Applications of General Equation.—For the solution of problems in hydrokinetics we have two fundamental equations, the equation of continuity (20) and the general equation for steady flow (21), usually known as Bernoulli's theorem. In most cases the following procedure may be employed:

1. Choose a datum plane through any convenient point.

2. Note at what sections the velocity is known or assumed. If at any point the cross-section is great as compared with its value elsewhere, the velocity will be so small that the velocity head may be disregarded.

3. Note at what points the pressure is known or assumed. In a body of water at rest with a free surface the pressure is

[1] L. M. Hoskins, "Hydraulics," page 119.

[2] W. R. Eckart, Jr., Inst. of Mech. Eng., Jan. 7, 1910.

known at every point. The pressure in a jet is the same as that in the medium surrounding the jet.

4. Note if there is any point where the three items of pressure, elevation, and velocity are known.

5. Note if there is any point where there is only one unknown quantity.

It is generally possible to write equation (21) between two points such that they fulfill conditions (4) and (5) respectively. Then the equation may be solved for the one unknown. If it is necessary to have two unknowns then equation (21) must be solved simultaneously with equation (20). The procedure is best shown by applications such as the following:

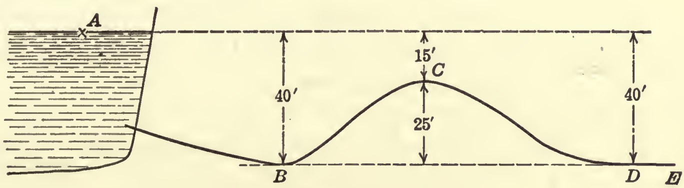


FIG. 60.

In Fig. 60 we have a pipe BCD which is 6 in. in diameter through which water flows from reservoir A. The diameter of the stream discharging freely into the air at E is 3 in. Let us assume that $n = 2$ in equation (21) so that the loss of head due to friction is proportional to the square of the velocity. Then $H' = kV^2/2g$, where V is the velocity in the pipe. Suppose that the roughness of the pipe and the lengths between the various points are such that the values of k from the reservoir to B, from B to C, from C to D, and from D to E are 2, 4, 4, and 1 respectively. Let it be required to find the pressure at C when flow takes place.

At C there is both an unknown pressure and an unknown velocity, hence we cannot immediately apply equation (21) as one equation is capable of determining only one unknown. Let us then follow the procedure outlined. The location of a datum plane is immaterial in the solution of the problem but it is usually convenient to take it through the lowest point in the figure and thus avoid negative values of z. Therefore let us assume a datum plane through E. In the reservoir we find that the velocity is negligible because of the large area as compared with

the area of the pipe. At a point A on the surface of the water we find the pressure to be atmospheric, which is also the case with the stream at E. Thus whatever the pressure of the atmosphere may be its effects can easily be shown to balance out and therefore we neglect it altogether. Hence at A we find that everything is known while at E the velocity head is the only unknown.

We shall apply equation (21), or its equivalent (23), between points A and E. We find that

$$\begin{aligned} H_{\mathrm{A}} &= 0 + 40 + 0 \\ H_{\mathrm{E}} &= 0 + 0 + V'^2/2g \\ H'_{\mathrm{A-E}} &= 11\, V^2/2g. \end{aligned}$$

Now V' is the velocity of the jet at E while V is the velocity in the pipe but one may be replaced in term of the other by equation (20). (It is seldom necessary to compute areas for this. It is both easier and more accurate to use the ratios of the areas, which means the ratios of the diameters squared.) Now $V' = FV/F'$, where F' is the area of the jet. But $F/F' = (6/3)^2 = 2^2 = 4$. Hence $V' = 4V$ and $V'^2 = 16V^2$. Replacing V' by V and substituting in equation (23) we have

$$40 - 11V^2/2g = 16V^2/2g.$$

Thus $V^2/2g = 40/27 = 1.48$ ft. We have now determined one of the unknowns at C.

We may next apply equation (23) between C and either A or E since we know the value of H at either of the latter points. The value of the effective head at C is $H = p + 25 + 1.48$ while $H'_{\mathrm{A-C}} = 6V^2/2g = 6 \times 1.48 = 8.88$ ft. Now from (23)

$$40 - 8.88 = H_C = p + 26.48.$$

Hence $$p = 4.64 \text{ ft.}$$

If the rate of discharge is also desired we can easily find that. Since $V^2/2g = 1.48$, $V = \sqrt{2g\,1.48} = 8.025\sqrt{1.48} = 9.78$ ft. per sec. Hence $q = 0.196 \times 9.78 = 1.92$ cu. ft. per sec.

EXAMPLES

1. Compute the pressures at B and D in Fig. 60.

2. Suppose that all other data for Fig. 60 remains unchanged except the diameter at C. What will this diameter be if there is a vacuum of 20 in. of mercury at C?

3. Suppose the diameter at C in Fig. 60 remains 6 in. and all other data is likewise unchanged except the elevation of C. How far above E can C be placed to produce a vacuum of 20 in. of mercury?

4. What fall from the surface of the reservoir in Fig. 60 to the outlet at E would be necessary to produce the same rate of discharge, if there were no loss due to friction?

49. PROBLEMS

1. In Fig. 60, with data as given in preceding article, what is the energy per cubic foot of water in the reservoir? What is the power transmitted past C? What is the power in the jet at E? What is the value of the power lost by friction?

2. Suppose all radiation of heat from the pipe in Fig. 60 could be prevented and that the temperature of the water in the reservoir is 32°F. What would be the temperature of the water in the jet? (778 ft.-lb. of work will raise 1 lb. of water 1°F.)

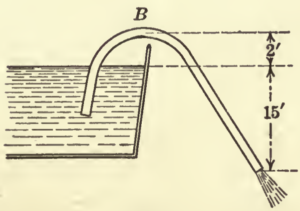


Fig. 61.

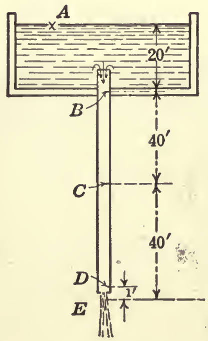


Fig. 62.

3. In the siphon shown in Fig. 61 the loss of head from the intake to B is 4 ft. and that from B to the discharge end of the pipe is 3 ft. Find the rate of discharge and the pressure head at B if the pipe is of a uniform diameter of 6 in.

Ans. $p = -14$ ft.

4. Suppose the discharge end of the siphon of Fig. 61 were 4 in. in diameter, other data remaining the same, what would be the rate of discharge and pressure at B?

5. If all data were as given in problem (4) except that the size of the stream discharging from the end of the siphon were not fixed, how large could the diameter of this be if the pressure at B is -25 ft.? What would then be the rate of discharge?

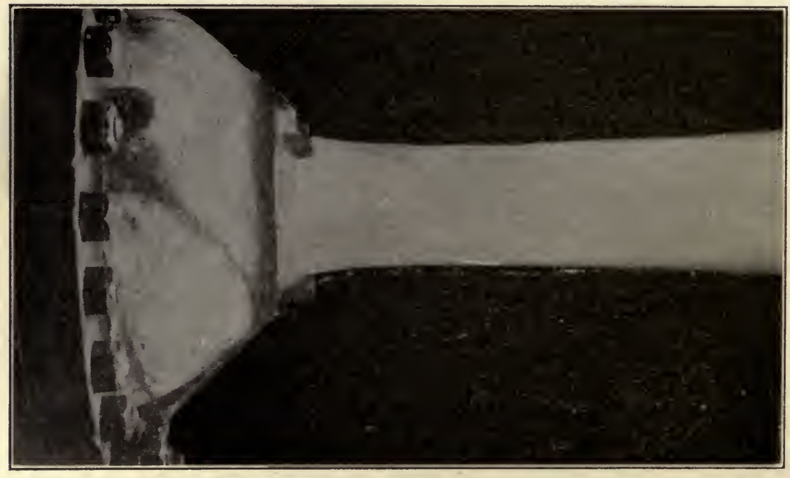
6. The diameter of the pipe in Fig. 62 is 4 in. and that of the stream discharging into the air at E is 3 in. Neglecting all losses of energy, what are the pressures at B, C, and D? (Velocity assumed negligible at A.)

CHAPTER VI

APPLICATIONS OF HYDROKINETICS

50. Definition of a Jet.—A jet is a stream bounded by a fluid of a different kind. The jets with which we are concerned in practical hydraulics are streams of water entirely surrounded by air. It is evident that the pressure to which the water in a jet is subjected is exactly equal to the pressure exerted upon its boundaries by the surrounding air.

51. Jet Coefficients.—Due to frictional resistance the actual velocity of a jet is always less than would otherwise be the case.



From a photograph by W. R. Eckart, Jr.

FIG. 63.—Jet from 7½-inch nozzle. (Head = 822 ft., velocity = 227.4 ft. per sec.)

The velocity which would be attained if friction did not exist may be termed the ideal velocity.[1] The ratio of the actual velocity to the ideal velocity is called the *coefficient of velocity.*

[1] This is frequently called "theoretical velocity" by others but the author feels that this is a misuse of the word "theoretical." Any correct and sensible theory should allow for the fact that friction exists and affects the result. Otherwise it is not theory but merely an incorrect hypothesis.

The area of the opening through which the jet issues is something that is readily determined, but in many cases the area of the jet cannot so readily be measured without special equipment. Hence it is desirable to know the relation between the area of a jet and the area of the opening through which it came. This factor, the ratio of the area of the jet to the area of the opening, is called the *coefficient of contraction.* The word "contraction" is used because the jet usually contracts and is smaller than the opening, as may be seen in Fig. 63. In case the jet does contract, the section of minimum area, the "vena contracta," is the section whose area is considered in the calculations. The velocity of a jet is also understood to be the velocity found at this point.

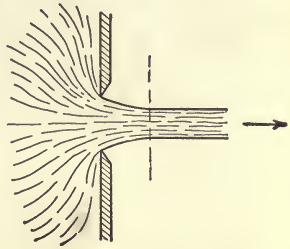


Fig. 64.—The "vena contracta."

The coefficient of contraction may be unity, indicating that the area of the jet is equal to the area of the opening from which

From a photograph by the author.

Fig. 65.—Discharge from end of a straight pipe. (Mixture of water, mud, and rocks building up the Calaveras earth dam.)

it issued. This is the case when the sides of the stream are parallel before it issues from the opening, as when the discharge is from the open end of a pipe as shown in Fig. 65. Of course after passing the point of minimum section the jet diverges again

due to the loss of velocity from frictional resistance. This is seen in Figs. 63, 66, and 67.

From a photograph by the author.
FIG. 66.—Jet from hydraulic giant sluicing out material for Calaveras earth dam.

From a photograph by the author.
FIG. 67.—Nearer view of jet in preceding figure.

The product of the coefficient of velocity and the coefficient of contraction is called the *coefficient of discharge.* It is the ratio of the actual rate of discharge to the ideal rate of discharge

that would be obtained if there were no friction and if the jet did not contract.

52. Flow through Orifice.—An orifice is any opening in the wall of a containing vessel. The only restriction is that the thickness of the wall shall be only a small fraction of the diameter or other linear dimension of the opening.

Let us write the general equation (23) between points (1) and (2) of Fig. 68, assuming the pressure is atmospheric at both points and that the area of the vessel is such that the velocity

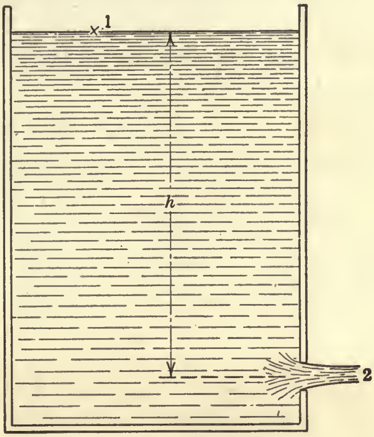


FIG. 68.

at (1) may be neglected. We shall also assume a loss of head between the two points which we shall consider as being proportional to the square of the velocity of the jet V and introduce such a factor k that $H'_{1-2} = kV^2/2g$. Thus we have

$$H_1 = 0 + h + 0, \qquad H_2 = 0 + 0 + V^2/2g.$$

Hence $H_1 - kV^2/2g = V^2/2g$.
From this, $V^2/2g = H_1/(1 + k)$ or,

$$V = \frac{1}{\sqrt{1 + k}}\sqrt{2gH_1} \tag{25}$$

If there were no frictional resistance to flow, the value of k would be zero. Thus the *ideal* velocity is $V = \sqrt{2gH_1}$. It is the ideal velocity that is to be multiplied by the coefficient of velocity to obtain the true velocity. Hence

$$c_v = \frac{1}{\sqrt{1 + k}} \tag{26}$$

In this equation we have the relation between the coefficient of velocity and the coefficient of loss. By squaring both sides and rearranging this may also be written

$$k = \frac{1}{c_v^2} - 1 \tag{27}$$

Since in this case $H_1 = h$, we may now write equation (25) in its more usual and more convenient form

$$V = c_v\sqrt{2gh} \tag{28}$$

53. Orifice in Case of Unequal Pressures.—In case the jet in Fig. 68 discharged into a medium under a different pressure from that existing upon the surface of the liquid in the vessel we should proceed as follows:

$$H_1 = p_1 + h + 0, \quad H_2 = p_2 + 0 + V^2/2g$$

Completing the solution as in the preceding article we should obtain

$$V = c_v\sqrt{2g(h + p_1 - p_2)} \tag{29}$$

54. Submerged Orifice.—For a submerged orifice as shown in Fig. 69 we could write for points (1) and (2):

$$H_1 = 0 + (h + y) + 0, \quad H_2 = y + 0 + V^2/2g.$$

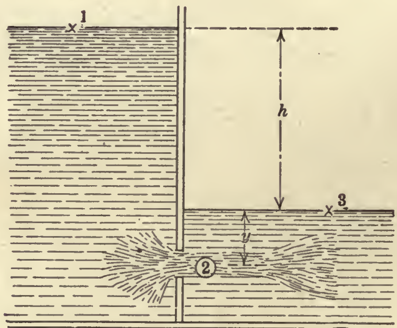


Fig. 69.

The pressure in the stream of water at (2) is equal to that upon its boundaries and that is equal to the depth y. It is evident that y cancels out so that for the submerged orifice we also have

$$V = c_v\sqrt{2gh} \tag{30}$$

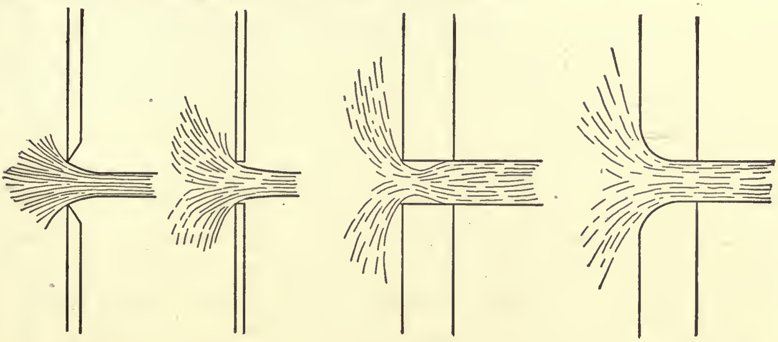
The coefficients for a submerged orifice would be different from those for an orifice discharging into the air. It is possible that the contraction coefficient would be materially larger and the velocity coefficient somewhat smaller.

EXAMPLES

1. In Fig. 69 determine the loss of head between (2) and (3), the velocity at the latter point being negligible. *Ans.* $V^2/2g$.

2. Knowing the loss of head between (2) and (3) add this to $kV^2/2g$ and solve for the velocity of flow through the orifice by writing an equation between (1) and (3).

55. Values of Orifice Coefficients.—The value of the coefficient of velocity is always less than unity, though it may often approach that value very closely. In some very well made nozzles and sharp-edged orifices the coefficient of velocity may be as high as 0.98 and occasionally 0.99 may be attained.



(a) (b) (c) (d)

FIG. 70.—Types of orifices.

A *standard orifice* is one with a sharp edge as in Fig. 70 (*a*). It is called a standard orifice because one will give practically the same results as another of the same size. Any other form of orifice such as (*c*) in Fig. 70 would give different results depending upon the thickness of the plate, the roughness of the material, etc.; hence the coefficients for each individual one would have to be determined if accurate computations were desired.

If the plate is thin and the inner corner is square and sharp, the orifice in Fig. 70 (*b*) may also be considered a standard orifice. But if the plate is too thick the condition in Fig. 70 (*c*) is met with and the velocity coefficient will be less than in the former case due to the greater frictional resistance the water encounters. Rounding the edge as in Fig. 70 (*d*) reduces the eddy losses and hence increases the coefficient of velocity slightly.

The contraction coefficient is much more sensitive to variations in the nature of the orifice than is the velocity coefficient. But it should be noted that contraction affects only the size of

the jet and not its velocity. The coefficient of contraction is the least in (*a*) and (*b*) and may be unity in (*c*) and (*d*). Hence the latter forms of orifices may discharge much more water than the former. Thus the type of orifice to be used depends upon whether one wishes a large discharge or wishes the maximum velocity. Of course if an orifice is used as a means of measuring rate of discharge only the standard orifice should be employed unless the special orifice can be calibrated.

If F and V denote the area and velocity of the jet respectively, while F_o is the area of the orifice, it may be seen that, since $c = c_c c_v$,

$$q = FV = (c_c F_o)(c_v\sqrt{2gH_1}) = cF_o\sqrt{2gH_1}.$$

Values of coefficients of discharge for standard circular and square orifices are given in Tables 1 and 2 respectively. These orifices are to be sharp edged and to be in vertical planes. The orifice should be so situated that it is flush with a flat wall on the water side free from all obstructions, projections, and sides for a distance in all directions of at least three times the diameter of the orifice. If this is not so the full contraction will not be obtained and the actual coefficient will be larger than given by the tables. The tables show that the coefficients are different for different sizes and for a given orifice vary with the head on the orifice, thus illustrating the impossibility of stating general values or laws that hold in all cases. It will be noted that the coefficients decrease as the head increases and that as higher values are attained the rate of decrease is much smaller. For heads above 100 ft. the values for 100 ft. may doubtless be applied with little error.

EXAMPLES

1. Find values of the coefficient of loss when the velocity coefficient has values of 1.00, 0.99, 0.95 and 0.80. Find c_v when k has values of 0, 0.5, 1.00.

2. Water issues from a vertical orifice (one in a vertical plane) under a head of 16 ft. The diameter of the orifice = 2 in. When measured it was found that $q = 33$ cu. ft. per min. What is the coefficient of discharge? If the coefficient of velocity is assumed to be 0.96, what is the value of the coefficient of contraction? What will be the diameter of the jet?

3. The discharge from an orifice under a head of 230 ft. was found to be 180 cu. ft. per min. The jet was found to be 2.16 in. in diameter while the diameter of the circular orifice was 2.25 in. What are the coefficients of velocity, contraction, and discharge?

4. What would be the rate of discharge from a standard circular orifice of 2 in. diameter under a head of 3 ft.? What would be the probable value of the jet velocity?

TABLE I.—DISCHARGE COEFFICIENTS FOR STANDARD CIRCULAR VERTICAL ORIFICES ACCORDING TO HAMILTON SMITH

Head on center of orifice in feet	Diameter of orifice in feet								
	0.02	0.03	0.04	0.05	0.07	0.10	0.20	0.60	1.00
0.4	......	......	0.637	0.631	0.624	0.618			
0.6	0.655	0.640	0.630	0.624	0.618	0.613	**0.601**	**0.593**	
0.8	0.648	0.634	0.626	0.620	0.615	0.610	0.601	**0.594**	**0.590**
1.0	0.644	0.631	0.623	0.617	0.612	0.608	0.600	**0.595**	**0.591**
1.4	0.638	0.625	0.618	0.613	0.609	0.605	0.600	**0.595**	**0.593**
2.0	0.632	0.621	0.614	0.610	0.607	0.604	0.599	**0.597**	**0.595**
2.5	0.629	0.619	0.612	0.608	0.605	0.603	0.599	0.598	**0.596**
3.0	0.627	0.617	0.611	0.606	0.604	0.603	0.599	0.598	**0.597**
3.5	0.625	0.616	0.610	0.606	0.604	0.602	0.599	0.598	0.596
4.0	0.623	0.614	0.609	0.605	0.603	0.602	0.599	0.597	0.596
6.0	0.618	0.611	0.607	0.604	0.602	0.600	0.598	0.597	0.596
8.0	0.614	0.608	0.605	0.603	0.601	0.600	0.598	0.596	0.596
10.0	0.611	0.606	0.603	0.601	0.599	0.598	0.597	0.596	0.595
20.0	0.601	0.600	0.599	0.598	0.597	0.596	0.596	0.596	0.594
50.0	0.596	0.596	0.595	0.595	0.594	0.594	0.594	0.594	0.593
100.0	0.593	0.593	0.592	0.592	0.592	0.592	0.592	0.592	0.592

TABLE II.—DISCHARGE COEFFICIENTS FOR STANDARD SQUARE VERTICAL ORIFICES ACCORDING TO HAMILTON SMITH

Head on center of orifice in feet	Side of square in feet						
	0.02	0.04	0.07	0.10	0.20	0.60	1.00
0.4	....	0.643	0.628	0.621			
0.6	0.660	0.636	0.623	0.617	**0.605**	**0.598**	
0.8	0.652	0.631	0.620	0.615	0.605	**0.600**	**0.597**
1.0	0.648	0.628	0.618	0.613	0.605	**0.601**	**0.599**
1.4	0.642	0.623	0.614	0.610	0.605	**0.601**	**0.598**
2.0	0.637	0.619	0.613	0.608	0.605	**0.604**	**0.602**
2.5	0.634	0.617	0.610	0.607	0.605	0.604	**0.602**
3.0	0.632	0.616	0.609	0.607	0.605	0.604	**0.603**
3.5	0.630	0.615	0.609	0.607	0.605	0.604	0.602
4.0	0.628	0.614	0.608	0.606	0.605	0.603	0.602
6.0	0.623	0.612	0.607	0.605	0.604	0.603	0.602
8.0	0.619	0.610	0.606	0.605	0.604	0.603	0.602
10.0	0.616	0.608	0.605	0.604	0.603	0.602	0.601
20.0	0.606	0.604	0.602	0.602	0.602	0.601	0.600
50.0	0.602	0.601	0.601	0.600	0.600	0.600	0.599
100.0	0.599	0.598	0.598	0.598	0.598	0.598	0.598

56. Flow through Short Tubes.—A short tube, whose length is not more than two or three diameters, may be treated in the same manner as the orifice. If its length is much greater it becomes a pipe, the consideration of which will be taken up in a subsequent chapter. As a measuring device the tube is not as valuable as the standard orifice, since the coefficients of the latter are known with greater accuracy. It is possible to apply to any standard orifice coefficients taken from tables and to have some assurance as to the accuracy of the result. But the tube

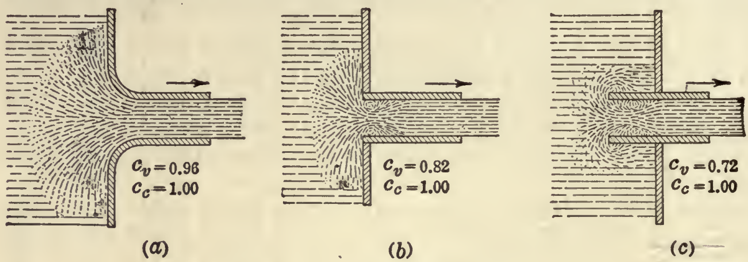


FIG. 71.—Coefficients for tubes.

cannot be standardized as accurately as the orifice and hence it is necessary to calibrate the tube itself if it is to be used for water measuring.

The coefficient of velocity is the highest in the case of the bell-mouthed tube, Fig. 71(a), since the eddy losses at entrance are reduced to a minimum. The greatest hydraulic friction loss is met with in the case of the re-entrant tube in Fig. 71(c); hence its velocity coefficient is the least. The contraction coefficients of all tubes are unity, provided the tubes have parallel sides.

EXAMPLES

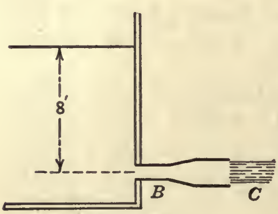


FIG. 72.

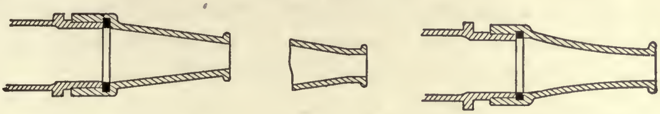
1. In Fig. 72 the diameter of the jet at C is 1.5 in. If the coefficient of discharge is 0.96, what is the rate of discharge?

2. At B in Fig. 72 the diameter is 1.0 in. while that at C is 1.5 in. What is the value of the velocity and pressure at B? (Assume no loss of head between B and C.)

3. In Fig. 72 if the pressure at B is to be absolute zero, what will be the velocity at that point? What would be the velocity at B if the tube were cut off at that section?

57. Flow through Nozzles.—A nozzle is a converging tube usually placed on the end of a pipe line or hose. It may be a

plain conical nozzle as in Fig. 73(a) or a smooth convex nozzle as shown in Fig. 73(c). The jet from a nozzle may undergo some



(a) Conical Nozzle (b) Straight Tip (c) Convex Nozzle

FIG. 73.—Standard Nozzles.

From a photograph by the author.

FIG. 74.—Jet from hydraulic giant washing out material for earth fill dam.

contraction or, if a small portion near the mouth is of uniform diameter as in Fig. 73(b), the water may leave in parallel lines and suffer no contraction.

Nozzles may be used as water-measuring devices the same as standard orifices, and are especially useful for that purpose when high heads are employed. They may also be used to furnish jets at high velocities for fire purposes, for power, or for hydraulic mining and similar work such as shown in Fig. 74.

In the case of a nozzle the water in the pipe leading to it usually flows with a velocity which is quite appreciable and it is desirable to consider this velocity of approach. Hence we shall derive an expression which takes this velocity head into account.

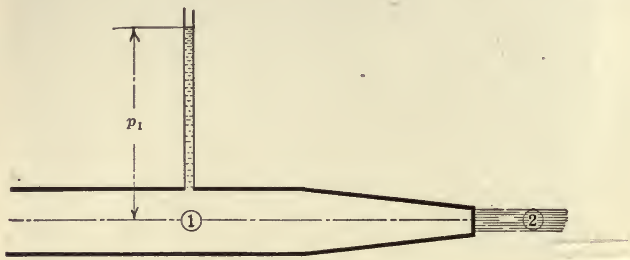


Fig. 75.

For points (1) and (2) of Fig. 75 we have $H_1 = p_1 + 0 + V_1^2/2g$, $H_2 = 0 + 0 + V_2^2/2g$, and $H'_{1-2} = kV_2^2/2g$. Now applying the general equation (23) we have

$$H_1 - kV_2^2/2g = V_2^2/2g.$$

From this

$$V_2 = \frac{1}{\sqrt{1+k}}\sqrt{2gH_1} = c_v\sqrt{2gH_1} \tag{31}$$

This is similar to the result for the orifice, but here we have $H_1 = p_1 + V_1^2/2g$ instead of $= h$ as in Art. 52. Inserting this value we have

$$V_2 = c_v\sqrt{2g\left(p_1 + \frac{V_1^2}{2g}\right)} \tag{32}$$

If the velocity in the pipe is known in some way this equation could be used directly. In case it is not known, we may proceed as follows:

Squaring both sides of equation (32) we have

$$V_2^2 = c_v^2 2gp_1 + c_v^2 V_1^2.$$

By the equation of continuity $V_1 = (F_2/F_1)V_2$. Substituting this in the above and transposing we have

$$[1 - c_v^2(F_2/F_1)^2]V_2^2 = c_v^2 2gp_1.$$

From this we obtain

$$V_2 = c_v\sqrt{\frac{2gp_1}{1 - c_v^2(F_2/F_1)^2}} \tag{33}$$

In this equation F_2 is the actual area of the jet at the point of minimum area. If the area of the nozzle opening be expressed by F_o, then $F_2 = c_cF_o$ and the latter expression could be inserted in (33). The rate of discharge is obtained by multiplying V_2 by F_2 or by c_cF_o as the case may be.

The velocity coefficients of well-made nozzles are very high, being practically equal to those of a standard circular orifice. We may reasonably assume an average value of the velocity coefficient of 0.98, though even this is often exceeded.[1]

The height to which a good fire stream can be thrown by a nozzle is from about two-thirds to three-fourths of the effective head at the base of the nozzle. The proportion is higher for large jets than for small ones, for smooth nozzles than rough ones, and for low pressures than for high pressures.

58. Efficiency of Nozzle.—Since a nozzle is frequently employed for power purposes, we may be interested in its efficiency. The efficiency of a nozzle may be defined as the ratio of the power in the jet to the power delivered to the nozzle. But we have seen that for a given rate of discharge, power is directly proportional to head. Thus referring to Fig. 75, we have

$$e = H_2/H_1.$$

But from equation (31) $H_2 = V_2^2/2g = c_v^2H_1$. From this it may be seen that

$$e = c_v^2 \tag{34}$$

This would be exactly true if all particles of water in the jet possessed the same velocity and hence the same kinetic energy. Actually V_2 is the average velocity of the jet, and it has been stated in Art. 47 that the true kinetic energy of a stream is greater than that obtained by using the square of the average velocity. Hence the true efficiency of a jet from a good nozzle may be about 1 per cent. more than the value given by (34).

[1] John R. Freeman, *Trans.* A. S. C. E., vol. 21, page 303 (1889); *Trans.* A. S. C. E., vol. 24, page 492 (1891).

W. R. Eckart, Jr., Inst. of Mech. Eng., Jan. 7, 1910.

V. R. Fleming, *Proc.* of Fifth Meeting of Ill. Water Supply Assoc., 1913.

R. L. Daugherty, "Hydraulic Turbines," page 69.

EXAMPLES

1. In Fig. 63 the actual measured diameter of the minimum section of the jet was $6\frac{13}{16}$ in., the area of the nozzle opening being 43.02 sq. in. Compute the coefficients of velocity, contraction, and discharge using the values of H_1 and V_2 given. What is the efficiency of the nozzle? What is the horsepower in the jet?

2. What is the value of the head lost in hydraulic friction in the nozzle of Fig. 63? What is the value of k?

3. The velocity of water in a 6-in. pipe is 12 ft. per sec. At the end of the pipe is a nozzle whose velocity coefficient is 0.98. If the pressure in the pipe at the base of the nozzle is 10 lb. per sq. in., what is the velocity of the jet? What is the diameter of the jet? What is the rate of discharge?

4. A jet 2 in. in diameter is discharged through a nozzle whose velocity coefficient is 0.98. In the pipe at the base of the nozzle there is a pressure of 10 lb. per sq. in., the diameter of the pipe at that point being 6 in. What is the velocity of the jet? What is the rate of discharge?

5. If the diameter of the jet in problem (4) were 1.0 in., all other data remaining the same, find the jet velocity and the rate of discharge.

59. Venturi Meter.—If water is caused to flow through the device shown in Fig. 76, the increased velocity through the "throat" will produce a corresponding pressure drop. This drop in pressure may be made to serve as a measure of the rate of discharge. Such an instrument is called a Venturi meter.

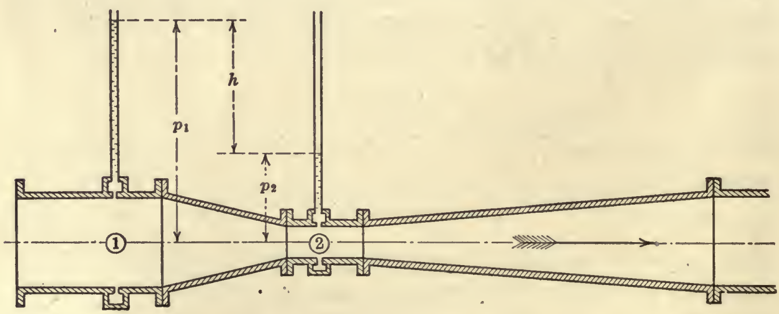


FIG. 76.—Venturi meter.

It may be seen that the Venturi meter is very similar in principle to the nozzle. In both there is an increase in velocity of the water accompanied by a corresponding pressure drop. And in both the rate of discharge may be found to be a function of the pressure drop. The only difference is that the pressure at the throat of the Venturi meter may be either somewhat greater or less than atmospheric, and the stream at that point is not a free jet but is expanded again to fill the pipe below the meter. Hence the

equations for the nozzle would seem to apply directly to the Venturi meter, with p_1 equal to the pressure drop in both cases.

Actually the coefficients for the Venturi meter are based upon a formula derived by a slightly different procedure. Thus we equate H_1 to H_2 assuming that H' is zero, and then introduce a coefficient as the very last step. Assuming the meter to be horizontal so that $z_1 = z_2$, we have

$$p_1 + V_1^2/2g = p_2 + V_2^2/2g.$$

From this,

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = p_1 - p_2 = h.$$

By the equation of continuity $V_1 = (F_2/F_1)V_2$, and hence

$$V_2 = \sqrt{\frac{2gh}{1 - (F_2/F_1)^2}}.$$

Since there is some slight loss of head between (1) and (2) the true velocity will be less than this and so we multiply it by a velocity coefficient. We then have

$$V_2 = c_v\sqrt{\frac{2gh}{1 - (F_2/F_1)^2}}. \tag{35}$$

This may be seen to differ from equation (33) in that the term $(F_2/F_1)^2$ is not multiplied by c_v^2, but both (33) and (35) could be made to yield the same numerical value by using a slightly different value of c_v for the two. Custom has based values of c_v upon (35) for the Venturi meter and upon (33) for the nozzle.

With the Venturi meter we desire q, not V_2, and hence, multiplying (35) by F_2 and replacing c_v by c, we have

$$q = cF_2\sqrt{\frac{2gh}{1 - (F_2/F_1)^2}} \tag{36}$$

For a given meter F_1 and F_2 are known quantities and, if $K' = F_2\sqrt{2g}/\sqrt{1 - (F_2/F_1)^2}$, this may be reduced to

$$q = cK'\sqrt{h} \tag{37}$$

The coefficient c may be assumed to be 0.985 for a new meter and 0.980 for an old one, the interior of which will be slightly rougher and perhaps reduced in area through incrustation. These factors will give a result that is very accurate. The coefficient is practically a constant, though there is some slight reason to

believe that it increases slightly with higher rates of discharge. If c is assumed constant for any given meter, it is convenient to replace cK' by K and we then have

$$q = K\sqrt{h} \tag{38}$$

The Venturi meter, invented by Clemens Herschel in 1886, affords a most valuable and accurate means of measuring water, especially in large quantities.[1] By a suitable recording device

Courtesy of Builder's Iron Foundry.

FIG. 77.—Venturi meter in wood pipe line.

it is possible to make a continuous record of the flow of water through any pipe line in which a Venturi meter is installed. The sole objection to its permanent use in a pipe is that it must necessarily cause some slight friction loss or resistance to flow. If this loss be expressed as $H' = kV_2^2/2g$, we find that values of k range from about 0.1 to 0.2. The higher values of k naturally go with smaller values of F_2/F_1.

The usual ratio of the diameter of the throat to the diameter of

[1] *Trans.* A. S. C. E., vol. 17, page 228 (1887).

the pipe is about 1 to 3, making the ratio $F_2/F_1 = 1/9$. But in order to reduce the resistance as much as possible and also to avoid producing pressures at the throat below atmospheric, it is quite common to make the diameters in the ratio of 1 to 2, making $F_2/F_1 = 1/4$. Of course this reduces the magnitude of h for a given rate of discharge and hence makes the readings less accurate, especially for very low discharges.

Courtesy of Builder's Iron Foundry.

FIG. 78.—Venturi meter of riveted steel.

A diverging stream is always less stable than a converging stream, that is it is more readily broken up into whirlpools and eddies, and hence more loss of energy takes place in the portion of the meter on the downstream side of the throat. In order to minimize this the down stream portion is made to taper much more gradually than the upstream side.

EXAMPLES

1. A Venturi meter with a 4-in. throat is to be used in a 12-in. pipe line. Assuming a value of $c = 0.985$, determine the value of K for this meter.

2. If a differential manometer employing mercury (sp. gr. = 13.57) were to be used, determine the value of K for the Venturi meter in problem (1), replacing h by y (Fig. 13) in inches of mercury.

3. Suppose the throat of the meter in problem (1) were to be 6 in. the pipe remaining 12 in. Compute the value of K.

4. Suppose that 5 cu. ft. per sec. is flowing through the Venturi meter. What are the values of h in problems (1) and (3)?

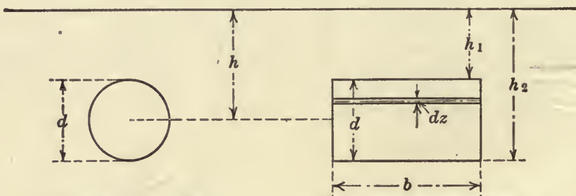
5. Water flows through a pipe line 6 ft. in diameter with a velocity of 7 ft. per sec. In this pipe line is installed a Venturi meter with a throat diameter of 2 ft. Assuming the value of k to be 0.12, what will be the loss of head caused by the meter? What will be the power lost?

60. Large Vertical Orifice.—In the case of an orifice whose vertical dimensions are large as compared with its depth below the free surface it is necessary to proceed as follows: Choose an elementary area dF such that all portions are at the same depth z below the free surface. Now by Art. 52 the rate of discharge through this strip may be expressed as

$$dq = c\sqrt{2gz}\,dF \tag{39}$$

The rate of discharge through the entire orifice may be obtained by integrating equation (39). Thus

$$q = c\sqrt{2g}\int z^{1/2}dF \tag{40}$$



(a) Circular Orifice (b) Rectangular Orifice

FIG. 79.

In the case of a rectangular orifice (Fig. 79) we may take $dF = bdz$ and after integrating obtain

$$q = c\tfrac{2}{3}\sqrt{2g}\,b(h_2^{3/2} - h_1^{3/2}) \tag{41}$$

If h = depth of center of gravity below the free surface we may write

$$h_2 = h + d/2;\ h_1 = h - d/2.$$

Expanding $(h + d/2)^{3/2}$ and $(h - d/2)^{3/2}$ by the binomial theorem and substituting in equation (41) we obtain

$$q = cbd\sqrt{2gh}\left[1 - \frac{d^2}{96h^2} - \frac{d^4}{2048h^4} - \cdot\cdot\cdot\right] \tag{42}$$

The expression in brackets is a rapidly converging series and its value is always less than unity. When $h = d$, the value of this factor is 0.989, while for $h = 2d$, its value becomes 0.997.[1] Thus for any head greater than $2d$ the rate of discharge may be obtained by the simpler formula $q = cF\sqrt{2gh}$.

[1] Russell, "Text-book on Hydraulics," page 60.

In similar manner the rate of discharge through a circular orifice of area F is given by the expression

$$q = cF\sqrt{2gh}\left[1 - \frac{d^2}{128h^2} - \frac{5d^4}{16,384h^4} - \cdot \cdot \cdot \right] \qquad (43)$$

It may be found that when $h = 2d$ the value of this series is 0.998, thus indicating that the use of the exact formula is unnec-

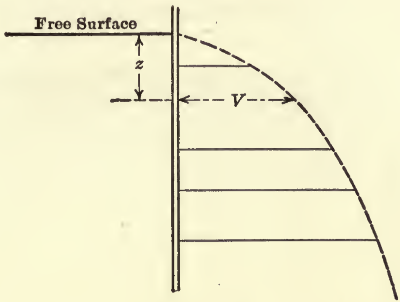


FIG. 80.

essary for heads above that value. In Tables I and II the coefficients in black-face type are to be used in the exact formulas, all other coefficients to be used in the formulas of Art. 52.

61. Weir.—A weir is a special form of orifice, its distinguishing feature being that it is placed at the water surface so that the head on its upper edge is zero. Thus the usual formulas for ori-

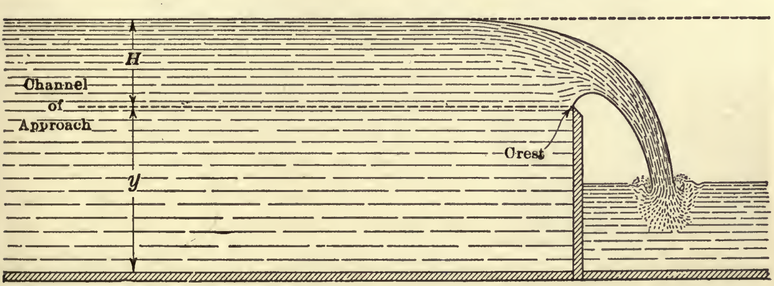
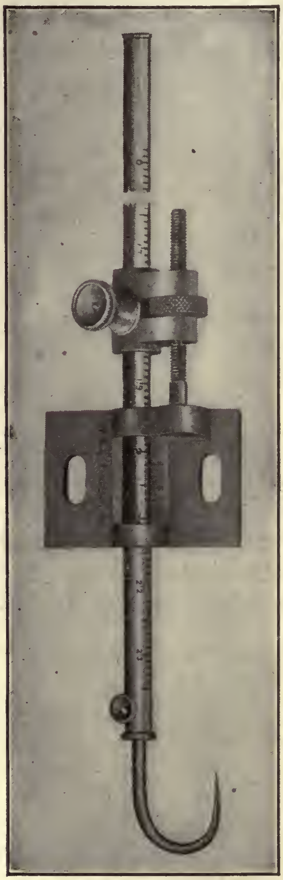


FIG. 81.—Weir.

fices given in Art. 52 can no longer be applied and the methods of Art. 60 must be employed. The weir is one of the most widely accepted standard devices for the measurement of water.

If it be assumed that the velocity of water through an orifice varies as the square root of the depth, the curve in Fig. 80 would give a true representation of the flow. However, the particles of

water at the surface of the weir opening do not remain at rest but flow with considerable velocity. It may also be observed that the level of the water at this point drops below its normal value, as shown in Fig. 81. Also it must be noted that the stream lines flowing through the weir are not necessarily normal to the plane of the weir; hence it is hardly correct to multiply their velocities by areas in the plane of the latter. For these and other reasons it is impossible to derive by theory weir formulas which are exactly correct, but they serve as expressions which may be made to yield correct results by the proper choice and use of experimental coefficients.

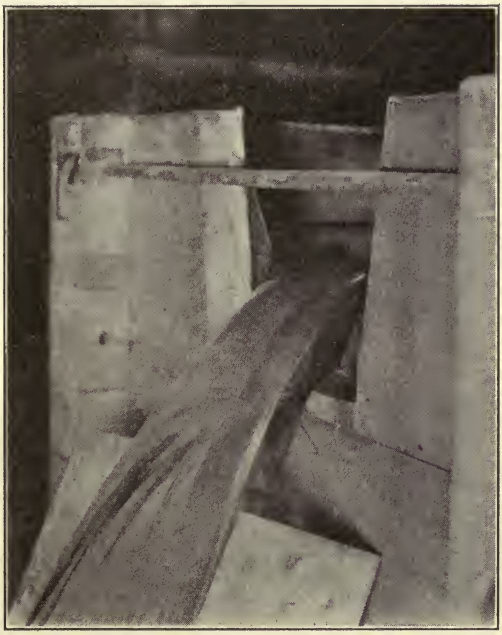


W. & L. E. Gurley.
FIG. 82.—Hook gage.

It might seem natural to measure the depth of water flowing over the crest of a weir, but in practice it is difficult to do this with any degree of accuracy. It is found more feasible to measure the elevation, above the weir crest, of the water surface at some distance back from the weir, where the water is relatively quiet. Thus all weir formulas express the rate of discharge as a function of H (Fig. 81). This measurement must be taken at a point far enough back to avoid the effects of the surface curve. This distance should be at least $6H$. The usual instrument for measuring H is the hook gage, one form of which is shown in Fig. 82. The gage should be mounted on some rigid support. In using it the sharp-pointed hook is submerged beneath the surface and then carefully raised until a slight distortion may be seen on the water surface. The hook should then be lowered until this distortion barely disappears. From this reading the value of H is obtained by subtracting the "hook gage zero," which is the reading of the gage when its point is just level with the crest of the weir, as the lower edge is called.

The Triangular Weir.—The triangular weir such as is shown in Fig. 83 is useful for measuring relatively small rates of discharge, as a reasonable value of H may be obtained by employing a sufficiently small vertex angle. But for discharges much above 2 or 3 cu. ft. per sec. excessively high values of H are necessary and other types of weir would then be used.

The Suppressed Rectangular Weir.—Probably the most common type of weir is one whose shape is rectangular. If the width of



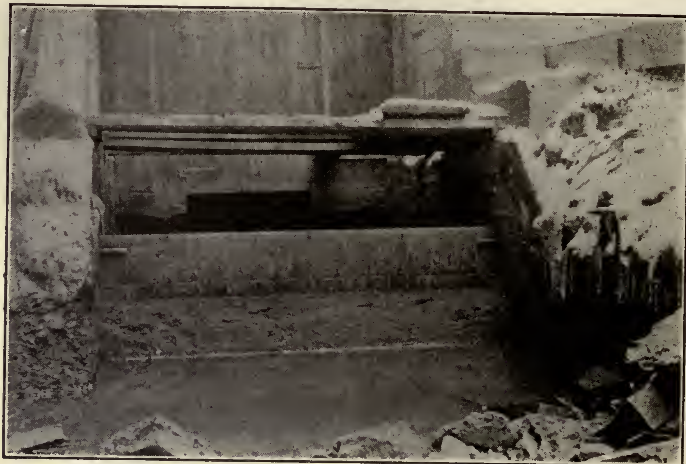
From a photograph by the author.

FIG. 83.—Discharge from a 60° triangular weir.

the weir is the same as that of the channel of approach, as in Figs. 84 and 85, the stream of water flowing over the crest will not undergo any lateral contraction, that is the end contractions are suppressed. With this type of weir it is customary to extend the sides of the channel beyond the crest so that the falling stream is bounded by them. If these sides are not so extended the stream will expand somewhat and the discharge for a given value of H will be slightly larger than in the standard type.

It is necessary in this or any type of weir to insure that the weir is "ventilated," that is that air has access to the under side of the

falling water. Otherwise the air will be gradually swept out and the water will tend to cling to the face of the weir instead of springing clear of it. For a given value of H the rate of discharge



From a photograph by the author.
FIG. 84.—Rectangular weir without end contractions.

From a photograph by the author.
FIG. 85.—Rectangular weir without end contractions.

would then be greatly increased and the usual coefficients would no longer apply.

In order to insure that the water shall spring clear, that is that

perfect crest contraction shall be attained, it is necessary to have a sharp edge on the weir plate. This may be produced by beveling the edge of a metal plate down to a knife edge. However, a

From a photograph by F. H. Fowler.

FIG. 86.—Rectangular weir with end contractions.

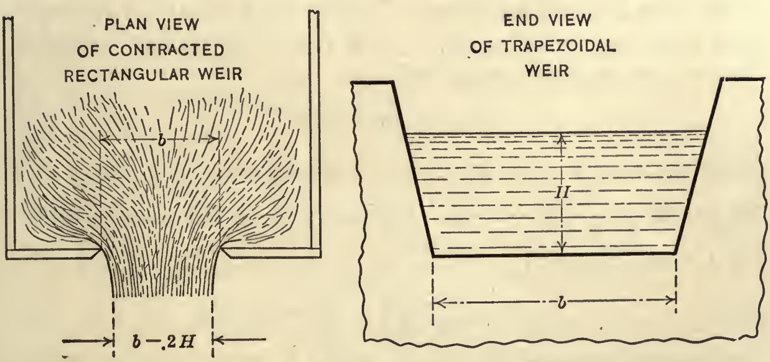


FIG. 87.

perfectly sharp square shoulder is just as good as a knife edge if the plate is not too thick for the water to clear the other shoulder. For very low values of H the knife edge would still permit crest contraction to take place when the flat edge of a plate would not.

Contracted Rectangular Weir.—When the width of the weir is less than that of the channel of approach, as in Fig. 86, the contraction that occurs at each end causes the real width of the stream of water to be less than that of the weir itself. Such a weir is called a contracted weir.

Trapezoidal or Cippoletti Weir.—The trapezoidal weir is one in which the sides of the notch are not vertical but diverge so that the width at the water surface increases. If the side slopes have the ratio of 1:4 the weir is called a Cippoletti after an Italian engineer of that name who proposed it. The advantage of this type of weir will be stated later.

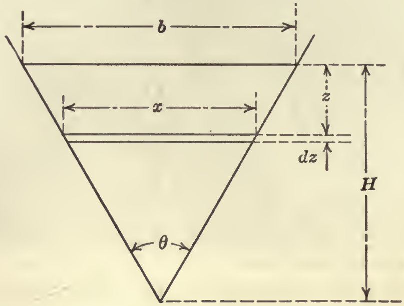


FIG. 88.

62. The Triangular Weir.—In Fig. 88 we have a triangular weir with any vertex angle θ. The rate of discharge through an elementary strip of area dF will be

$$dq = c\sqrt{2gz}\,dF.$$

Now $dF = xdz$ and by similar triangles $x:b = (H - z):H$. Hence $dF = \frac{b}{H}(H - z)dz$. Substituting in the above we have for the entire notch

$$q = c\sqrt{2g}\,\frac{b}{H}\int_0^H (H - z)z^{1/2}dz.$$

Integrating between limits we have

$$q = c\sqrt{2g}\,\frac{b}{H}\,[2/3 H^{5/2} - 2/5 H^{5/2}].$$

But $b = 2H \tan \theta/2$. Inserting this and reducing we have

$$q = \frac{8}{15}c\sqrt{2g}\tan\frac{\theta}{2}H^{5/2} \tag{44}$$

This expression for any given weir may be reduced to

$$q = KH^{5/2} \tag{45}$$

In Figs. 89 and 90 may be found experimental values of K for several triangular weirs. The 54°, 60°, and one of the 90° weirs are in the laboratory of Sibley College.[1] The two lower curves in Fig. 90 were plotted from data in a very valuable paper by James Barr.[2] The weir for which the very lowest curve was constructed

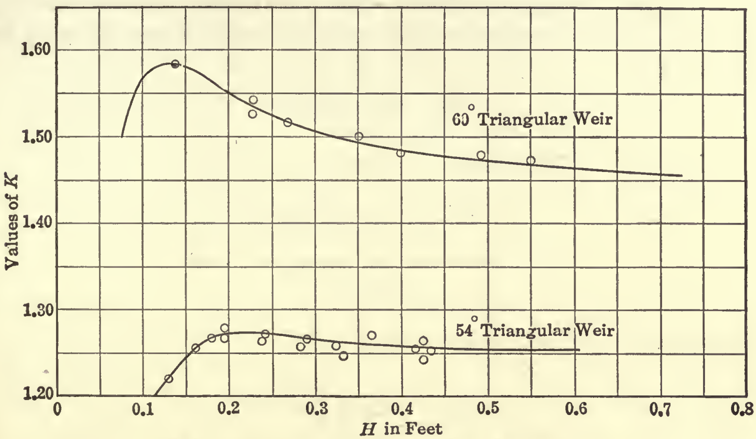


FIG. 89.—Coefficients of triangular weirs.

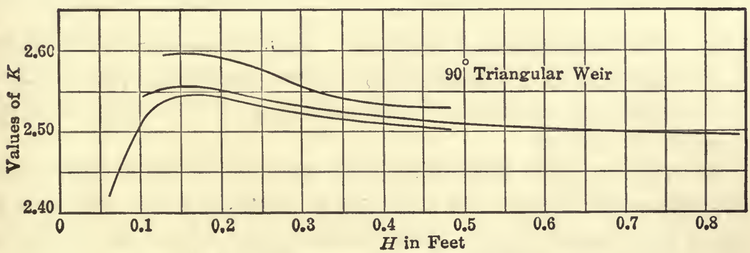


FIG. 90.—Coefficients of 90° triangular weirs.

had a very fine sharp edge, while the other weir had a square corner and a thickness of about 1/16 in. Both of these weirs have values of K below that of the Sibley College weir but the difference, of about 1 per cent., may be due to different methods of measurement of the water and of H.

[1] *Engineering News*, vol. 73, page 636 (1915).

[2] "Experiments upon the Flow of Water over Triangular Notches." *Engineering*, Apr. 8 and 15, 1910.

These curves show either that the discharge does not vary as the five-halves power of H or that c is not a constant. Thompson, who first employed the triangular weir, chose for K a value of 2.54. The value of 2.65 that so many writers persist in giving is entirely too high and is based upon Thompson's first experiments which were not accurately performed.

(In finding $H^{5/2}$ on some slide rules, it is well to note that $H^{5/2} = H^2\sqrt{H}$.)

63. The Rectangular Weir.—For the rectangular weir in Fig. 91 the discharge through the elementary strip of area dF may be given by

$$dq = c\sqrt{2gz}\,bdz.$$

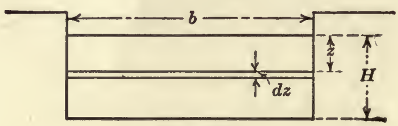


FIG. 91.

Integrating between limits we have

$$q = c\sqrt{2g}\,b\int_0^H z^{1/2}\,dz$$

$$q = \frac{2}{3}c\sqrt{2g}\,bH^{3/2} \tag{46}$$

This is the fundamental formula. Many variations of it have been suggested in an attempt to express the value of c, which is not necessarily a constant for all values of H. (It may be noted that $H^{3/2} = H\sqrt{H}$.)

64. The Francis Weir Formula.—About 1850 Mr. James B. Francis made some very accurate investigations of the flow of water over weirs.[1] As a result of his experiments he selected a value of 0.622 for c in equation (46), so that for a suppressed weir we have

$$q = 3.33bH^{3/2} \tag{47}$$

With a contracted weir he concluded that the effect of each contraction was to reduce the effective width of the weir by $0.1H$. Thus for a contracted weir we have

$$q = 3.33(b - 0.1\,nH)H^{3/2} \tag{48}$$

[1] "Lowell Hydraulic Experiments."

The usual type of contracted weir will have two end contractions, giving n a value of 2 in equation (48), but we might have a weir with one end contracted and one end without contraction. Equation (48) is strictly empirical and applicable only within limits. If H is greater than one-third b it is impossible for perfect end contraction to occur and hence the conditions upon which (48) is based no longer exist.

When the cross-section area of the channel of approach is relatively small, there may be a velocity of flow in it that is high enough to affect the result. This velocity is called the velocity of approach. Francis corrected for this by replacing $H^{3/2}$ in both (47) and (48) by $[(H + h_v)^{3/2} - h_v^{3/2}]$, where h_v is the velocity head in the channel. In practical work the last term is often dropped. There is no real good theoretical foundation for any expression involving velocity of approach. A modified Francis formula for the suppressed weir is

$$q = 3.33b\ (H + \alpha h_v)^{3/2} \tag{47a}$$

in which α is given values ranging from 1 to 2. If the velocity of water in the channel at the section where H is measured is uniform over the cross-section, a value of 1 should be used for α. But if the surface velocity is much higher than the bottom velocity, the value of α should be greater than unity. This is because the true velocity head which affects the discharge over the weir is greater than the h_v which is based upon the average velocity in the section. The value of α is sometimes taken as the ratio of the surface velocity to the average velocity. Note that the area of the section is the product of the total width of the channel by $(H + y)$ in Fig. 81. In using equation (47a) first solve (47) to obtain an approximate value of q. Divide this by the area of the section, where the hook gage is located, and from this velocity an approximate value of V, and hence h_v, can be obtained. This αh_v is to be added to H and a new and somewhat larger value of q calculated. From this a new value of h_v could be computed, and so on. However, after about two such solutions it would be found that further solution would alter the result very slightly.

65. The Bazin Weir Formula.—Bazin, in France, made a valuable series of experiments upon weirs without end contractions and with high velocities of approach. From these he devised a weir formula which expresses the effect of velocity of approach in a much less awkward manner than the Francis formula. His most

accurate formula is rather complicated, but for practical work the following approximate formula is sufficient:

$$q = \left[3.25 + \frac{0.0789}{H}\right]\left[1 + 0.55\left(\frac{H}{H+y}\right)^2\right]bH^{3/2} \qquad (49)$$

The quantity y indicates the height of the weir crest above the bottom of the channel and thus introduces the effect of velocity of approach into the formula in an indirect manner.

66. Comments on Weirs.—The formula (48) for the contracted weir is applicable only to the standard Francis weir, whose limiting proportions are shown in Fig. 92. Unless there is sufficient space left at the two ends and at the bottom of the weir, perfect contraction will not be obtained. These dimensions may be

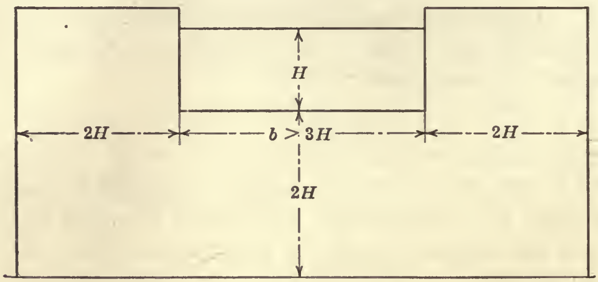


FIG. 92.—Limiting proportions of standard contracted weirs.

made greater than the values given, but never less. The height of the crest above the bottom should preferably be at least $3H$. Francis estimated that a height of $2H$ would increase the discharge about one per cent. If sufficient space cannot be had to secure perfect end contraction, the end contractions should be entirely suppressed, or one of them suppressed and all the space given to the other end. There is no known coefficient or method for dealing with imperfect contraction. Also the value of H should not exceed one-third of the width b.

For the suppressed weir there are no standard dimensions to be observed. Experiments at Cornell University have indicated that the Francis coefficient in equation (47) is applicable for heads up to at least 5 ft. and for heads down to 0.3 ft. For $H = 0.2$ ft. it should be increased 3 per cent., and for $H = 0.1$ ft. it should be increased 7 per cent.

If the area of the channel of approach exceeds $6bH$ it can be shown that the velocity of approach is negligible. Hence it is

seen that velocity of approach need not be considered in a contracted rectangular weir. But with a suppressed weir the depth of the channel would have to be $6H$, and that is not often the case. Hence most suppressed weirs have a velocity of approach that needs to be considered. The Francis coefficient was based upon work with weirs having a velocity of approach less than 1 ft. per sec. When the velocity of approach is high, the formula of Bazin should be applied.

The most accurate type of weir is a suppressed weir with such a deep channel of approach that the velocity of approach is negligible. A contracted weir for which the velocity of approach is negligible is about in the same class with a suppressed weir with a moderate velocity of approach. End contractions have been held to be a source of error and there appears to be no truly rational way to correct for them. The least desirable type of weir is the one with a high velocity of approach because of the difficulty not only of reading H accurately but also of allowing for the effect of this velocity in a scientific manner.

It almost goes without saying that a weir should be set with its crest level and its plane vertical. An inclination upstream decreases and an inclination downstream increases the discharge for a given H. The crest should be sharp and in good condition.

In using weirs for accurate work it is desirable to study the original experiments upon which the formulas are based and use the formula that has been derived under circumstances most nearly like those in hand. And it is likewise desirable to duplicate the original investigator's methods. The hook gage, for instance, should be located in the same way and at the same distance from the weir. Unless these precautions are followed one has no assurance that the coefficients given fit his own case.[1]

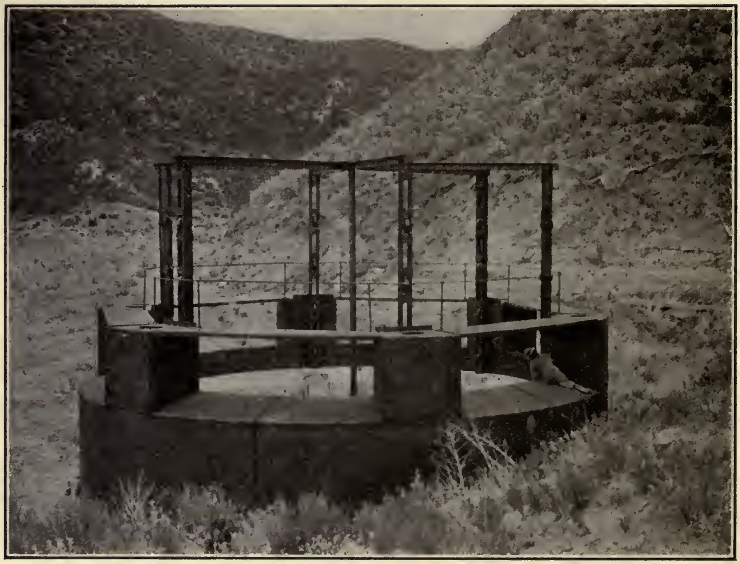
67. The Cippoletti Weir.—In order to avoid the trouble of correcting for end contractions, the sides of the Cippoletti weir are given such a batter (1 :4) that they add enough to the effective width of the stream to offset the contraction $0.2H$ of the con-

[1] For information on weirs see:
"Weir Experiments, Coefficients, and Formulas," by R. E. Horton, U. S. G. S. Water Supply and Irrigation Paper, No. 150, Revised No. 200.
Hughes and Safford, "Hydraulics."
Parker, "The Control of Water."

tracted Francis weir. Thus computations may be made upon the basis of the width b at the crest by the following formula

$$q = 3.367bH^{3/2} \tag{50}$$

68. Special Weirs.—There are other types of weirs for special purposes. Thus we have a floating circular intake weir in Fig. 93. The purpose of this is not for accurate water measurement, but rather to float in such a manner in the reservoir that a certain



From a photograph by F. H. Fowler.

Fig. 93.—Floating circular intake weir for Los Angeles aqueduct.

depth of water continually flows through or over its passageways and down the intake in the center. Thus no matter how the water level in the reservoir rises and falls a uniform quantity will automatically be delivered to the intake.

69. The Pitot Tube.—Among other water-measuring devices is the Pitot tube. This is an instrument which indicates the velocity of water at a point. From the velocity the rate of discharge may be obtained.

The principle of the Pitot tube is illustrated in Fig. 94 and its theory will be discussed in a subsequent chapter. For an open stream only a single tube is necessary, but in a stream of water under pressure a second tube is necessary to record the pressure

alone. The quantity desired is the difference between the two readings, which we shall call h. It can be shown by correct theory that if h is the value in feet of water of the dynamic pressure exerted by the impact of the stream against the opening of the tube, the velocity of the water is given by

$$V = \sqrt{2gh} \tag{51}$$

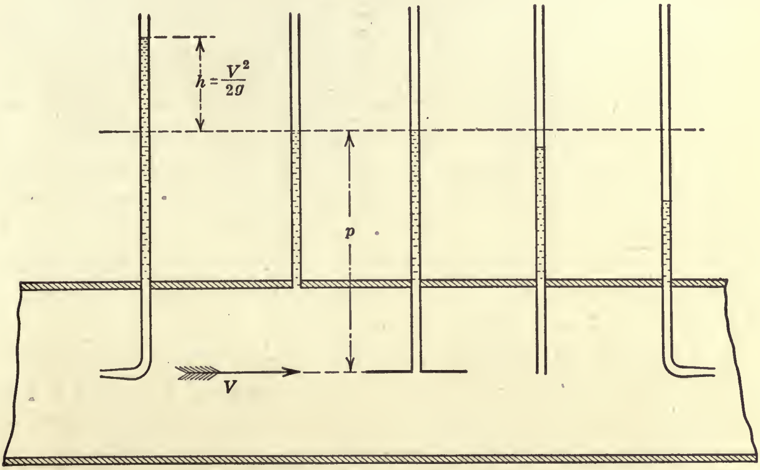


FIG. 94.

This has been found by experiment to be true when there is smooth stream line flow, but in case of turbulent flow we should introduce a coefficient whose value is about 0.977, so that we should write

$$V = 0.977\sqrt{2gh} \tag{52}$$

The fact that this coefficient is anything less than unity is not because our theory is at fault nor because of any defect in the instrument itself, but is due to the fact that the instrument records the true velocity at the point while we desire, for practical purposes, the axial component of velocity. Hence the factor is designed to give us the axial component of velocity (*OB* in Fig. 52 rather than *OD*).[1]

In using the Pitot tube it is often convenient to divide a cross-section of the stream up into parts of equal area and to determine the velocity in the center of each area. The average velocity of

[1] L. F. Moody, "Measurement of the Velocity of Flowing Water." *Proc. of the Engineer's Soc. of W. Penn.*, vol. 30, page 319 (1914).

the stream will be the average of the observed velocities. But if the areas are not equal the average of the velocities will have no significance. It will then be necessary either to plot a curve from which velocities at other points may be taken or to multiply each observed velocity by the area which it may be assumed to represent. The total rate of discharge of the entire stream is the sum of all such partial discharges. Thus

$$q = \Sigma F'V' \tag{53}$$

where F' is a portion of the total area and V' is the velocity through that area. If the average velocity is desired, it can be obtained by dividing the rate of discharge by the total area.

70. The Current Meter.—For moderate velocities such as are found in canals and natural streams the current meter is well adapted. It consists of a wheel, as in Fig. 95, or in other types a screw, which is rotated by the action of the water. By calibration the relation is determined between the velocity of the water and the rate of rotation of the meter.

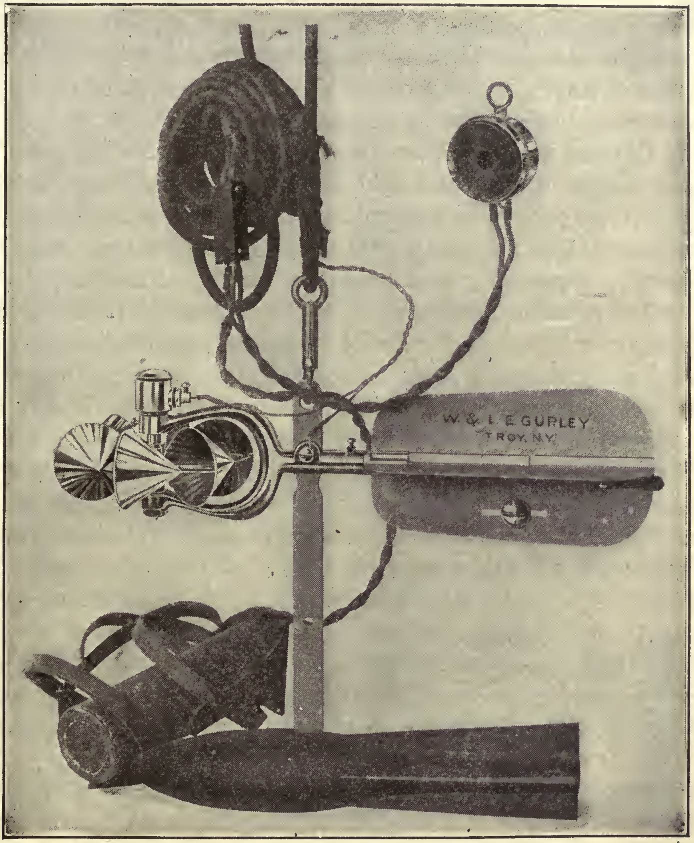
In many current meters each revolution is recorded by a click in a telephone receiver at the ear of the observer, the click being produced by the wheel making an electric contact every revolution. In most meters the contact is not made so frequently, every ten revolutions being the number commonly recorded. Other types of meter have some form of mechanical recording device. It is generally better to determine the time necessary for a given number of revolutions rather than to attempt to find the number of revolutions made in some specified time, owing to the difficulty of estimating fractions of a revolution or fractions of the number of revolutions that may be recorded as a unit.

Current meters may be roughly divided into two classes, those with the axis vertical, as in Fig. 95, and those with the axis horizontal.

In comparatively shallow water the meter may be rigidly fastened to a rod, and in this case the weight and tail, as shown in Fig. 95, are unnecessary. But for deeper water where the meter is suspended by a cable the latter are required to hold the meter in the proper position.

Generally it is desired to find the velocity of the water flowing across some sectional area. If the stream lines are not perpendicular to the area in question, it is the normal component of the

velocity that is desired rather than the value of the actual velocity. It may be seen that the type of meter shown in Fig. 95 will rotate with equal velocity no matter from which horizontal direction the water may come. It will also be rotated by a current that is vertical, or parallel to its axis. And in any case the



Courtesy of W. & L. E. Curley.

Fig. 95.—Current meter.

rotation is always in the same direction. Thus this meter tends to record the value of the velocity regardless of its direction.

In other types, generally with the axis horizontal and the wheel made in some form similar to a screw propeller, the meter records only the component of the velocity parallel to its axis. And in case the meter is located in a portion of the stream where an

eddy causes a reverse current the meter will then give a negative reading, since it will be rotated in the opposite direction. Such a type of meter is more accurate in all cases where the flow is irregular or turbulent. However, the type shown in the figure is of excellent mechanical construction and is widely used. For many cases where the stream flow is fairly regular and extreme accuracy is not required, it is quite satisfactory.

In using the current meter the velocities are determined at a number of different points and the total discharge of the entire stream computed in the same manner as in Art. 69.

71. Comments on Measurement of Water.—The accurate measurement of rate of discharge is one of the most difficult problems in practical hydraulics. The only positive way of measuring rate of discharge is to weigh the amount of water discharged in a given time or to determine its volume in suitably calibrated tanks or reservoirs. The former method is applicable only for relatively small rates of discharge, and facilities for the latter are seldom to be had. Also in the latter method the effect of leakage, evaporation, and other factors may sometimes prove troublesome.

The methods that are usually employed are the ones that have been given in this chapter. They are all indirect in that we assume the velocity or the rate of discharge to be a function of some other quantity which can be measured.

The discharge of water from any tank can be measured by an orifice, tube, or nozzle. When a stream of water flows in an open channel it may be caused to flow over a weir or its velocity throughout any cross-section may be found by a current meter, by floats, or other means. For a stream of water confined within a closed pipe we may use a Pitot tube to determine the velocity across a cross-section or cause the water to flow through a Venturi meter. At the end of a pipe line we might place a nozzle which would also permit the rate of discharge to be obtained. The discharge from a nozzle may be computed or it may be measured directly by determining the velocity of the jet with a Pitot tube. The means of measurement that is to be used depends upon the circumstances.

In addition to the methods of measurement that have been described in this chapter, there are other methods, especially chemical methods. One of these is simply a matter of discharging a small quantity of highly colored liquid into the intake of a pipe line and noting the time that it takes for the discoloration to

be noted at the other end. Knowing the length of pipe it is easy to compute the velocity of the water. Another valuable method consists of adding a strong salt solution at a known definite rate. Samples of water are taken at a down stream section and analyzed. Knowing the strength of the solution used, its rate of discharge, and the amount of dilution in the main stream the rate of discharge in the latter may be determined This method has been used in some cases with a high degree of accuracy and it may offer an easy, cheap, and convenient way of measuring rate of discharge of large quantities of water.[1]

Where water flows over a spillway dam the latter may be used as a special type of weir. The same weir formula as given in equation (46) may be applied, if the proper value of the coefficient is known. Since the spillway crest may be of various shapes and dimensions, it is not a standard piece of apparatus like the sharp crested weir. Hence the value of the coefficient has to be determined for each case either by calibrating the spillway in question or using the results of observations upon another spillway of similar form.

72. Discharge under Varying Head.—If the head varies, the rate of discharge will likewise vary and the total discharge in a given time, or the time required for a given total discharge, must be determined as follows.

Let Q = the total volume in cubic feet of any given body of water, while q = cu. ft. per second as usual. Then

$$q = dQ/dt \text{ or } dQ = qdt.$$

Suppose that into this body of water in question there is an inflow at the rate of q_1 cu. ft. per second, while water flows out at the rate of q_2 cu. ft. per second. It then follows that the change in the total volume in any time dt is

$$dQ = q_1dt - q_2dt.$$

Also let A = the area of the water surface of the body in question while dz = the change in the level of the surface. Then

$$dQ = Adz$$

Equating these two expressions for dQ we have

$$Adz = q_1dt - q_2dt \tag{54}$$

Now either q_1 or q_2 or both may be variable and functions of z the variable height of the water surface, or one of the two may

[1] "Chemihydrometry and Precise Turbine Testing" by B. F. Groat in the *Trans.* A. S. C. E., Vol. lxxx, p. 951 (1916).

have a constant value or be equal to zero. For instance if the water is discharged through an orifice or pipe line of area F under the head z, we may write

$$q_2 = cF\sqrt{2gz},$$

while if it overflows a weir or spillway dam of width b we have

$$q_2 = Kbz^{3/2}$$

In the former case z might correspond to the h in Fig. 96 while the value of c would be determined from the principles of Art. 74 and subsequent articles in case the discharge takes place through a pipe line. In the case of flow over a spillway the z would be the height of the surface of the water above the crest or in other words would correspond to the H in Fig. 81 or of equation 46. And in like manner q_1 may also be some function of z.

Equation (54) is perfectly general and if it is possible to express A, q_1, and q_2 as mathematical functions of z, it may then be possible to solve the problem by integration. In other cases the integral may be evaluated by graphical methods. For example from equation (54) we may write

$$t = \int_{z_1}^{z_2} \frac{A\,dz}{q_1 - q_2}.$$

By integration this will give us the time required for the water level to change from z_1 to z_2. If it cannot be integrated by calculus we may do it graphically by computing values of q_1 and q_2 and plotting values of $A/(q_1 - q_2)$ against corresponding values of z. The area between this curve and the z axis is the value of the integral. Of course without actually plotting it, the value of the area may be computed by various rules of approximation.

73. PROBLEMS

1. What is the rate of discharge of a 54° triangular weir when $H = 0.400$ ft.? With the same value of H what would be the rate of discharge of a 90° triangular weir?

2. What would be the value of H for a rate of discharge of 2.0 cu. ft. per sec., if a 60° triangular weir is used? Assume $K = 1.45$.

3. The width of the weir in Fig. 84 is 7.573 ft. Neglecting velocity of approach, what is the rate of discharge when $H = 1.200$ ft.?

4. In problem (3), if the value of y is 2.85 ft., solve for the rate of discharge by using the Francis formula with $\alpha = 1.0$. Solve with $\alpha = 2.0$.

5. Solve problem (4) by the use of Bazin's formula.

6. Assume that the weir in Fig. 86 is also 7.573 ft. in width. What would be the rate of discharge when $H = 1.200$ ft.?

7. Assume that a Pitot tube and a piezometer tube are connected to two sides of a differential manometer containing mercury. Suppose the Pitot tube is placed in such positions in the stream, which is 10 in. in diameter, that it measures the velocities in five areas of equal magnitude. Suppose these five readings on the differential manometer are 1.50, 2.15, 2.84, 3.62, and 4.05 in. of mercury. Find the rate of discharge of the stream.

8. Suppose that a ship lock in a canal has a uniform area of water surface at all depths of water, what would be the integration of equation (54)?

9. Suppose that a ship lock in a canal is of uniform rectangular cross-section and that it is 300 ft. by 90 ft. by 40 ft. deep. Suppose that the water from this lock is discharged through a tunnel which is 3 ft. in diameter, the coefficient of discharge being 0.50. If the initial head under which water discharges is 35 ft., how long will it take for the level to drop 25 ft.?

10. In the problem (9) how large would the tunnel have to be to permit the water level to drop from 35 ft. to 10 ft. in 15 min.?

11. Water enters a reservoir at a uniform rate of 150 cu. ft. per second and flows out over a spillway whose length of crest is 100 ft. The value of K for this spillway is 3.45. Areas of water surface at various elevations above the crest of the spillway are given in the adjoining table. (*a*) Find the time required for the level to drop from 3 ft. to 1 ft. above the crest. (*b*) Find the final elevation after equilibrium is established. (*c*) How long a time will it take for equilibrium to be established?

Ans. (*a*) 2052 seconds. (*b*) 0.573 ft. (*c*) Infinite time theoretically.

z (ft.)	A (sq. ft.)
3.00	860,000
2.50	830,000
2.00	720,000
1.50	590,000
1.25	535,000
1.00	480,000

12. The spillway of a reservoir is 40 ft. long and is of such a form that $K = 3.50$. There is a constant inflow into the reservoir of 300 cu. ft. per second. Areas of water surface are given in the adjoining table. (*a*) What will be the height of the water surface for equilibrium? (*b*) Starting with the water level 3 ft. below the crest of the spillway, how long will it take for the water to rise until the height of the water surface is 1.50 ft. above the crest?

Ana. (*a*) 1.66 ft. (*b*) 3 hrs. 49 min.

z (ft.)	A (sq. ft.)
−3.0	500,000
−2.0	530,000
−1.0	560,000
0.0	600,000
+0.5	650,000
+1.0	700,000
+1.5	740,000

CHAPTER VII

FLOW THROUGH PIPES

74. Loss of Head in Pipe Friction.—In dealing with such devices as the orifice, nozzle, Venturi meter, etc., we have compensated for the effect of frictional resistance to flow by the introduction of velocity coefficients. This is feasible because all of these devices can be standardized so that the coefficients which have been determined for one may be applied to another of the same type. We might have velocity coefficients for pipes also if the latter were more nearly alike. But actually pipe lines differ from each other in length, size, degrees of roughness, and other respects to such an extent that the application of velocity coefficients is impractical. Therefore it is necessary to proceed on a different basis.

In Art. 44 it is stated that the loss of head which always accompanies flow may be expressed as

$$H' = k\frac{V^n}{2g} \tag{55}$$

In the case of a pipe line, or any water conduit of any length, it is apparent that the loss of head between two sections is a function of the distance between them and that the factor k should correspond to the roughness of the surface of the channel. Mathematical analysis as well as experimental evidence indicates that, other quantities being equal, the friction is less in a large conduit than in a small one. It is also found that hydraulic friction is independent of the pressure and the temperature effect is so slight that it can be neglected.

In order to express k as a function of the size of the channel we need some dimension which can be used for all shapes of cross-section. In the case of a circular pipe alone we might use the diameter, but this would not be applicable for other shapes. The quantity that is used for this purpose is the ratio

$$m = \frac{\text{area of water cross-section}}{\text{length of wetted perimeter}} \tag{56}$$

This is a linear dimension and is called the "hydraulic mean depth" or the "hydraulic radius." Its physical meaning, so far as it has any, is that m is the depth of water necessary on a plane surface of width equal to the length of the wetted perimeter, so that the imaginary volume of water thus formed shall be equal to the actual volume.

Mathematical analysis and experimental investigation have led to an approximate empirical formula

$$k = \frac{f}{4}\frac{l}{m} \tag{57}$$

where f is a friction cœfficient which depends upon the roughness of the surface as well as upon other factors.

Experimental evidence indicates that the value of n in equation (55) varies from about 1.75 to 2.00, the former holding for very smooth surfaces and the latter for rough ones such as the interiors of iron and steel pipes after years of service.[1] As shown in Art. 84 it is possible to select values of n for different kinds of surfaces but ordinarily this is not done since it is difficult to express degree of roughness with any precision and often the degree of roughness cannot be known or estimated. Hence for the sake of simplicity in computation the value $n = 2.00$ is usually used.

Assuming $n = 2.00$ and inserting the value of k given by (57) in (55), we have,

$$H' = \frac{f}{4}\frac{l}{m}\frac{V^2}{2g} \tag{58}$$

If the channel is a circular pipe flowing full of water the value of m is

$$m = \pi r^2/2\pi r = r/2 = d/4.$$

Inserting this value of m in equation (58) we have for a circular pipe full of water,

$$H' = f\frac{l}{d}\frac{V^2}{2g} \tag{59}$$

If equation (59) were entirely correct we should expect that f would depend only upon the roughness of the surface. But we have already seen that the loss of head does not always vary as the square of the velocity. If the actual exponent of V is less than 2, the value of f in equation (59) would have to decrease with increasing velocity. But for rough surfaces, where the

[1] For velocities below the critical the loss of head varies as the first power of the velocity. It is then also a function of temperature.

exponent is practically 2, the value of f in (59) should be independent of the velocity. Fortunately the variation of f with V is not very great in most cases of actual practice and may be neglected in the present treatment. It has also been found that the loss of head does not vary inversely as the first power of d but rather as $d^{1.25}$. Thus if equation (59) is used, the value of f should be made to decrease as d increases.

According to the experiments of Darcy the value of f for new, clean, cast-iron pipes may be given by

$$f = 0.02 + \frac{0.02}{d''} \tag{60}$$

For old, corroded, cast-iron pipe the values given by (60) should be doubled, but it is impossible to formulate any definite law by which the value of f should be increased with age. It depends to some extent upon the chemical composition of the water carried. In the case of a smooth wood-stave pipe the value of f should be somewhat less than that given by (60) and it does not increase with age. For a riveted steel pipe the values of f are slightly greater than given by (60). But roughness cannot be expressed mathematically and the selection of f for any given case is largely a matter of judgment.[1]

It should be noted that in (60) the value of d'' is in inches, but f is an abstract number. In equation (59) we have another abstract number, the ratio of the length to the diameter. Thus l and d should both be in the same units.

EXAMPLES

1. What will be the loss of head in a 10-in. pipe line 2,000 ft. long, when the velocity of the water in the pipe is 6 ft. per sec.?
Ans. 29.5 ft.

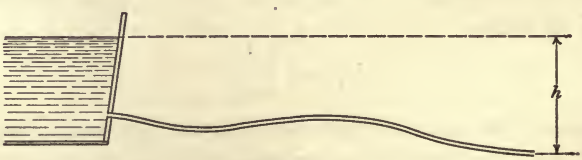


FIG. 96.

2. Suppose that the pipe of problem (1) is shown in Fig. 96, that there are no losses save those due to pipe friction, and that the size of the stream discharging at the end is the same as that of the pipe. What value of h will be required to produce the flow in problem (1)? *Ans.* 30 ft.

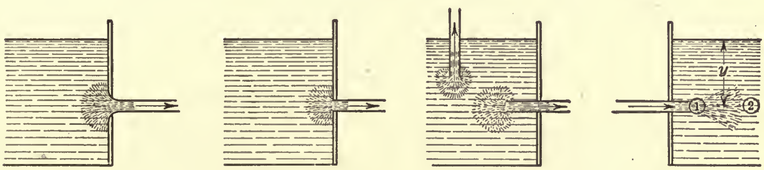
[1] Unless otherwise specified, all problems in this text will be based upon the value of f given by (60) for the sake of uniformity.

3. Suppose we were to express the velocity of discharge from the pipe line in Fig. 96 as $V = c_v\sqrt{2gh}$. With the same data as in problem (1) what would be the value of c_v?

Ans. $c_v = 0.1365$.

4. What will be the pressure at a point 1,000 ft. from the end of the pipe in Fig. 96, if the point is located 10 ft. above the mouth of the pipe? The length of pipe is 2,000 ft., the diameter is 6 in., and the value of h is 50 ft.

75. Loss of Head at Entrance.—Whenever the velocity of a flowing stream is abruptly altered there will be eddy currents set up which will cause a certain loss of head due to the internal friction of the particles of water against each other. Thus when water flows into a pipe from a reservoir, the loss of head within the first few feet may be much greater than that due to pipe friction alone in that same distance. This additional



(a) $k=0$ (b) $k=0.5$ (c) $k=1.0$ (d) $k=1.0$

FIG. 97.—Entrance and discharge losses.

loss is called entrance loss. It is estimated to be about the same as the loss of head in a short tube such as those shown in Fig. 71. Using the velocity coefficients which are there given and determining values of k from them, it may be found that for a bell-mouthed intake the entrance loss is very small and so it is usually neglected. For a pipe that is flush with the surface of the reservoir wall, such as Fig. 97(*b*), the value of k is taken to be 0.5, while for a projecting pipe, such as in Fig. 97(*c*), the value of k is assumed to be 1.0. These are not the precise values that are obtained from the values of the velocity coefficients, but are close enough for practical purposes when it is realized that the entrance losses are often very small as compared with the other losses of head.

If V indicates the velocity in the pipe itself, the losses of head at entrance may be assumed to be:

For a non-projecting pipe

$$H' = 0.5\frac{V^2}{2g} \tag{61}$$

For a projecting pipe

$$H' = 1.0 \frac{V^2}{2g} \tag{62}$$

76. Loss of Head at Discharge.—In the case of a pipe discharging into a body of water at rest, as in Fig. 97(*d*), the entire kinetic energy of the stream may be lost,[1] for considering the body of water in Fig. 97(*d*) to be so large that the velocity at (2) is negligible we may write,

$$H_1 = y + 0 + V^2/2g, \qquad H_2 = y + 0 + 0.$$

Then

$$H'_{1-2} = H_1 - H_2$$

or

$$H' = \frac{V^2}{2g} \tag{63}$$

77. Loss of Head in Nozzle.—Although a nozzle does not produce an abrupt change of velocity, it nevertheless causes a

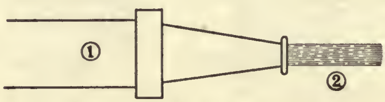


FIG. 98.—Loss in nozzle.

certain loss of head by virtue of which its velocity coefficient is less than unity. For Fig. 98 we may write $H' = H_1 - H_2$. Since $V_2 = c_v\sqrt{2gH_1}$,

$$H_1 = \frac{1}{c_v^2}\frac{V_2^2}{2g}$$

and

$$H_2 = \frac{V_2^2}{2g}.$$

Therefore for the nozzle

$$H' = \left(\frac{1}{c_v^2} - 1\right)\frac{V_2^2}{2g} \tag{64}$$

giving for k the value

$$k = \frac{1}{c_v^2} - 1$$

exactly as in the case of the orifice in Art. 52. Note that in equation (64) the loss of head in the nozzle is based upon the jet velocity.

[1] Unpublished experimental work by L. F. Moody of Rensselaer Polytechnic Institute indicates that in some cases at least there may be a certain amount of diffusion so that only about 70 per cent. of the kinetic energy is lost, the rest being converted into pressure.

78. Other Minor Losses of Head.—When there is an abrupt contraction of the stream as in Fig. 99(*a*), there is a loss of head $H' = kV_2^2/2g$, where k has the values given in the following table:[1]

TABLE III

$\frac{F_2}{F_1}$	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
k	0.362	0.338	0.308	0.267	0.221	0.164	0.105	0.053	0.015

When there is an abrupt enlargement of the stream as in Fig. 99(*b*), theory and experiment indicate that the loss of head may be approximately represented by

$$H' = (V_1 - V_2)^2/2g$$
$$= \left(\frac{F_2}{F_1} - 1\right)^2 \frac{V_2^2}{2g}$$

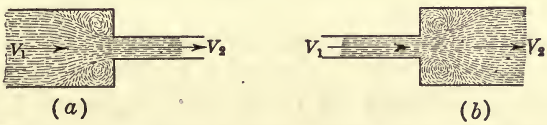


FIG. 99.—Losses due to abrupt change in cross-section.

In the case of an obstruction in a pipe the loss of head is usually assumed to be largely that due to expansion after passing through the constricted part and hence is computed by the method just given.

It may be noted that all of the losses of head may be represented as some function of the velocity head. In many cases the loss may be small as compared to other losses in the same pipe and hence its omission involves little error. As a general rule it may be stated that any feature in the pipe line which disturbs or changes the velocity of flow induces some additional friction loss. Exact and reliable equations or coefficients for many of these losses are lacking and no attempt will be made to give more of them here.[2]

[1] L. M. Hoskins, "Hydraulics," page 74. After data from Weisbach.

[2] See Hughes and Safford, "Hydraulics;" Gibson, "Hydraulics and Its Applications;" Lea, "Hydraulics."

79. Flow through Long Pipe Line.—The strict application of the general equation

$$H_1 - H_2 = H'$$

would require us to express H' as a function of all the various losses that might exist in a certain pipe line. This procedure is followed in the case of short pipes but in the case of a long pipe line, whose length is at least one thousand diameters, it will usually be found that the loss in pipe friction alone renders the others insignificant. For we have just seen that all the losses in a pipe line may be expressed in the form

$$H' = k\frac{V^2}{2g}$$

and in the case of entrance, discharge, and other similar minor losses values of k are either less than unity or but very little greater. But for pipe friction alone we have seen that

$$k = f\frac{l}{d}$$

and if l is only great enough the magnitude of the quantity fl/d may make all other values of k negligible by comparison. In view of the uncertainty of the exact value of f as well as other loss factors, too great a degree of refinement is unwarranted.

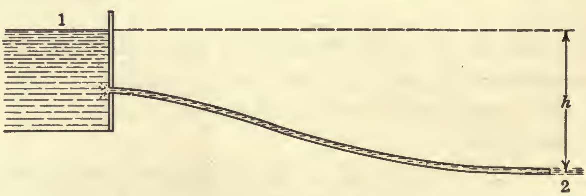


FIG. 100.

Assume a pipe line discharging freely into the air at (2) in Fig. 100. We may write for this

$$H_1 = h, \qquad H_2 = V^2/2g.$$

If the length be great enough all other losses save those due to pipe friction may be neglected and hence between (1) and (2), $H' = f\frac{l}{d}\frac{V^2}{2g}$. Thus from the general equation we have

$$h - \frac{V^2}{2g} = f\frac{l}{d}\frac{V^2}{2g}.$$

Therefore $h = \left(1 + f\frac{l}{d}\right)\frac{V^2}{2g}$. However, if the length be great enough $\left(f\,\frac{l}{d}\right)$ may be so much greater than unity that the expression

$$h = f\frac{l}{d}\frac{V^2}{2g} \tag{65}$$

is often used. But it should be noted that this is applicable only when the pipe line is long and when the velocity of the stream at (2) is no greater than that in the pipe itself. Equation (65) is equivalent to the assumption that $H' = h$.

Inspection of equation (65) shows that for a given head and length of pipe, the velocity will vary somewhat with the diameter of the pipe. For by equation (60) f decreases as d increases and in (65) the ratio l/d also becomes smaller with larger diameters; therefore the entire coefficient of $V^2/2g$ becomes smaller as the size of the pipe increases. Hence for the same value of h, V will increase as the diameter of the pipe increases, and it may be shown that V varies as $d^{0.5 \text{ to } 0.6}$.

EXAMPLES

1. Suppose that in Fig. 100 the pipe projects into the reservoir at entrance and discharges freely into the air at (2), the size of the jet being equal to the diameter of the pipe. If h = 40 ft., d'' = 12 in., and l = 50 ft., compute the rate of discharge considering all losses.

Ans. 22.7 cu. ft. per sec.

2. In problem (1) if l = 1,000 ft., all other data remaining the same, compute the rate of discharge considering all losses. Compute the rate of discharge by the approximate method, neglecting minor losses.

Ans. 8.2 cu. ft. per sec.; 8.56 cu. ft. per sec.

3. Suppose in Fig. 100 that a nozzle on the end of the pipe line discharges a jet which is 2.5 in. in diameter. Assume the velocity coefficient of the nozzle to be 0.95. If h = 260 ft., d'' = 10 in., and l = 5,000 ft., find the rate of discharge.

Ans. 3.45 cu. ft. per sec.

4. Find the rate of discharge in problem (3) if the nozzle were removed so that the pipe would discharge freely a stream of 10 in. diameter.

Ans. 6.11 cu. ft. per sec.

5. In problem (3) find the pressure head in the pipe at the base of the nozzle.

Ans. 176.8 ft.

80. Hydraulic Gradient.—If a piezometer tube be erected at B in Fig. 101, the water will rise in it to some height BB' equal to the pressure head existing at that point. If the lower end of the pipe were closed so that no flow could occur, the

height of this column would evidently be BM. The drop from M to B' that is found when flow takes place is due to two factors, one of these being that a portion of the pressure has been converted into the velocity head which the water has at B and the other that there has been a loss of head through friction between A and B.

If a series of water piezometers were erected along the pipe line, the water would rise in them to various levels. The line drawn through the summits of such an imaginary series of water columns is called the *hydraulic grade line* or the *hydraulic gradient.* It is seen that this line is an indication of the pressure

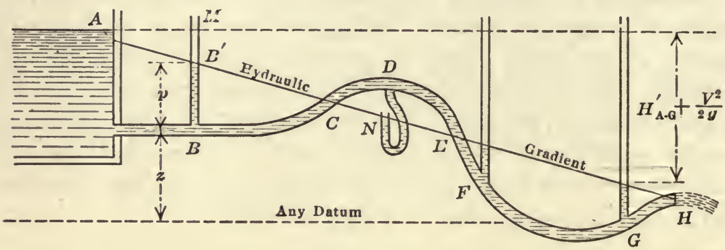


FIG. 101.—Hydraulic gradient.

variation along the pipe. Thus at any point the vertical distance from the pipe line to the hydraulic gradient is the pressure at that point. Since at C this distance is zero it follows that at C the pressure is atmospheric. And at D the line is below the pipe indicating that at the point in question the pressure is below that of the atmosphere and is equal to $-DN$. The advantage of the construction of the hydraulic gradient is that it gives a very clear picture of the pressure variation along a pipe line. Also in practical applications the profile of a proposed pipe line should be drawn to scale. Then by computing a few points only the hydraulic gradient can be drawn and from it the pressures at all points can be readily measured.

The vertical distance from the hydraulic gradient to the level of the water surface at A represents $H' + V^2/2g$. Hence the position of the hydraulic gradient is independent of the position of the pipe line. Thus it is not always necessary to compute pressures at various points in order to plot the gradient. Instead values of $H' + V^2/2g$ may be laid off below the proper horizontal line, and this procedure is often more convenient. It is usually necessary to locate only a very few points and often only two,

the terminal points, are sufficient. For example if Fig. 101 represents the profile of a pipe of uniform diameter drawn to scale, the hydraulic gradient can readily be drawn as follows. At the intake to the pipe there will be a drop below the level of the surface of the water which should be laid off equal to the sum of $V^2/2g$ plus the entrance loss. At H the pressure is known to be zero gage pressure and hence the gradient must pass through the end of the pipe. In the case shown the hydraulic gradient is practically a straight line and hence may be drawn at once from these two points. The location of other points, as B', may be computed if desired. In the case of a long pipe line the velocity may be such that the drop in the gradient at the entrance is very small and hence the error is very slight if the gradient is drawn as a straight line from the surface of the water above the intake to the lower end of the pipe.

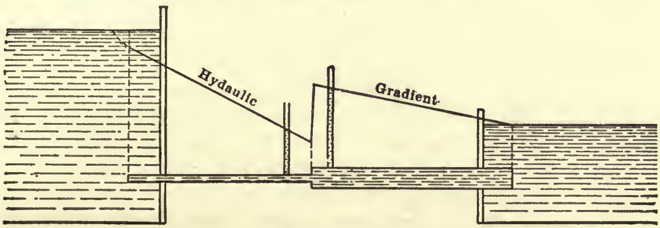


Fig. 102.—Hydraulic gradient.

The hydraulic gradient is not necessarily a straight line. For a pipe of uniform diameter it will be a straight line only if the pipe itself is straight. If the pipe is of uniform diameter the drop in the hydraulic gradient along its length is then a measure of the loss of head and this will be proportional to the horizontal distances in the figure only when the latter in turn are proportional to the actual lengths of pipe. But for ordinary amounts of curvature the hydraulic gradient will deviate but very little from a straight line. Of course if there are losses of head aside from those due to ordinary pipe friction there will be abrupt drops in it, and any variations in velocity head due to changes in diameter affect the hydraulic gradient.

It may be seen that if the velocity head is constant the drop in the hydraulic gradient between any two points is the measure of the loss of head between those two points. And the slope of the gradient is a measure of the rate of loss. Thus in Fig. 102 the rate of loss is much less in the larger pipe than in the smaller one. If the velocity changes, the hydraulic gradient might actu-

ally rise in the direction of flow as may be seen in both Figs. 102 and 102*a*. Additional illustrations of the hydraulic gradient for other cases are to be seen in Figs. 105, 108, and 109.

It is sometimes instructive to represent not only the variation of pressure head but also the variation of total head. If any arbitrary datum plane is assumed, the vertical distance from it to any point in the pipe represents the elevation head for that point. And the vertical distance from this point to the hydraulic

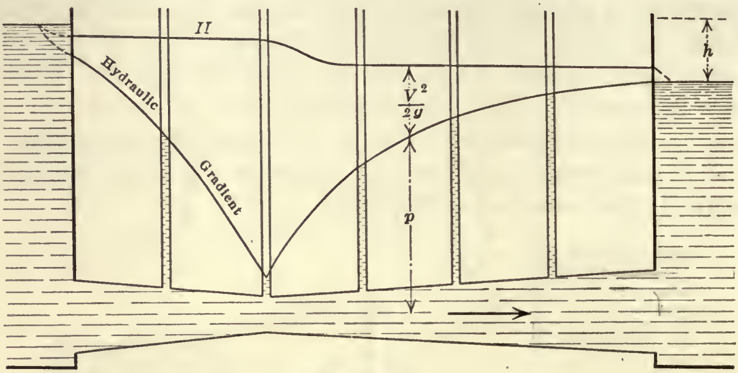


FIG. 102a.

gradient represents the pressure head. Hence the vertical distance from the datum plane to the hydraulic grade line represents the sum of pressure head plus elevation head. If to this we add the velocity head a curve is obtained, as shown in Fig. 102*a*, the ordinates of which represent total head or energy. And, as in the case of the hydraulic gradient, the location of this total head curve is independent of the position of the pipe and may be plotted by laying off values of loss of head below a horizontal line. The particular one shown, plotted from experiments made by the author, shows that the chief loss of head in a Venturi meter takes place just beyond the throat. The total loss of head between the two tanks is h and both entrance and discharge losses are here represented.

The drop in the gradient at entrance to a pipe depends both upon the velocity head and the entrance loss. But in Fig. 102 the gradient ends at the water surface directly above the discharge end of the pipe. This means that the pressure at this end is equal to the depth of water. The discharge loss means not a loss of pressure head, but a loss of velocity head and hence it is not shown by the gradient. If the velocity head were not all

lost, a portion of it would have to be converted into pressure head and this could take place only if the pressure at the end of the pipe were less than in the still water some distance away. And if this were so the water surface would have to be as shown in Fig. 102*b*. In other words the case would be similar to what would be found if the end of the pipe were produced in a diverging form as shown by the dotted lines.

It may be observed that the hydraulic gradient in all cases represents what would be the free surface, if one could exist and maintain the same conditions of flow.

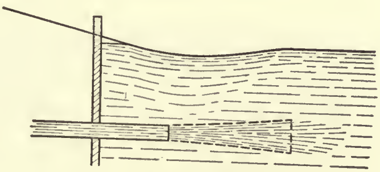


Fig. 102b.

EXAMPLES

1. Draw the hydraulic gradients for problems (3) and (4) of Art. 79.

2. In Fig. 102 assume the pipe to be of a uniform diameter of 20 in. and 500 ft. in length. The difference in level of the two water surfaces is 30 ft. Consider all losses. Find the distance the hydraulic gradient drops below the surface of the water at a point just within the pipe at entrance.

Ans. 7.23 ft.

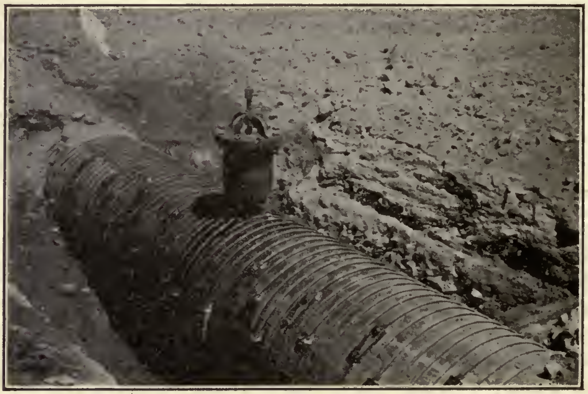
3. In Fig. 101 suppose the horizontal distance from the intake of the pipe to D is 300 ft. and to H is 900 ft. The vertical distance from the reservoir level to D is 20 ft. and to H is 100 ft. Suppose that at H the water does not discharge freely into the air but the conditions are such that the pressure at H is 52 ft. Draw hydraulic gradient neglecting slight drop at entrance, plot profile of pipe line (sketching portions BC and EFG at pleasure), and find pressure head at D.

Ans. 4 ft.

81. Effect of Air at Summit.—In Fig. 101 is shown a pipe line having a "summit" at D, which is above the hydraulic gradient, indicating that the pressure at this point is less than atmospheric. In practice this would be avoided, for not only might the excess external pressure cause this portion of the pipe to collapse, but the accumulation of air at this point might interfere with or even stop the flow entirely. All ordinary water carries air in solution and readily gives it up at a point of low pressure so that air would collect in time, though it were all expelled by some means in the beginning. Therefore in designing a pipe line, whenever any portion of it is found to be above

the hydraulic gradient, an attempt would be made to change the profile so that this may be avoided. In case this is impossible then provision must be made for exhausting the air occasionally, if full flow is to be maintained.

If the summit is below the hydraulic gradient, air could still collect, though not so readily since water under pressure tends to absorb air. But under such conditions it is very easy to release the air, since it will escape if an opportunity is offered it. A valve for such a purpose is shown in Fig. 103. Such valves usually have a float, the dropping of which, as air collects



Courtesy Redwood Manufacturers Co.

FIG. 103.—Air valve on wooden pipe line.

and lowers the water surface, causes a valve to open. When the air escapes, the water level rises and the float closes the valve again. The valve in Fig. 103 is also constructed so as to admit air into the pipe in case a vacuum should accidentally occur in any way. This will prevent the pipe from collapsing in such an event. In many cases it is highly desirable that pipe lines be furnished with suitable air valves for both these purposes.

In Fig. 104 is shown how a vacuum might accidentally occur, when normally the pipe is under a positive pressure. We have seen that the greater the velocity of flow through a pipe line the less the pressure will be at any point. Hence if some event, such as the bursting of the pipe at C, permits a larger flow of water, the hydraulic gradient will be much steeper than normal. This means that it will be lowered, and it may be lowered sufficiently to be below portions of the pipe as in Fig. 104.

Also if the admission of water to a pipe line is shut off by the closure of a gate valve at intake, the water which is already in the pipe will tend to run out. As no more water can get in to take its place, a vacuum might be created unless air were admitted. Hence, some device is usually provided just below the valve at the intake, and in some cases at other points, to admit air upon such an occasion.

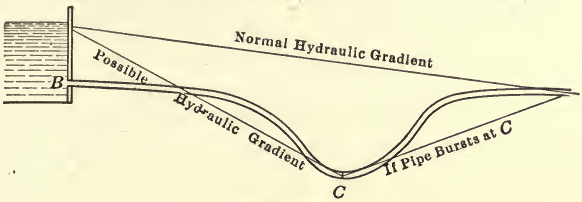


FIG. 104.

82. Hydraulic Slope.—If the velocity in a pipe line is constant the drop in the hydraulic gradient is equal to the loss of head. The ratio

$$s = \frac{H'}{l} \tag{66}$$

is called the *hydraulic slope.* If there were no loss of head the hydraulic gradient for a pipe of uniform diameter would be a horizontal line. Hence the steepness of this line, or the magnitude of s, is a measure of the rate of loss.

83. Chezy's Formula.—Equation (58), expressing the loss of head due to pipe friction may often be written and used in another form known as Chezy's formula. Thus (58) is

$$H' = \frac{f}{4}\frac{l}{m}\frac{V^2}{2g}.$$

Solving this equation for V^2 we have

$$V^2 = \frac{8g}{f} m \frac{H'}{l}$$

For a given channel $8g/f$ may be a constant whose value we shall indicate by c^2 so that

$$c = \sqrt{\frac{8g}{f}} \tag{67}$$

By equation (66) $H'/l = s$, the slope of the hydraulic gradient, hence

$$V = c\sqrt{ms} \tag{68}$$

It may be seen that equation (68) can be used only for a long pipe line to which equation (65) could be applied. If it is desired to consider other losses of head aside from pipe friction Chezy's formula cannot be used.

84. Other Formulas for Pipe Friction.—It has already been pointed out that equation (59) is known not to be correct in form but is widely used because of ease of computation, its defects being covered by suitable values of f. The true equation is of the form

$$H' = f' \frac{l}{d^x} V^n \tag{69}$$

In this equation f' would be independent of both d and V and would depend only upon the nature of the pipe wall. The value of x is given as 1.25, though occasionally other values differing slightly from this are to be found. The value of n ranges from about 1.75 to 2.00 depending upon the nature of the surface. In practical cases the selection of f' and n is about as difficult as the choice of f in equation (59). On account of the greater ease of computation with equation (59) it is likely that it will continue to be used and that more precise ways of expressing f will be determined.[1]

In regard to equation (60) it has been established that values of f for cast-iron pipes are really somewhat less than given by Darcy's experiments, but this discrepancy should be looked upon as a small factor of safety. The formula itself errs on the side of safety.

We also have various exponential forms of Chezy's formula, equation (68), and in these also the exponents are variables but the coefficient itself depends only upon the nature of the surface. The one most commonly used in this country is the Hazen-Williams formula in which values of the exponents for average conditions are used. This equation is of the form

$$V = c'm^{0.63}s^{0.54} \tag{70}$$

In order to facilitate computation by this formula a set of tables has been prepared and also a special slide rule constructed.[2]

Moritz gives the following as the practice of the U. S. Reclamation Service;

$$q = c''d^{2.7}(1000\ s)^{0.555}$$

[1] For a method of doing this see article by R. Biel, *Zeit. des Ver. deut. Ing.*, June 27, July 4, 1908.

[2] Williams and Hazen, "Hydraulic Tables."

where c'' has the values given Table IV. Since s is the slope of the hydraulic gradient, $1000\,s$ will be the drop of the hydraulic gradient in feet per thousand feet of pipe.

TABLE IV

Values of c''

Wood stave pipe	1.35
Cast iron pipe	1.31
Concrete pipe	1.24
Riveted steel pipe	1.18

Although the Kutter and Bazin formulas for determining the coefficient c in equation (68) were intended by their authors to be applied to open channels, they have been widely used for closed pipes as well. This has been due to the fact that experimental data has been lacking for many cases of pipe lines, especially in large sizes, and the Kutter and Bazin formulas are of a general nature that would seem to make them fit a wide range of conditions. These formulas are given in the following chapter.

85. Values of Friction Factors.—Values of f given by equation (60) are applicable only to pipes under 2 or 3 ft. in diameter since the largest size used by Darcy in his experiments was about 20 in. It has also been stated that equation (60) gives values of f that are now known to be somewhat too high for smooth, cast-iron pipes, but are conservative values that can be used in design. In Table V will be found some values that are more nearly correct, though these values are for large pipes. The form of equation (60) shows that for large pipes the value of f will be nearly independent of the diameter, and if the pipe surface be sufficiently rough for the loss of head to vary as V^2, it will be independent of the velocity also.

Though the values given in Table V are for large pipes only, they will serve to give an idea of the correct values of f even in the case of small pipes. It is difficult to give values of f that can be applied to small pipes because variations in the roughness of small pipes are more serious in their effects. Furthermore the walls of a metal pipe become covered with tubercles or scaly deposits in the course of time. After a depth of about 2 in. has accumulated this action ceases. But a deposit of 2 in. is much more serious in reducing the capacity of a small pipe than it is in the case of a large pipe.

Values of c in the table are average values for use in Chezy's formula, equation (68), though methods of computing c may be found in the next chapter. The values of c' in Table V are for use in the Hazen-Williams formula, equation (70). These values are merely selected as typical for the classes of surface given in the table. The values of n in Table V are to be used in the formulas of Kutter and Manning, which are to be found in the following chapter.

TABLE V

	f	c	c'	n
New, smooth, cast-iron pipe	0.015	130	170	0.011
Wood-stave pipe, new or old	0.018	120	160	0.012
New riveted pipe	0.022	110	145	0.013
Old, tuberculated, cast-iron pipe or riveted pipe	0.026	100	130	0.014
Any old and rough pipe	0.040	80	105	0.019

86. Size of Pipe for Given Discharge.—It is possible to find the diameter of pipe necessary for a given rate of discharge by direct solution, but a fifth degree equation is involved.[1] It will therefore be found slightly easier and simpler to solve for the diameter by a method of "cut and try" rather than to attempt the solution of the higher degree equation, though the latter is not difficult.

The procedure of "cut and try" is first to assume any diameter that seems reasonable and by the usual methods compute the actual rate of discharge that this diameter would give. Then compare this with the value of q desired and if it is too large or too small assume a new size of pipe and repeat until the computed rate of discharge is approximately equal to that specified. Of course one should use only commercial sizes of pipe and not attempt to get a diameter to a fraction of an inch. Naturally if only commercial sizes of pipe are used the computed rate of discharge may not agree precisely with the value desired, and so the size pipe that gives the nearest to that should be chosen. If the conditions of the problem are such that at least a certain rate of discharge must be obtained, then a size pipe should be

[1] If f is expressed by (60) this equation will be of the sixth degree.

used which will give a slightly greater value than this rather than one just under it. In the problems of the text sizes of pipe are used in whole inches. Actual standard pipe dimensions are given on page 263.

If good judgment is used, one should be able to get the correct answer within two or three trials at most. In order to do this it is necessary to carefully compare the rate of discharge computed with the value specified and then to estimate how much the area of the pipe might need to be increased or diminished to yield the proper result. In doing this it must be borne in mind that the velocity is not the same in pipes of different sizes. The larger the pipe the higher the velocity of flow and hence it will discharge more in proportion to its area than a smaller pipe. This is not a matter that is worth making any computations on for this purpose, but it might be noted that, all other things being equal, the rate of discharge varies about as $d^{2.6}$. Therefore in assuming a new diameter one should not go quite so far from the former value as one would if the velocity remained the same in value, in which case q would vary as d^2.

As an example, suppose it is desired to find the diameter of pipe necessary to discharge 9 sec. ft. with a total fall of 50 ft. in a distance of 2 miles. Since the length is so great we may use the approximate formula, equation (65). Suppose we assume $d'' = 10$ in., then $f = 0.0220$ and

$$0.0220 \frac{10{,}560 \times 12}{10} \frac{V^2}{2g} = 50.$$

Solving this we find that

$$V = 3.40 \text{ ft. per sec.}$$

and

$$q = 0.545 \times 3.40 = 1.85 \text{ sec. ft.}$$

The value desired, of 9 sec. ft., is 9/1.85 or about 5 times the result obtained. If the velocity were unchanged we should require a pipe whose area was 5 times that of a 10-in. pipe. But we know that the velocity will be somewhat larger with a larger pipe, and so we would assume a pipe whose area is about 4 times that of the 10-in. pipe, trusting that the increase in velocity will make up the difference. Now this pipe would have a diameter the square of which would be about 4 times 10^2 or 400.

So we use 20 in. as a second trial value. For a 20-in. pipe, $f = 0.0210$ and

$$0.0210 \frac{10{,}560 \times 12}{20} \frac{V^2}{2g} = 50.$$

Solving this we obtain

$$V = 4.92 \text{ ft. per sec.}$$

and

$$q = 2.18 \times 4.92 = 10.74 \text{ sec. ft.}$$

This answer would appear to be satisfactory as it is on the safe side and is not much larger than the value desired. The next size pipe below this is 18 in. and we know that the rate of discharge through it must be less than $\frac{18^2}{20^2} \times 10.74 = 8.7$. (The actual value for the 18-in. pipe is 8.20.) Hence we would conclude that a 20-in. pipe would be required unless it were allowable for the capacity to be somewhat less than 9 sec. ft.

EXAMPLE

1. What must be the diameter of pipe necessary to discharge 6.5 sec. ft. under a head of 120 ft. if the length of pipe is 5,000 ft.

87. Power Delivered by a Pipe.—In Fig. 105 consider a point C which is located near the end of a pipe line. When no flow occurs, due to the closure of a valve or other device

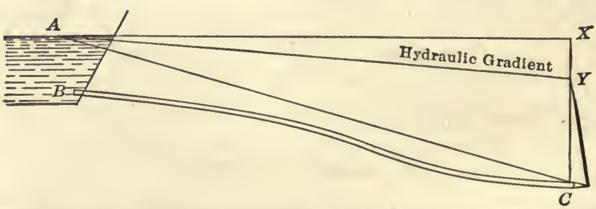


FIG. 105.—Varying hydraulic gradient with different rates of discharge.

beyond C, the pressure at C is a maximum, being equal to CX. But when flow occurs the pressure at C drops to the value CY, and the greater the rate of discharge the steeper will be the hydraulic gradient and the less will be the pressure at C. If the nozzle, or other device beyond C, be removed entirely making C a point at the very end of the pipe, the pressure will then be reduced to zero. In Fig. 106 are shown the decrease in pressure head at C and the increase in velocity head at C as

the rate of discharge is caused to increase by opening wider whatever device is below C. Now the total head at C is the sum of the pressure head and the velocity head, but it is seen to continually decrease with increasing discharge until it reaches a minimum value which is the velocity head when the pipe is wide open.

We have seen that power is a function of both q and H and may be expressed as wqH_C. In the case under consideration H decreases as q increases. When q is zero H_C is a maximum

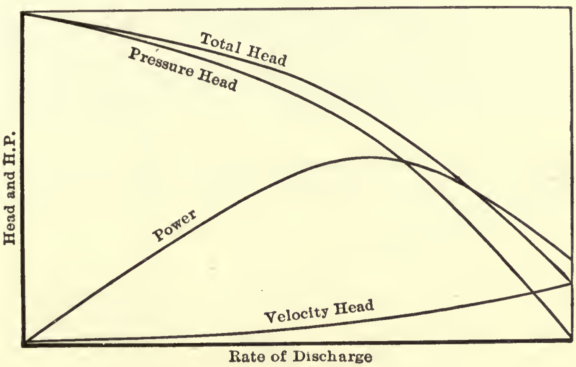


FIG. 106.—Head and power delivered by a pipe.

but the power is zero. And when q is a maximum the power is small due to the small value of H_C. Somewhere between these two extremes the product of these two variables reaches a maximum as shown by Fig. 106. It can be shown that the power is a maximum when the flow is such that one-third of the static head is consumed in friction, provided H' varies as V^2.

The efficiency of a pipe line may be defined as the ratio of the power delivered to the power supplied. But power is proportional to head, and hence the efficiency is H_C/h, where $h = CX$ in Fig. 105. In the case of maximum power delivered one-third of CX has been consumed in friction, hence the efficiency is only $66\frac{2}{3}$ per cent. If economy of water is no object it would be desirable to transmit power under these conditions as the cost of the pipe line would be small in proportion to the power delivered. But under usual conditions it is undesirable that one-third of the energy of the water be wasted, and hence such a size of pipe line would be employed that it could deliver the water available with a loss of only a few per cent. With ordinary

8

power-plant practice the efficiency of the pipe lines leading to the turbines is about 95 per cent.

EXAMPLES

1. Find the power delivered in the jet in problem (3) of Art. 79.

2. What is the efficiency of the pipe line (and nozzle) in problem (3) of Art. 79?

Ans. 61.5 per cent.

3. A pipe line 2,000 ft. long is 5 ft. in diameter. If the fall from the reservoir to the end of the pipe is 120 ft., what is the maximum amount of power the pipe could deliver?

4. What amount of power would the pipe in problem (3) deliver if its efficiency were 95 per cent.?

5. What size pipe would be required to deliver the water discharged in problem (3) if the efficiency of the pipe were to be 90 per cent.?

88. Pipe Line with Pump.—In case a pump lifts water from one reservoir to another, as in Fig. 107, it not only does work in lifting the water the height z but it also has to overcome the

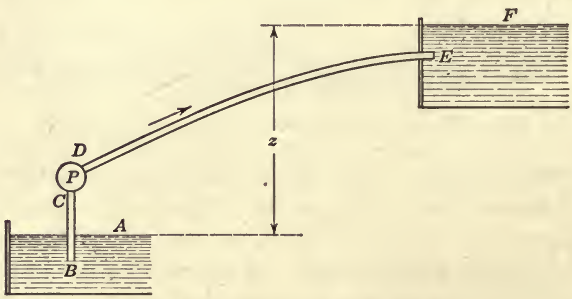


FIG. 107.—Pipe line with pump.

friction loss in the suction and discharge piping. This friction head is equivalent to some added lift so that the effect is the same as if the pump lifted the water a height $z + H'$, without loss. Hence the power delivered to the water by the pump is

$$W(z + H') \tag{71}$$

The power required to run the pump is greater than this, depending upon the efficiency of the pump. Although the pump actually lifts the water a height z, it is said to work against a head h whose value is

$$h = z + H' \tag{72}$$

In case the pump discharges a stream of water through a nozzle, such as in Fig. 108, the water has not only been lifted a height z but it has also received kinetic energy proportional to $V_2^2/2g$, where V_2 is the velocity of the jet. Thus the power delivered to the water by the pump is

$$W\left(z + \frac{V_2^2}{2g} + H'\right) \tag{73}$$

And the head against which the pump works is now

$$h = z + \frac{V_2^2}{2g} + H' \tag{74}$$

The difference between the two cases in Figs. 107 and 108 is really slight. In equation (71) we have considered the velocity

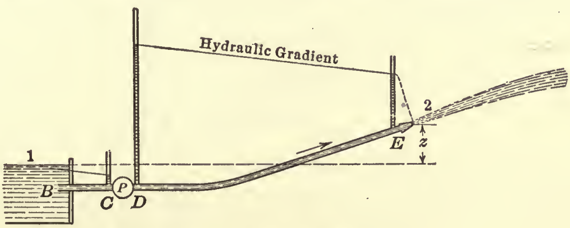


FIG. 108.—Pipe line with pump.

head at E to have been lost, while in equation (73) the velocity head in the jet has not yet been lost. Thus H' in (71) includes the velocity head of discharge while the H' in (73) does not.

EXAMPLES

1. A 10-in. pipe line is 3 miles long. If 4 cu. ft. of water per sec. are to be pumped through it, the total actual lift being 20 ft., what will be the horsepower required if the pump efficiency is 70 per cent.?

Ans. 240 hp.

2. In Fig. 107 assume $d'' = 10$ in., $BC = 20$ ft., $DE = 3{,}000$ ft., and $z = 135$ ft. If $q = 7$ sec. ft. and the pump efficiency is 80 per cent., what is the power required?

Ans. 340 hp.

3. In problem (2), if the elevation of C above the water surface is 13 ft. and that of D is 15 ft., compute the pressures at C and D.

Ans. $p_C = -19.48$ ft., $p_D = +323$ ft.

4. In Fig. 107 assume $d'' = 3$ in., $BC = 20$ ft., $DE = 200$ ft., and $z = 70$ ft. The elevation of C above the water surface is 15 ft. (*a*) If the pressure at C is to be -25 ft., what is the rate at which water is pumped? (*b*) If the efficiency of the pump is 60 per cent., what is the power required?

Ans. (*a*) $q = 0.613$ cu. ft. per sec. (*b*) 15 hp.

5. When a certain pump is delivering 1.0 cu. ft. of water per sec., the pressure gage at D (Fig. 107) reads 20 lb. per sq. in., while a vacuum gage at C reads 10 in. of mercury. The pressure gage is 2 ft. higher than the vacuum gage. If the diameter of the suction pipe is 4 in. and that of the discharge pipe is 3 in., find the power delivered to the water.

Ans. 7.23 hp.

89. Pipe Line with Turbine.—The type of machine that is usually employed for converting the energy of water into mechanical work is called a turbine. In flowing from the upper body of water in Fig. 109 to the lower, the water loses its potential energy due to the elevation z. This energy, which the

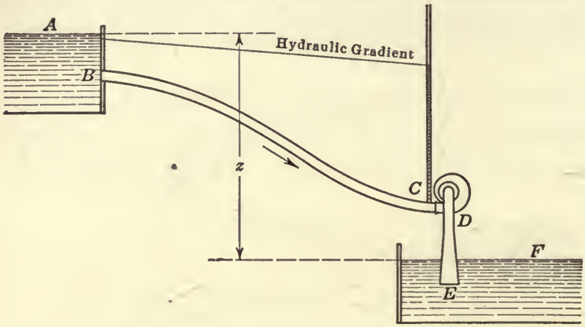


FIG. 109.—Pipe line with turbine.

water loses, is expended in two ways. A part of it is consumed in hydraulic friction in the pipe and the rest of it is delivered to the turbine. Of that which is delivered to the turbine, a portion is lost in hydraulic friction within the machine and the rest is converted into mechanical work.

The power delivered to the turbine is decreased by the friction loss in the pipe line, and its value is given by

$$W(z - H') \tag{75}$$

The power delivered by the machine is less than this depending upon both the hydraulic and mechanical losses of the turbine. The head under which the turbine operates is

$$h = z - H' \tag{76}$$

In the case of a turbine the only loss of head H', which is deducted, is that in the supply pipe. The draft tube, as the conduit which leads the water away from the turbine is called, is considered an integral part of the machine and hence h should cover losses in it as well as in the turbine case itself.

In applying these equations it should be noted that the particular location of the turbine is immaterial so long as it is not set so high above the lower water level that the pressure at the top of the draft tube approaches absolute zero in value. But as long as this is avoided the turbine can make use of the entire fall to the lower water level by the use of an air-tight draft tube. The higher the turbine is situated, within the limit specified, the less the pressure will be at intake but this is offset by an increased suction on the discharge side.

EXAMPLES

1. In Fig. 109 assume $d'' = 12$ in., $BC = 200$ ft., and $z = 120$ ft. The entrance to the pipe at the intake is flush with the wall. (*a*) If $q = 8$ sec. ft., what is the head supplied to the turbine? (*b*) What is the power delivered by the turbine if its efficiency is 75 per cent.?

Ans. (*a*) $h = 112.2$ ft. (*b*) 76.5 hp.

2. In problem (1) if the elevation of C above the water level is 18 ft., what is the pressure head at that point?

Ans. 92.56 ft.

3. A turbine operating under a total fall of 120 ft. ($z = 120$ ft.), is supplied with water through 300 ft. of 8-in. pipe. If the rate of discharge be such that 30 ft. of head is lost in friction in the pipe, what will be the power delivered to the turbine?

89a. Equation of Energy with Turbine or Pump.—The general equation of energy, derived in Art. 44, may be applied equally well to a pipe line in which there is any form of turbine or pump between the two sections considered. But the equation should always be applied with the water flowing from point (1) to point (2) regardless of the relative positions of these two points. In the preceding article H' represents the energy lost by the water in pipe friction, while h represents the energy lost by the water within the turbine. (Of the latter a part is lost within the turbine in hydraulic friction and a part is converted into mechanical work, but it is all lost so far as the water is concerned.) Hence we may write for the turbine

$$H_1 - H_2 = H' + h.$$

This is really equivalent to equation (76) where A and F correspond to points (1) and (2) in the above equation, for $H_A - H_F = z$.

In the case of the pump the h represents energy put into the water by the pump between points C and D in Fig. 107 and hence

it is a negative loss. We may thus write for the pump

$$H_1 - H_2 = H' - h.$$

This is equivalent to equation (72) where $H_A - H_F = -z$ or to equation (74) where $H_1 - H_2 = -(z + V_2^2/2g)$.

As an illustration let us consider a special case where a turbine of known capacity is placed in a pipe line of known dimensions and it is then desired to determine the rate of discharge. Since in the pipe line $H' = kV^2/2g$, we may write $H' = Aq^2$ where A is a constant whose value may be determined from the dimensions of the pipe. It may be shown (Art. 149) that the rate of discharge through any turbine may be expressed as $q = k\sqrt{h}$ and k would be known for a given turbine. Hence we may write $h = (q/k)^2 = Bq^2$, where B is another constant whose value can be determined. Now referring to Fig. 109

$$H_1 - H_2 = H_A - H_F = H' + h$$

$$z = Aq^2 + Bq^2$$

$$q = \sqrt{\frac{z}{A + B}}.$$

After q is determined the net head on the turbine may readily be found and everything is then known. The method can readily be extended to other combinations.

EXAMPLES

1. Assume the total fall from one body of water to another to be 120 ft. The water is conducted through 200 ft. of 12 in. pipe with the entrance flush with the wall. At the end of the pipe is a turbine and draft tube which discharged 5 cu. ft. of water per second when tested under a head of 43.8 ft. in another location. What would be the rate of discharge through the turbine and the net head on it under the present conditions.

Ans. $q = 8$ cu. ft. per second. $h = 112.2$.

2. Compare the above with problem 1 of Art. 89.

89b. Economic Size of Pipe.—Where the physical conditions fix the value of the head to be lost in pipe friction, the size of pipe for a given rate of discharge is to be determined as in Art. 86. But in case the pipe line is to deliver the water from a pump the friction head may have any value whatever, while if it supplies water to a turbine the head lost may also be of any magnitude up to the value of the total head available minus the velocity head in the pipe. Practically, however, it should be restricted to less than one-third the total head. See Fig. 106.

If we assume the rate of discharge to be constant, it is clear that the larger the pipe the less the velocity of flow and hence the less the value of the head lost. Since lost head means a waste of power in pumping water, or a loss of power which might otherwise be developed by a power plant, the problem becomes one of determining the proper value to be assigned to this item.

The larger the pipe the more it costs as is shown in Fig. 109*a*. The values plotted in this curve, however, are a certain percentage of the total cost being the annual fixed charges and including

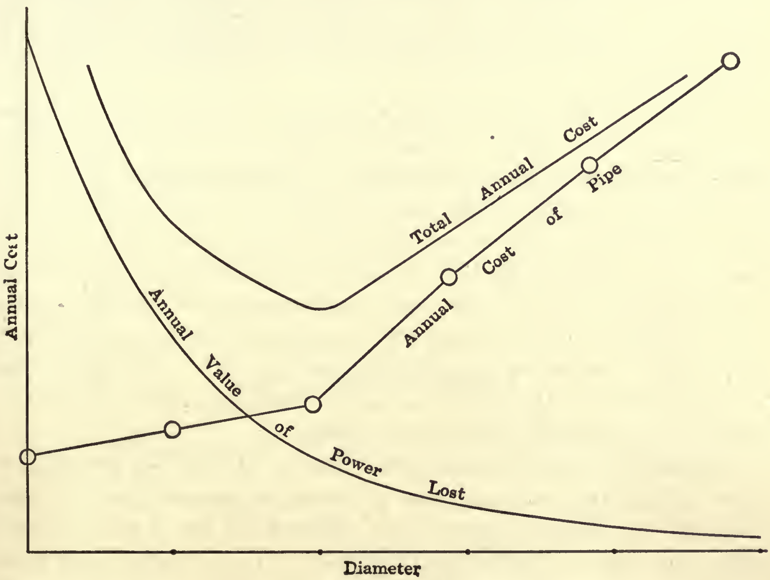


FIG. 109a.

interest on the investment, depreciation, etc. This curve in general is a discontinuous function since the costs of different commercial sizes do not follow a mathematical equation. Also the curve is subject to abrupt breaks where the increasing size may compel the change from one type of pipe to another of different construction. For each size of pipe the loss of head may be determined and hence the amount of power lost. If this horse-power lost is then multiplied by the annual value of a horse-power a second curve showing the annual loss due to pipe friction may be plotted. The sum of these two items is the total annual cost

of the pipe line. The size for which this total is a minimum is the most economical.

It must be pointed out that an accurate solution of this problem may be difficult in practice, especially in the case of a water power plant. The chances are that the rate of flow through the pipe line will not be constant, since the load on the plant will vary, and hence the load factor must be known before one can compute the total amount of power lost per year. And even then it is hard to fix the exact money value of such power lost. However, it is probable that in most cases a sufficiently accurate solution may be obtained by considering the flow constant and taking the value of a horse-power per year to be lower as the load factor is lower.

It must also be noted that in some cases the economic solution may not be the best. In case the value of a unit of power is small and the fixed charges are high the resulting pipe size would be relatively small and the loss of head large. This means that the velocity of the water would be high and, as will be seen later, this may cause trouble due to surges and water hammer when any change is made in the flow in governing the turbine. Also when the loss of head is large the variation in head from full load to no load is large, as may be seen in Fig. 106. This may also be undesirable in the operation of the turbine. Hence for these reasons a larger size pipe may be used.

It should be clearly understood that in Art. 87 the size of the pipe is fixed and varying rates of discharge are assumed to flow through it. In the present case the size of the pipe is varied while the rate of discharge is constant. In the latter case also the power delivered by the pipe varies, increasing as the diameter of the pipe increases. If the problem is one of delivering a fixed amount of power, it may be seen that the larger the pipe the higher the net head on the turbine and the less the water consumption. From Art. 87 it follows that the smallest size of pipe that can be used for a given amount of power regardless of economy of water is such that the loss of head is one third the static head. The best size of pipe may be determined in a manner similar to that shown in Fig. 109*a*, if a money value can be assigned to a cubic foot of water.

EXAMPLE

1. A water supply of 300 cu. ft. per second is available for a power plant under a static head of 1200 ft. The penstock is of riveted steel ($f = 0.022$)

and 7000 ft. long. Assume the fixed charges on investment to be 10% per year and the annual value of a horse-power to be $20 under the conditions of operation, treating the case as though the flow were constant. Fill in the table and determine the most economical size.

Ans. 90 in.

Diam. in.	Cost	V	H'	Annual value of power lost	Annual fixed charges	Total annual cost
70	$192,000					
80	250,000					
90	317,000					
100	390,000					

89c. Compound Pipes.—In the case of flow through compound or parallel pipes, such as in Fig. 109**b**, we have the following

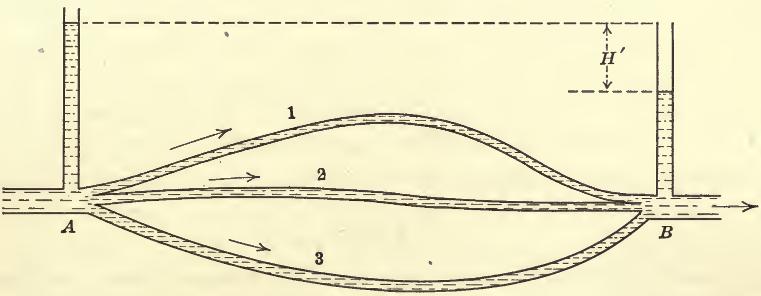


FIG. 109b.

fundamental relations. The sum of the flow through all the compound pipes equals the total flow in the main. And since the pressures at A and B are common to all the pipes it follows that the loss of head in each pipe is the same. Or

$$1.\ q_0 = q_1 + q_2 + q_3.$$
$$2.\ H' = \text{same for all.}$$

Applying Bernoulli's theorem to any section of any pipe we obtain $H' = (f\frac{l}{d} + n)\frac{V^2}{2g}$, where n is a factor to account for any

minor losses and in long pipes may be neglected as in Art. 79. Solving for V we obtain

$$V = \sqrt{\frac{2gH'}{f\frac{l}{d} + n}}$$

$$q = FV = F\sqrt{\frac{2gd}{fl + nd}}\sqrt{H'} = K\sqrt{H'}$$

Since K is a constant for any given pipe we may compute its value in each case and write

$$\begin{aligned} q_1 &= K_1\sqrt{H'} \\ q_2 &= K_2\sqrt{H'} \\ q_3 &= K_3\sqrt{H'} \\ \hline q_0 &= K_0\sqrt{H'} \end{aligned}$$

where $K_0 = K_1 + K_2 + K_3$.

If the value of H' be given and all dimensions of the pipes are known it is then easy to find the rate of discharge in each separate pipe. If the total rate of discharge, q_0, be given the value of H' may be computed and then the flow in each pipe can be found. If the dimensions of one or more of the pipes are unknown, however, a solution by trial may be necessary. If any water is supposed to be withdrawn between A and B, it will then be necessary to combine this problem with that in Art. 89*d*.

EXAMPLE

1. In Fig. 109*b* suppose that water enters at A from a large standpipe and that B is located 50 ft. above a given datum plane. The three pipes are of the following dimensions: 1200 ft. of 6-in. pipe, 1000 ft. of 8-in. pipe, and 1200 ft. of 10-in. pipe, while the diameter at B is 16 in. If 14 cu. ft. of water per second are delivered at B under a pressure head of 100 ft., what must be the elevation of the water surface in the standpipe above the datum plane?

Ans. 242 ft

89d. Branching Pipes.—Suppose the water flowing in pipe AB in Fig. 109*c* divides at B, a portion flowing through BC into the reservoir shown, while the rest flows on through pipe BD to some destination not shown. Suppose the pressure at D to be indicated by a piezometer column. (In reality both branches are similar since the condition would be practically the same if the second pipe discharged at D into a reservoir whose water surface

were the same as the top of the water in this tube.) We have here two fundamental relations also. The flow in the main AB is equal to the sum of the flow in the branches. And the pressure at B is a value common to all three pipes. That is

1. $q_0 = q_1 + q_2$.
2. p_B (or h) = common to all.

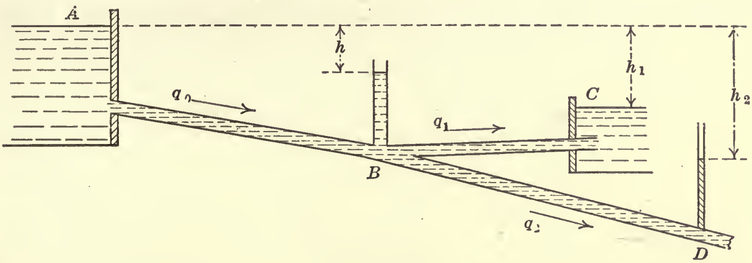


FIG. 109*c*.

Applying Bernoulli's theorem, as in the preceding article, we have

$$q_0 = K\sqrt{h}$$
$$q_1 = K_1\sqrt{h_1 - h}$$
$$q_2 = K_2\sqrt{h_2 - h}$$

This problem is not so readily solved as the one in the preceding article because the factor under the radical sign is different for each pipe. Also in the former case there may be different rates of flow and hence different values of H'. But in this case there is only one value of p_B or h for equilibrium and hence there can be only one rate of flow, if other dimensions are fixed.

The solution of this problem is illustrated by Fig. 109*d*. The value of h at which equilibrium is attained is given by the intersection of curves for q_0 and $q_1 + q_2$. It should be noted that if the conditions are such that h is greater than h_1 for instance the flow in BC would be opposite to that assumed and the curve for q_1 would then be as indicated by the dotted line. In this case values of q_1 should then be added to q_0.

EXAMPLE

1. Suppose that in Fig. 109*c*, AB consists of 1500 ft. of 12-in. pipe, BC of 800 ft. of 6-in. pipe, and BD of 1200 ft. of 8-in. pipe. The value of h_1 is

20 ft., B is 35 ft. below the level of the water surface at A, and D is 60 ft. below. When the pressure head at B is 25 ft., find the values of the flow in each pipe and the pressure head at D.

Ans. $q_0 = 3.49$, $q_1 = 0.817$, $q_2 = 2.67$, $p_D = 12.6$ ft.

89e. Pipe with Laterals.—Assume a pipe main from which water is withdrawn by laterals along its course. Then either V or d or both must vary. In such a case the loss of head between any two points may be determined as follows. Differentiating the expression for loss of head we obtain an expression for the loss of head in any infinitesimal distance dl. Thus

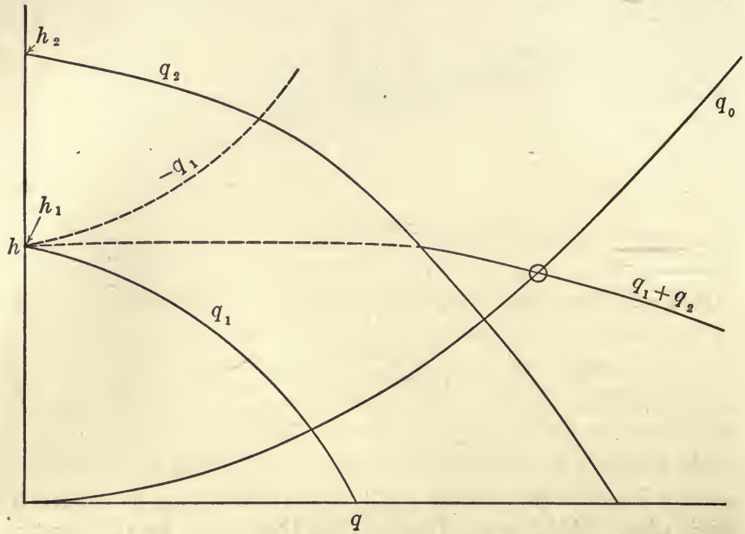


FIG. 109d.

$$dH' = \frac{fd(l)}{d}\frac{V^2}{2g}.$$

The integration of this between the proper limits of l will give us the value of the head lost in that distance. Thus

$$H' = \frac{1}{2g}\int f\frac{V^2}{d}\,d(l). \qquad (76a)$$

If it is possible to express f, V, and d as functions of l the integration of the above equation will give the value of H'. If an integration by calculus is not possible values of fV^2/d may be plotted as a function of l. The area between this curve and the axis for l is the value of the integral.

Special Case.—If the pipe is of uniform diameter and the laterals are uniformly spaced and may be assumed to take off water uniformly along its length, the above may readily be integrated. If the velocity of the water entering the length considered is V_1 and that leaving it is V_2 while the total length is l, the above conditions give us

$$\frac{d(l)}{dV} = \frac{l}{V_2 - V_1},$$

since the velocity decreases uniformly along the length of pipe. Substituting this value of $d(l)$ in equation (76*a*) we have

$$H' = \frac{f}{2gd} \times \frac{l}{V_2 - V_1}\int_{V_1}^{V_2} V^2 dV$$
$$= \frac{1}{3}\frac{fl}{2gd}\frac{V_1^3 - V_2^3}{V_1 - V_2}$$

If the terminal of the main is a dead end so that the value of V_2 is zero, this expression is further simplified and indicates that the loss of head is one-third the loss that would exist if the entire amount of water entering at (1) flowed clear through the pipe and discharged at (2).

EXAMPLE

1. In Fig. 109*d*, suppose that the branch *BD* were closed at *D* and discharged uniformly through laterals along its length. What would then be the pressure head at *D*.

Ans. 37.7 ft.

From a photograph by the author.

FIG. 110.—Cast-iron pipe line.

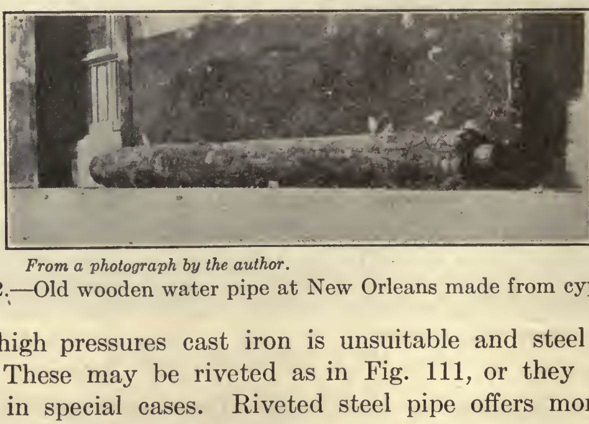
90. Construction of Pipe Lines.—Cast-iron pipes have been employed for the last 200 years and are very satisfactory for ordinary water-works purposes where moderate heads are employed. They are very durable and require but little atten-

tion. While it is sometimes used under higher pressures, cast iron is not considered desirable for heads above 400 ft. nor is it suitable for very large diameters on account of low tensile strength and possible defects in casting. For temporary pur-

From a photograph by the author.

Fig. 111.—Riveted steel pipe under head of 1300 ft. leading to Drum power house of Pacific Gas & Elec. Co. in California.

poses or for cheaper installations pipes are sometimes made of very light weight riveted steel, usually coated with some material in order to enable them to resist corrosion.



From a photograph by the author.

Fig. 112.—Old wooden water pipe at New Orleans made from cypress log.

For high pressures cast iron is unsuitable and steel pipe is used. These may be riveted as in Fig. 111, or they may be welded in special cases. Riveted steel pipe offers more resistance to flow than a new cast-iron pipe on account of the

projecting rivet heads and the overlapping of the plates, but an old riveted steel pipe and an old cast-iron pipe are about the same since both become coated alike with tubercles. A steel pipe is not considered as durable as a cast-iron pipe, but for high heads it is necessary to use it.

For heads under 200 or 300 ft. wood-stave pipe offers many advantages. It is cheaper than a metal pipe for the same service. The resistance to flow is less than a riveted steel pipe

Courtesy of Redwood Manufacturers Co.

FIG. 113.—Construction of wood-stave pipe.

and about the same as a new, smooth, cast-iron pipe, but it has the advantage that its capacity does not decrease with age. The early types of wooden pipe used were simply hollow logs as shown in Fig. 112. Some of these were used for many years. Modern wood pipe is generally built up of staves as shown in Fig. 113. The staves are so arranged that the joints are "broken." In order to make a water-tight joint, a thin steel tongue is inserted in a saw cut across the end of each stave. This piece of steel is slightly wider than the stave so that when the bands are tightened up it will sink into the staves on either side a distance of about ⅛ in. or more. In the life of a wood-

stave pipe the encircling metal bands often have to be renewed. It is essential that a wooden pipe be kept filled with water, if it is to have a long life, as wood does not rot rapidly if it is kept continually wet or continually dry. It rots the worst when it is exposed to alternations in these conditions. The life of a wood pipe is not as long as that of a heavy cast-iron pipe but it

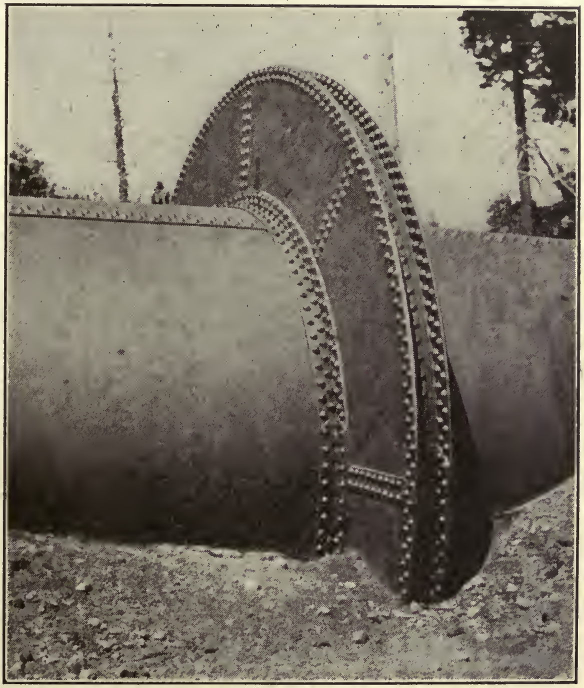
From a photograph by the author.

FIG. 114.—Curves in wood-stave pipe. In the Sierra Nevada Mts. of California.

may be as long as that of a steel pipe. However, statistics of these matters are lacking and subject to much dispute. The wood pipe is free from corrosion and from electrolysis and is not attacked by acids in the water. Hence, it is often used for carrying liquids that could not be handled by a metal pipe. It is possible to introduce broad sweeping curves into a wood-

stave pipe without any special devices or fittings, as shown in Fig. 114.

Metal pipes are subject to expansion and contraction due to temperature changes and provision must often be made for this. In the case of a cast-iron pipe line the amount of play afforded at each joint is usually sufficient. But a riveted-steel pipe line has no such flexibility and expansion devices may be employed.



From a photograph by the author.

FIG. 115.—Expansion joint in 8.5 ft. riveted steel pipe under low head.

One type of expansion joint is shown in Fig. 115, which is suitable only for low pressures. It may be seen that the circular plates can spring enough to permit the necessary endwise motion of the pipe. For higher pressures a joint such as in Figs. 116 and 117 may be used.

When water is lifted by a pipe line to a greater height than the initial water level, as in Fig. 118, the pipe is called a siphon. Of course it is necessary to exhaust the air by some means in order to start the flow, and if the flow is to continue

From a photograph by the author.

FIG. 116.—Expansion joint in high pressure riveted steel pipe line.

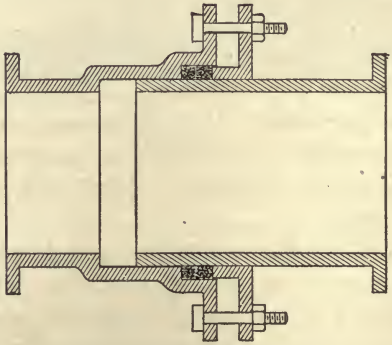


FIG. 117.—Expansion joint.

the air which collects at the summit must be removed from time to time. There are times when such a device cannot be avoided.

By analogy a pipe line such as shown in Fig. 119 is called an "inverted siphon," and it is usually found where it is necessary

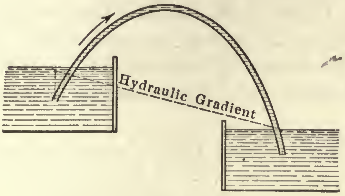


FIG. 118.—Siphon.

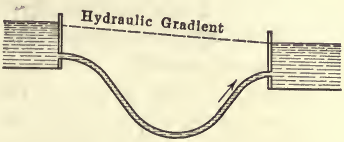


FIG. 119.—Inverted siphon.

From a photograph by the author.

FIG. 120.—Riveted steel siphon. Lake Spaulding development of Pacific Gas & Elec. Co.

to carry water across a valley or depression as in Fig. 120. However, it is quite common to call a pipe so situated simply a "siphon."

91. PROBLEMS

1. A pipe line 850 ft. long discharges freely into the air under a fall of 40 ft. (Assume a projecting pipe at entrance.) (*a*) If $d'' = 6$ in., find the rate of discharge. (*b*) If $d'' = 12$ in., find the rate of discharge.

Ans. (*a*) $q = 1.55$ sec. ft. (*b*) $q = 8.85$ sec. ft.

2. Suppose a pipe line runs from one reservoir to another, both ends of the pipe being under water. Assume the intake end is non-projecting. If the difference in water levels is 110 ft., the length of pipe is 500 ft., and the diameter 10 in., what will be the rate of discharge? What will the capacity be when the pipe is old?

3. In problem (2) a point C in the pipe is located 120 ft. below the level of the surface of the water in the upper reservoir and 300 ft. from the intake. What will be the pressure at that point? Draw the hydraulic gradient.

Ans. 49.5 ft.

4. A pipe line 800 ft. long discharges freely at a point 150 ft. below the water level at intake. The pipe projects into the reservoir. The first 500 ft. is 12 in. in diameter and the remaining 300 ft. is 8 in. in diameter. Find the rate of discharge.

Ans. $q = 9.25$ sec. ft.

5. The junction of the two sizes of pipe in problem (4) is 120 ft. below the surface of the water level. Find the pressure just above C and just below C, where C denotes the point of junction. Assume a sudden contraction at this point.

6. A jet of water is discharged through a nozzle at a point 200 ft. below the water level at intake. The jet is 4 in. in diameter and the velocity coefficient of the nozzle is 0.90. If the pipe line is 12 in. in diameter, 500 ft. long, with a non-projecting entrance, what is the pressure at the base of the nozzle?

Ans. 177.8 ft.

7. It is desired to deliver 3 cu. ft. of water per sec. at a point 10,000 ft. distant with a loss of head of 150 ft. What size pipe would be required?

8. What would be the probable capacity of the pipe in problem (7) when it was old?

9. A pump delivers water through 300 ft. of 4-in. fire hose to a nozzle which throws a 1-in. jet. The velocity coefficient of the nozzle is 0.98 and the value of f for the hose may be assumed to be 0.025. The nozzle is 20 ft. higher than the pump. It is required that the velocity of the jet be 70 ft. per sec. What will be the necessary pressure at the pump?

10. The steel siphon shown in Fig. 120 is 8.5 ft. in diameter. It is 1,900 ft. long and carries 300 cu. ft. of water per sec. What must be the difference in water level at the two ends? (It is arranged as in Fig. 119.)

11. The pipe line shown in Figs. 116 and 191 has an average diameter of 62 in., is 6,272 ft. long, and the difference in level between the power house and the intake is 1,375 ft. When the pipe delivers 300 cu. ft. of water per sec., what is its efficiency?

12. What is the horsepower delivered to the plant in problem (11)?

CHAPTER VIII

UNIFORM FLOW IN OPEN CHANNELS

92. Open Channels.—An open channel is one in which the stream is not completely enclosed by solid boundaries and therefore has a free surface subjected only to atmospheric pressure. The flow in such a channel is not dependent upon some external head but rather upon the slope of the channel and of the water surface.

The principal types of open channels are: natural streams or rivers, artificial canals, and sewers, tunnels, or pipe lines not completely filled.

From a photograph by the author.

FIG. 121.—Canal of the Pac. Gas & Elec. Co. with one bank rock lined.

The accurate solution of problems of flow in open channels is much more difficult than in the case of pressure pipes. Not only is reliable experimental data more difficult to secure, but there is a wider range of conditions than is met with in the case of pipes. Practically all pipes are round, but the cross-sections of open channels may be of any shapes from circular to the irregular forms of natural streams. It is probable that the shape of the

cross-section affects the flow in a way that is not covered by the factor, m, the hydraulic mean depth (see page 97). In pipes the degree of roughness ordinarily ranges from that of new, smooth,

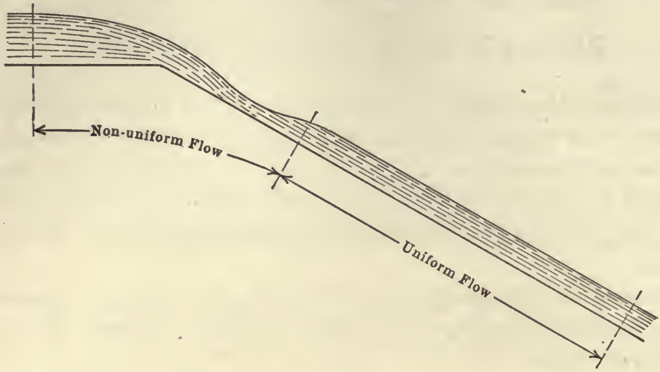


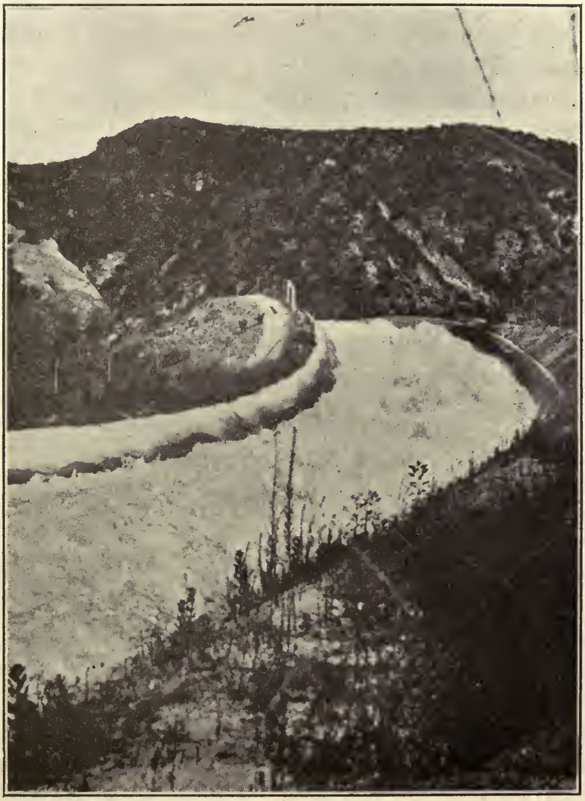
Fig. 122.

cast-iron or wood-stave pipes on the one hand to that of old corroded pipes on the other. But with open channels the surfaces vary from smooth timber (Fig. 123) to the rough and

From a photograph by the author.
Fig. 123.—Non-uniform flow in wooden flume.

irregular beds of some rivers. Hence the choice of friction factors is attended with greater uncertainty in the case of open channels than in the case of pipes.

93. Uniform Flow.—If the shape and size of any water cross-section is identical with that of every other section in the length of channel under consideration, the flow is said to be *uniform.* Such cases are shown in Figs. 54 and 121. Uniform flow must not be confused with *steady* flow. The former requires that the conditions at any time be the same from place to place; the



From a photograph by the author.

FIG. 124.—Cascade on Los Angeles aqueduct.

latter requires that the conditions at every section be constant with respect to time. We might have steady flow for both uniform and non-uniform flow as shown in Fig. 122. Uniform flow is obtained only when a channel is uniform for a considerable distance so that the water has a chance to adjust itself. The channel in Fig. 123 is uniform but the flow is non-uniform in the portion shown because the water has just entered it and has not

yet attained a condition of equilibrium. The conditions are similar to the upper portion of the channel shown in Fig. 122. On the other hand the flow is non-uniform in Fig. 124 because the slope of the channel varies.

94. Hydraulic Gradient.—It is quite evident that in the case of an open channel the hydraulic gradient coincides with the water surface. For if a piezometer tube be attached to the side of the channel the water will rise in it until its surface is level with that of the water in the channel.

95. Equation for Uniform Flow.—The equation that is most generally used for steady uniform flow in open channels is one that is also often used for flow in long pipes. It is

$$V = c\sqrt{ms} \tag{68}$$

the derivation of which was given in Art. 83. In this formula, c is a coefficient dependent upon the roughness of the surface in contact with the water, and it is also often given as a function of other variables as well. The quantity, m, is the hydraulic mean depth, or hydraulic radius, and its value is given by

$$m = \frac{\text{area of water cross-section}}{\text{length of wetted perimeter}^{1}} \tag{56}$$

In the case of an open channel, s is the slope of the water surface, according to Art. 94.

Recognizing that the velocity does not vary exactly as the square root of m or of s, exponential formulas such as equation (70) are sometimes used. But if equation (68) is employed it is seen that c must then be a function of m and s, since equation (68) does not involve the correct exponents of m and s.

96. Kutter's Formula for c.—The formula for c that has probably been more widely used than any other is that of Kutter and Ganguillet, two Swiss engineers. This formula is based upon a wealth of data from small artificial canals up to natural streams as large as the Mississippi, and for this reason it is believed to be applicable to a wide range of conditions. But any formula that attempts to cover too large a field must necessarily be a mere average of a number of scattered values and, though giving approximate values at least for any combination of factors, it

[1] The wetted perimeter is only that portion of the channel section that is in contact with the water. The width across the free surface of the water should not be included.

cannot be expected to give exact values in individual cases. Hence too great reliance must not be placed upon values given by the use of this or any other such empirical formula.

In Art. 95, it was pointed out that, since equation (68) is not a true expression of the law of flow, the value of c must be a function of m and s as well as the roughness of the surface. The formula of Kutter takes all three factors into consideration. It is

$$c = \frac{41.65 + \dfrac{0.00281}{s} + \dfrac{1.811}{n}}{1 + \left(41.65 + \dfrac{0.00281}{s}\right)\dfrac{n}{\sqrt{m}}} \tag{77}$$

In this expression the factor n is a coefficient of roughness, values of which are given in Table VI.

TABLE VI.—VALUES OF n IN KUTTER'S AND MANNING'S FORMULAS

Nature of surface	n
Planed and smoothly laid timber	0.009
Planed timber, not perfectly true	0.010
Wood-stave pipes	0.011
Smooth cement	0.011
Smooth iron pipes	0.011
Rough timber, good brickwork	0.013
Slightly rough iron pipes	0.015
Rough brickwork, cut stones	0.015
Good rubble masonry	0.017
Tuberculated iron pipes	0.017
Rough brick and stonework	0.017
Smooth earth channels	0.017
Coarse gravel, well packed	0.020
Large earth channels, good condition	0.022
Small earth channels, good condition	0.025
Channels in fair condition	0.030
Channels in bad order, with weeds, etc	0.035
Channels encumbered with drift	0.045

In order to save tedious computation when equation (77) is used, various sets of tables have been published and also a number of graphical solutions have been devised.[1] In Table VII will be found values of c determined by equation (77). Inter-

[1] One of the simplest of these is the diagram published by Karl R. Kennison.

TABLE VII.—VALUES OF *c* COMPUTED FROM KUTTER'S FORMULA

Slope s	n	Hydraulic mean depth, m, in feet												
		0.2	0.4	0.6	0.8	1	1.5	2	3	4	6	8	10	15
0.00005	0.009	100	124	139	150	158	173	184	198	207	220	228	234	244
	0.010	87	109	122	133	140	154	164	178	187	199	206	212	220
	0.011	77	97	109	119	126	139	148	161	170	182	189	195	205
	0.012	68	88	98	107	114	126	135	148	156	168	175	181	189
	0.013	62	79	90	98	104	116	124	136	145	156	163	169	179
	0.015	51	66	76	83	89	99	107	118	126	137	144	149	158
	0.017	44	57	65	71	77	87	94	104	111	122	129	134	142
	0.020	35	46	53	59	64	72	79	88	95	105	111	116	125
	0.025	26	35	41	46	49	57	62	71	77	85	91	96	104
	0.030	21	28	33	37	40	47	51	59	64	72	78	82	90
	0.035	18	24	28	31	34	40	44	50	56	63	68	72	79
0.0001	0.009	112	136	149	158	166	178	187	198	206	215	221	226	233
	0.010	98	119	131	140	147	159	168	178	186	195	201	205	212
	0.011	86	106	118	126	137	144	151	162	169	178	184	188	195
	0.012	76	95	105	114	120	130	138	149	155	164	170	174	181
	0.013	69	86	96	103	109	120	127	137	143	152	158	162	169
	0.015	57	72	81	88	93	103	109	119	125	134	139	143	150
	0.017	48	62	70	76	81	89	96	104	111	119	124	128	135
	0.020	39	50	57	63	67	75	81	89	94	102	107	111	118
	0.025	29	38	44	48	52	59	64	71	76	84	88	92	98
	0.030	23	31	35	39	42	48	53	59	64	71	75	78	85
	0.035	19	25	30	33	35	41	45	51	55	61	66	69	75
0.0002	0.009	121	143	155	164	170	181	188	200	205	213	218	222	228
	0.010	105	125	138	145	151	162	170	179	185	193	198	201	207
	0.011	93	112	122	131	136	146	154	163	168	176	182	185	190
	0.012	83	100	111	118	123	133	140	149	155	162	167	170	176
	0.013	74	91	100	107	113	122	129	137	143	150	155	158	164
	0.015	61	76	85	91	96	105	111	119	125	132	137	140	145
	0.017	52	65	73	79	83	91	97	105	111	117	122	125	131
	0.020	42	53	60	65	69	77	82	89	94	100	105	108	113
	0.025	31	40	46	50	54	60	64	72	76	82	87	89	95
	0.030	25	32	37	41	44	49	54	59	63	69	73	76	82
	0.035	21	27	31	34	37	42	45	51	55	60	64	67	72
0.0004	0.009	126	147	157	166	172	183	190	199	204	211	215	219	224
	0.010	110	129	140	148	154	164	170	179	184	191	196	199	203
	0.011	97	115	126	133	138	148	154	162	168	175	179	183	187
	0.012	87	104	113	121	125	135	141	149	154	161	165	168	172
	0.013	78	94	103	110	115	124	130	138	142	149	153	157	162
	0.015	65	79	87	93	98	106	112	119	124	130	135	138	143
	0.017	54	68	75	81	85	93	98	105	110	116	120	123	128
	0.020	44	55	62	67	70	78	83	89	94	99	104	107	110
	0.025	32	42	47	51	55	61	65	71	76	81	85	88	92
	0.030	25	33	38	42	45	50	54	59	63	69	73	75	80
	0.035	21	27	31	35	37	42	45	51	55	60	64	66	70
0.0010	0.009	129	150	161	169	175	184	191	199	204	211	214	218	222
	0.010	113	131	142	150	155	165	171	179	184	190	194	197	202
	0.011	99	117	127	134	139	149	155	163	168	174	178	181	186
	0.012	89	105	115	122	127	136	142	149	154	160	164	167	171
	0.013	81	96	104	111	116	124	134	138	142	149	152	155	160
	0.015	66	80	88	94	99	108	112	119	124	130	134	136	141
	0.017	57	69	76	82	86	93	98	105	110	116	120	122	127
	0.020	45	56	63	68	71	78	83	89	93	99	103	105	110
	0.025	34	43	48	52	56	62	66	71	75	81	85	87	91
	0.030	27	34	39	42	45	50	54	59	63	68	72	74	78
	0.035	22	28	32	35	38	43	46	51	54	59	63	65	68
0.0100	0.009	130	151	162	170	175	185	191	199	204	210	213	217	222
	0.010	114	133	143	151	156	165	171	179	184	190	193	196	200
	0.011	100	119	129	135	141	149	155	162	167	173	176	180	184
	0.012	90	107	116	123	128	136	142	149	154	160	163	166	170
	0.013	81	98	106	112	117	125	130	138	142	148	151	154	159
	0.015	66	80	89	95	100	108	112	119	124	129	133	134	138
	0.017	57	70	77	82	87	94	99	105	109	115	118	121	126
	0.020	46	57	64	68	72	79	83	89	93	99	102	105	108
	0.025	34	44	49	53	56	62	66	71	76	81	84	86	90
	0.030	27	35	39	43	45	51	55	59	63	68	71	74	77
	0.035	22	29	33	35	38	43	46	51	55	59	62	65	68

mediate values may be found by interpolation with as much accuracy as the conditions warrant.

Although c is a function of the slope, it will be found that its variation with values of s is not great. The difference between values of c for $s = 0.0001$ and $s = 0.0010$ is a matter of from 10 to 15 per cent. at the very most. The equation also shows that as s increases in value its influence upon the value of c decreases. For all values from $s = 0.0010$ up to $s = 0.100$, or even greater, the change in the value of c is negligible. See Fig. 125, which is constructed for several values of m when $n = 0.017$. Similar results would be obtained with any other value of n.

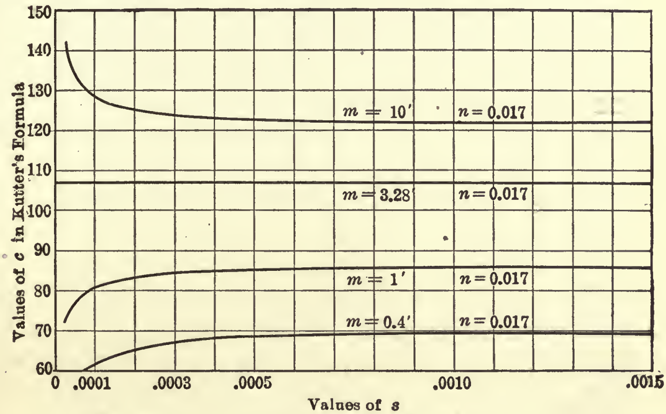


FIG. 125.—Relation between c and s.

From the range of experiments upon which it is based, Kutter's formula would appear to be applicable for values of m up to 10 ft., for velocities up to 10 ft. per sec., and for slopes greater than $s = 0.0001$. Outside of these limits reliable data are lacking and Kutter's formula should be used with caution.

97. Manning's Formula for c.—Since the value of c is affected by the slope to a very small extent only, as shown by Fig. 125, both Manning and Bazin disregard its influence altogether and propose formulas in which c is independent of s. This results in much simpler expressions. The formula of Manning gives almost the same values as that of Kutter and is probably as accurate as the circumstances warrant. It is

$$c = \frac{1.49}{n} m^{0.17} \tag{78}$$

in which n is the same as in Kutter's formula. Values of n are given in Table VI.

Equation (78) gives a better idea of the way c varies with n and m than can be obtained from an inspection of equation (77). Since these two equations give values of c which are approximately equal to each other, it follows that in Kutter's formula c varies approximately inversely as n and directly as $m^{0.17}$.

For practical use it is better to compute V directly, rather than to determine c separately by Manning's formula. Substituting the value of c given by equation (78) in equation (68) we obtain

$$V = \frac{1.49}{n} \sqrt[3]{m^2} \sqrt{s}. \tag{79}$$

Values of $m^{2/3}$ may be found on page 263.

98. Bazin's Formula for c.—For small artificial channels with values of m less than 3 ft. and for velocities of flow of not more than 4 ft. per sec., the formula of Bazin is considered excellent. It is

$$c = \frac{157.6}{1 + \frac{n'}{\sqrt{m}}} \tag{80}$$

where n' depends upon the roughness of the surface. It will be noted that this formula does not give c as a function of s. In Table VIII will be found values of n' for use in equation (80).

TABLE VIII.—VALUES OF n' IN BAZIN'S FORMULA

Nature of surface	n'
Smooth cement, or planed wood	0.109
Rough lumber, cut stone, and brickwork	0.290
Rubble masonry	0.833
Earth channels with regular surfaces	1.54
Ordinary earth channels	2.35
Rough earth channels with boulders and weed-grown sides	3.17

99. Construction of Open Channels.—Inspection of the expression $V = c\sqrt{ms}$ shows that, for a given slope and degree of roughness, the velocity increases as m increases. This is also accentuated by the fact that the value of c also increases as m increases or, as shown by equation (79), V varies as $m^{2/3}$. Therefore for a given area of water cross-section the rate of discharge will be a maximum when m is a maximum. Or for a given rate of discharge the cross-section area will be a minimum when the design is such as to make m a maximum.

From equation (56) it may be seen that the value of m will be a maximum for a given area when the length of the wetted perimeter is a minimum Now of all geometric figures, the circle has the least perimeter for a given area. Hence a semicircular open channel will discharge more water than one of any other shape, assuming that the area, slope, and roughness of surface are the same. Semicircular open channels are often built of pressed

From a photograph by the author.

FIG. 126.—Open channel with steep slope. Future power site on Los Angeles Aqueduct. Drop in elevation = 524 ft.

steel and other forms of metal, but for other types of construction such a shape is impractical. (The open channel in Fig. 126 has a semicircular steel lining. This channel has a wooden covering over it but that does not convert it into a pressure conduit.)

For wooden flumes the rectangular shape is usually used. Of all rectangles the square has the least perimeter in proportion to

area and hence for an open channel the depth of the water should be half the width.

Canals excavated in earth must have a trapezoidal section, and of all trapezoids the half hexagon will have the largest value of m. But the angle θ cannot always be made equal to 60° for other reasons. The slope of the sides must be such that the angle θ is

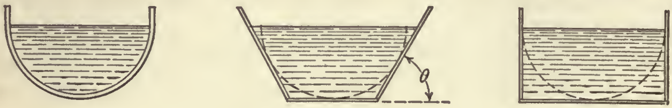
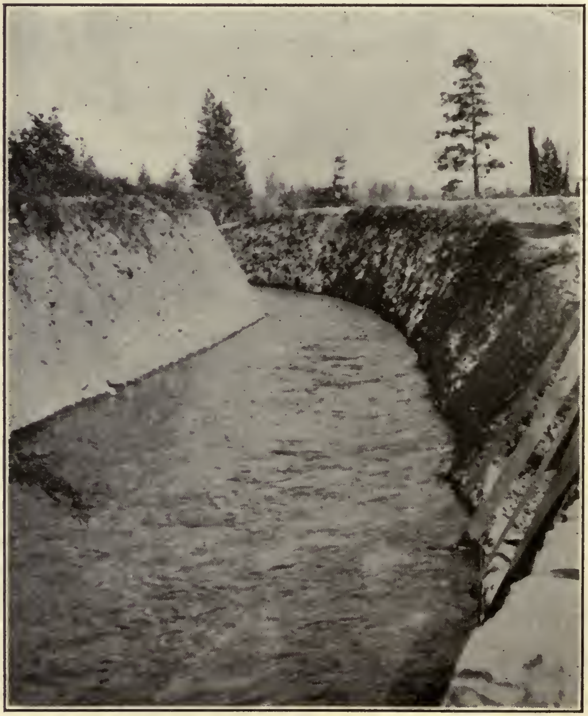


Fig. 127.



From a photograph by the author.

Fig. 128.—Unlined canal with steep banks. In the Sierra Nevada Mts.

less than the "angle of repose" of the material of which the banks are composed, otherwise the latter will cave in. In Fig. 128 the angle θ is made much greater than 60° in order to save a considerable amount of excavation in a deep cut, the firm character of the soil permitting such steep sides.

Whatever the value of the angle, it will be found that the best proportions will be obtained when the sides are tangent to a semicircle whose center lies in the water surface.

But other forms of cross-section are often used either because they have certain advantages in construction or are desirable from other standpoints. Thus oval or egg-shaped sections are common for sewers and similar channels where there may be large fluctuations in the rate of discharge. It is desirable that the velocity, when a small quantity is flowing, be kept high enough to prevent the deposit of sediment, and when the conduit is full the velocity should not be too high on account of wearing the lining of the channel.

100. Non-uniform Flow in Open Channels.—As a rule uniform flow is found only in artificial channels of regular shape and slope, although even under these conditions the flow for some distance may be non-uniform as shown in Fig. 123. But with a natural stream the slope of the bed and the form and size of the cross-section usually vary to such an extent that true uniform flow is rare. Hence the application of the equations given in this chapter to natural streams can be expected to yield results which are

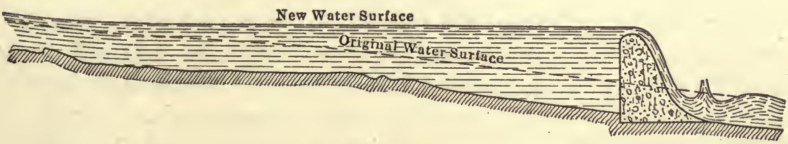


FIG. 129.

only rough approximations to the truth. In order to apply these equations at all it is necessary to divide the stream up into lengths within which the conditions are approximately the same. A satisfactory and reliable treatment of the problem of non-uniform flow in open channels is lacking.

In the case of pressure conduits we have dealt with uniform and non-uniform flow without drawing any distinction between them. This is possible because in a closed pipe the area of the water cross-section, and hence the velocity, is fixed at every point. But in an open channel these conditions are unknown and the stream adjusts itself to the size of cross-section that the slope of the hydraulic gradient requires.

In either artificial or natural streams non-uniform flow may be produced in a variety of ways, each one of which leads to a differ-

ent hydraulic phenomenon. The one case that is of the most practical importance is where a dam or other obstruction is placed across a flowing stream (see Fig. 129). It is desirable to determine how far up-stream the "backwater" created by this dam will extend, or at a given point, how far the water level will be raised.

A mathematical treatment of this case is given in various books, but it rests upon assumptions of conditions which are rarely fulfilled in natural streams, so that its accuracy for most practical cases is doubtful.[1] In view of the fact that it is only an approximation, it seems fully as satisfactory to apply the simple equation, $V = c\sqrt{ms}$, to the case. In order to do this the stream must be divided up into various lengths within which the flow may be assumed to be fairly uniform. Then for each length an average value of m and s should be used and the solution completed for that length. Of course the solution for one portion must be consistent with that for other portions, that is, the same rate of discharge must exist for all of them.

101. Stream Gaging.—The determination of the rate of discharge of a stream for any given depth of water is termed *stream gaging*. It may be seen that the rate of discharge of a stream could be computed from the formula, $V = c\sqrt{ms}$, if the flow is uniform and the cross-section area, the hydraulic mean depth, and the slope of the water surface are known. But a more accurate determination of discharge can be made by measuring the velocity directly, by passing the water over a weir, or by other means.

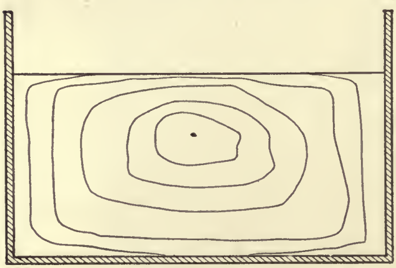


FIG. 130.

In a straight portion of an artificial channel the velocity might vary as shown in Fig. 130. These curves are velocity contours or curves of equal velocity. Within the area enclosed by the curve the velocity is higher than that of a point on the curve. Outside the enclosed area the velocity is less than that on the curve.

[1] The formula derived by calculus to fit this case is based upon the assumptions that the slope of the bed is uniform, that the form of the water cross-section is uniform except that its depth varies, and that the stream is very broad as compared to its depth.

It may be seen that the velocity of the water varies from side to side and from top to bottom. If there is a bend in the channel, or if the bed is irregular, as in natural streams, these velocity curves are often very irregular and distorted from the forms shown here. It is, therefore, necessary to determine the velocity at a number of different points across the section of the stream.

The instrument that is commonly used for this purpose is the current meter, described in Art. 70. In using the current meter or any other device it is customary to divide the stream up into sections as in Fig. 131 and to determine the contour of the bed, so that the area may be computed. If then the average velocity is determined for some section such as $ABCD$ the discharge through this section will be the product of this velocity and the area $ABCD$. The sum of all such partial discharges gives the total rate of discharge of the entire stream.

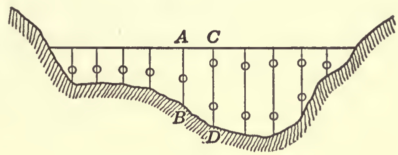


Fig. 131.

In finding the average velocity in the area $ABCD$ it is customary to take it as the average of the velocity measured in the line AB and the velocity measured in the line CD. But, as shown in Fig. 132, the velocity varies from A to B or from C to D, and hence we should determine the average velocity in each vertical line. This might be done by taking a number of observations so that curves similar to that in Fig. 132 could be plotted. But a study of a number of such curves has shown that in general the average velocity in a vertical line is found at about 0.6 the depth. Hence if the current meter be set at that depth, the velocity determined by it may be assumed to be the mean velocity. Of course this is only an approximation. To insure a higher degree of accuracy than a single observation could give, measurements are often taken at 0.2 the depth and 0.8 the depth. The mean of these two values will be approximately the average velocity. Thus, in an actual stream gaging, observations would be made at the points indicated by the circles in

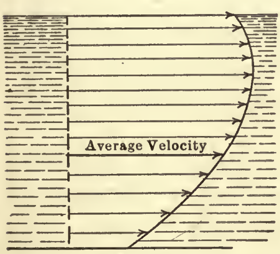


Fig. 132.

Fig. 131. Further details of this topic are not within the scope of this text.[1]

Sometimes floats are used but such procedure is less accurate. However they are often applicable when other methods are not feasible, such as during floods. If surface floats are used, the average velocity may ordinarily be taken as about 0.9 that of the surface velocity. But the velocity at the surface is greatly affected by the wind.

102. Rating Curve.—If a natural stream is to be used for water supply or power purposes, it is necessary to determine the amount of water it can be depended upon to furnish. Since the flow will usually be subject to wide fluctuations during a long period of time it is necessary to make an extended series of observations upon it.

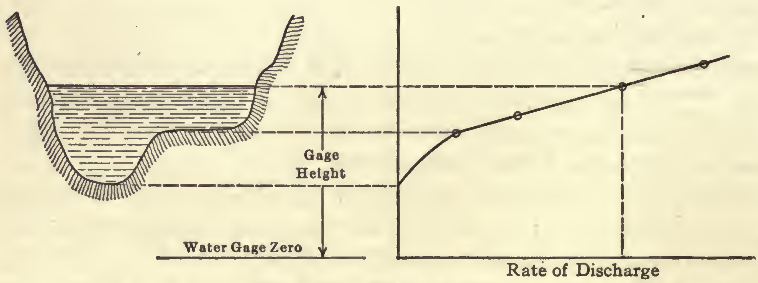


FIG. 133.—Rating curve.

The level of the surface of the water in a stream is called the gage-height, and may be measured above any arbitrary point. Thus the gage-height does not necessarily coincide with the depth of the stream.

It is apparent that for a given stream, the rate of discharge will be a function of the gage-height. If the rate of discharge of the stream be determined for several gage-heights a curve, such as in Fig. 133, may be constructed. This curve is called the *rating curve*, and from it the value of q for any height of water can be obtained.

Thus in making a study of the stream it is necessary to make only a record of the gage-heights. From the rating curve the quantity of flow can then be determined. This gage-height might simply be read and recorded once a day by an observer, or by means of a float and clockwork a continuous record could be

[1] See Hoyt and Grover, "River Discharge."

obtained which would show all the variations in the flow. Fig. 87 shows such a gaging station.

103. PROBLEMS

1. A circular conduit of smooth cement is exactly half full of water. The diameter is 4 ft. and the slope is 1 ft. per 10,000 ft. Compute the rate of discharge by the formulas of Kutter, Manning, and Bazin.

Ans. q = 8.38, 8.51, and 8.92 sec. ft. respectively.

2. A rectangular flume of timber slopes 1 ft. per 1,000 ft. Compute the rate of discharge if the width is 6 ft. and the depth of water 3 ft.

Ans. 114 sec. ft.

3. What would be the rate of discharge in problem (2) if the width were 3 ft. and the depth of water 6 ft.? Which of the two forms would require less lumber?

4. A rectangular channel of rubble masonry is 6 ft. wide, the depth of water is 3 ft., and the slope of 1 ft. per 1,000 ft. Compute the rate of discharge and compare with that in problem (2).

Ans. 65 sec. ft.

5. A semicircular channel of rubble masonry with a slope of 1 ft. per 1,000 ft. will give what discharge when flowing full if its diameter is 6.55 ft.? Compare the cross-section areas and amounts of lining required with that in problem (4).

Ans. 65 sec. ft.

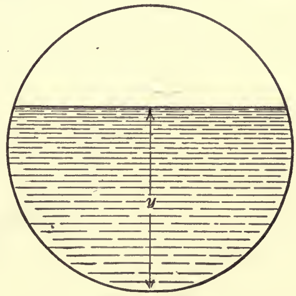


FIG. 134.

6. A circular conduit of concrete (n = 0.012) is 10 ft. in diameter and slopes 1.6 ft. per 1,000 ft. (See Fig. 134). The following table gives values of wetted perimeter and area of water cross-section for various depths of water in the conduit. Find values of V and q for the various depths in the table. What value of y gives the highest velocity? What value of y gives the highest rate of discharge?

Depth, y	Wetted perimeter	Area, F	m	$\sqrt{m}$	c	V	q
1.0	6.44	4.09					
3.0	11.59	19.82					
5.0	15.71	39.27					
8.0	22.14	67.36					
9.0	24.98	74.45					
9.5	26.91	77.07					
10.0	31.42	78.54					

7. The amount of water to be carried by a canal excavated in firm gravel is 370 sec. ft. It has side slopes of 2:1 (horizontal component is two times vertical component) and the depth of water is to be 5 ft. or less (Fig. 135).

If the slope is 2.5 ft. per mile, what must be the width at the bottom? (This problem can best be solved by trial.)

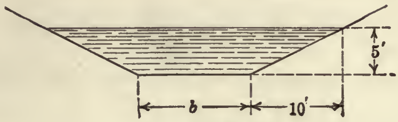


FIG. 135.

CHAPTER IX

HYDRODYNAMICS

104. Dynamic Force Exerted by a Stream.—Whenever the velocity of a stream of water is changed either in direction or in magnitude, a force is required. By the law of action and reaction an equal and opposite force is exerted by the water upon the body which produces this change. This is called a dynamic force in order to distinguish it from static forces due to the pressure of the water.

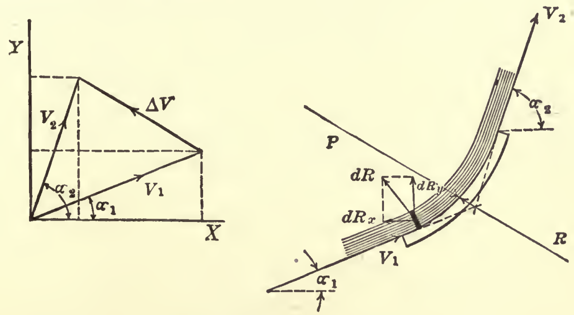


FIG. 136.

Let the resultant force exerted by any body upon the water be denoted by R and its components by R_x and R_y. Let dR be the force exerted upon the elementary mass shown in Fig. 136. Then, since resultant force equals mass times acceleration,

$$dR = dm\frac{dV}{dt}.$$

The summation of the forces acting upon all such elementary masses along the path will give the total force exerted upon the water by the entire body at any instant. But it is necessary to express dm as a function of V or vice versa before this can be integrated. Let the time rate of flow be dm/dt. Then in an interval of time dt there will flow past any section the mass $(dm/dt)dt$, which will be the amount considered. Hence we may write

$$dR = \left(\frac{dm}{dt}\cdot dt\right)\frac{dV}{dt} = \frac{dm}{dt}\left(\frac{dV}{dt}\cdot dt\right)$$

But $(dV/dt)dt = dV$. Our discussion here shall be restricted to

the case where the flow is steady in which case dm/dt is constant and equal to W/g. Therefore

$$dR = \frac{W}{g} dV.$$

In general these various elementary forces will not be parallel and, since integration is an algebraic and not a vector summation, it is necessary to take components along any axes in order to integrate the above. Thus

$$R_x = \frac{W}{g}\int_1^2 dV_x = \frac{W}{g} V_x \Big]_1^2$$

Now at point (1) the value of V_x is $V_1 \cos \alpha_1$ and at (2) it is $V_2 \cos \alpha_2$. Inserting these limits and noting from Fig. 136 that $V_2 \cos \alpha_2 - V_1 \cos \alpha_1 = \Delta V_x$ we have

$$R_x = \frac{W}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1) = \frac{W}{g}\Delta V_x$$

If P indicates the value of the force exerted by the water, which is equal and opposite to R, we shall have

$$P_x = \frac{W}{g}(V_1 \cos \alpha_1 - V_2 \cos \alpha_2) = -\frac{W}{g}\Delta V_x \qquad (81)$$

In similar manner the y component of P will be

$$P_y = \frac{W}{g}(V_1 \sin \alpha_1 - V_2 \sin \alpha_2) = -\frac{W}{g}\Delta V_y \qquad (82)$$

Since $P = \sqrt{P_x^2 + P_y^2}$ and $\Delta V = \sqrt{\Delta V_x^2 + \Delta V_y^2}$, the value of the resultant force is

$$P = \frac{W}{g}\Delta V \qquad (83)$$

The direction of R will be the same as that of ΔV and the direction of P will be opposite to it. It is because P and ΔV are in opposite directions that the minus sign appears in the last terms of equations (81) and (82). Note that ΔV is the *vector* difference between V_1 and V_2.

104a. Dynamic Force (Second Method).—The preceding derivation pictures the total dynamic force exerted by a flowing stream to be the vector sum of all the elementary forces exerted along its path at any instant. The following derivation makes it clear that the total force depends solely upon the initial and terminal conditions and is independent of the path. (Of course the numerical value of the terminal velocity would be affected by friction losses which might be different for different paths.) The former method is based on the principle that resultant force equals mass times acceleration. The second method is based on

the principle of force and momentum, which may be stated as follows: The time rate of change of the momentum of any system of particles is equal to the resultant of all external forces acting on the system. Thus instead of $R = mdV/dt$ we write $R = d(mV)/dt$.

Consider the portion of a filament of a stream in Fig. 136a which is between two cross-sections A and B at the beginning of a time interval dt, and between the cross-sections A' and B' at the end of the interval. Denote by ds_1 and ds_2 the distances moved during the interval by particles at A and B at the beginning.

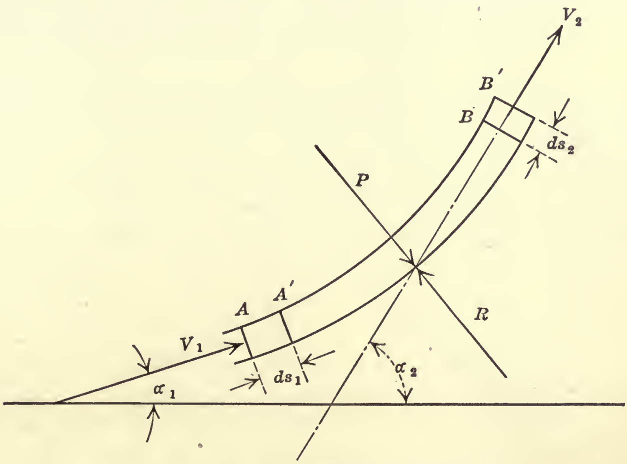


FIG. 136a.

ning. Let F_1 be the cross-section area at A, V_1 the velocity of the particles, and α_1 the angle between the direction of V_1 and any convenient x axis. Let the same letters with subscript (2) apply to B.

At the beginning of the interval the momentum of the portion of the filament under consideration is the sum of the momentum of the part between A and A' and that of the part between A' and B. At the end of the interval its momentum is the sum of the momentum of the part between A' and B and that of the part between B and B'. In the case of steady flow the momentum of the part between A' and B is constant. Hence the change of momentum is the difference between the momentum of the part between B and B' and that of the part between A and A'. Noting that $wF_1ds_1 = wF_2ds_2$, since the flow is steady, the

change in the x component of the momentum during dt is then

$$\frac{wF_1ds_1}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1).$$

If the rate of flow be denoted by W, then

$$wF_1ds_1 = Wdt$$

and the time rate of change of the x component of the momentum is

$$\frac{W}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1).$$

Denoting by R_x the x component of the resultant force which changes the momentum,

$$R_x = \frac{W}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1) = \frac{W}{g}\Delta V_x.$$

From this point the treatment is the same as the last paragraph of the preceding article.

This method has the advantage that it may readily be extended to the case where the flow is unsteady, if desired.

104b. Dynamic Action upon Stationary Body.—In order to find the dynamic force exerted by a stream upon a stationary object, we have merely to find the value of ΔV, assuming the rate of discharge to be known. The following special forms of equations (81) and (82) will often be more convenient. If the x axis be taken parallel to V_1 and the angle between V_1 and V_2 be denoted by θ, Fig. 137 ($\theta = \alpha_2 - \alpha_1$ and $\cos \alpha_1 = 1.0$),

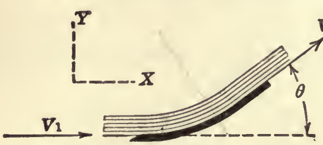


Fig. 137.

$$P_x = -\frac{W}{g}\Delta V_x = \frac{W}{g}(V_1 - V_2 \cos \theta) \tag{84}$$

$$P_y = -\frac{W}{g}\Delta V_y = -\frac{W}{g}V_2 \sin \theta \tag{85}$$

In certain special cases a stream will be equally divided so that the P_y for one half will be equal and opposite to the P_y for the other half. Hence in this special case $P = P_x$. It may be noted also that friction in flow over the body tends to decrease the numerical value of V_2. This increases the value of P_x if θ is less than 90° but decreases it if θ is greater than 90°.

EXAMPLES

1. In Fig. 137 assume that $\theta = 60°$, and that the stream striking the body is a jet 2 in. in diameter with a velocity of 100 ft. per sec. If the frictional loss is such as to reduce the velocity of the stream leaving the

body to 80 ft. per sec., find: (*a*) the component of the force in same direction as the jet, (*b*) the component of the force normal to the jet, (*c*) the magnitude and direction of the resultant force exerted by the water.

Ans. (*a*) 254 lb. (*b*) 293 lb. (*c*) 388 lb. at 49° 08′ with direction of jet.

2. Suppose the jet in problem (1) struck a flat plate normally, what would be the value of the force exerted upon the plate?

Ans. 423 lb.

3. Suppose the jet in problem (1) were completely reversed in direction, or that $\theta = 180°$. If V_2 were 100 ft. per sec., what would be the component of the force in the same direction as the jet? (Compare with problem (2).) What would be the component normal to the direction of the jet?

Ans. 846 lb

4. Suppose that in problem (3) the value of V_2 were reduced to 80 ft. per sec. as in problem (1). What would be the value of the force exerted? (Compare with problem (3).)

Ans. 761 lb.

105. Force Exerted upon Pipe.—When a flowing stream is confined there may be static forces due to pressure as well as dynamic forces due to changes in velocity. Consider the water to be flowing to the right in Fig. 138. Since the velocity is increased from V_1 to V_2, the dynamic force exerted upon the water, according to equation (83), will be

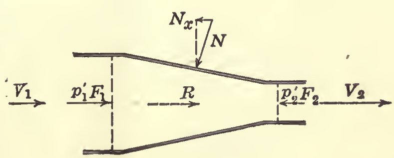


FIG. 138.

$$R = \frac{W}{g}(V_2 - V_1).$$

This force, producing the acceleration of the water, must be the resultant of all the forces acting. The real forces acting upon the volume of water shown are the pressures upon the two ends p'_1F_1 and p'_2F_2 exerted by the rest of the water, and the force N exerted by the pipe walls.[1] If there were no friction this force would be normal to the walls, but actually it will be inclined somewhat from the normal because it must have a frictional component. Let the component of N parallel to the axis of the pipe be denoted by N_x. It may be seen that R must be in the same direction as V_1 and V_2 in Fig. 138. Hence the sum

[1] The N shown in Fig. 138 represents the force for an element only. For a pipe of circular cross-section the resultant force exerted by the walls must be axial.

of all the forces parallel to the axis of the pipe must equal R. Therefore

$$R = p'_1F_1 - p'_2F_2 - N_x.$$

Inserting the value of R given above, it follows that

$$N_x = p'_1F_1 - p'_2F_2 - \frac{W}{g}(V_2 - V_1) \tag{86}$$

It must be remembered that N_x is assumed to be the axial component of the force exerted upon the water by the conical portion of pipe. The force exerted by the water upon the pipe is equal and opposite to this. That is, its magnitude is given by equation (86) but it acts toward the right.

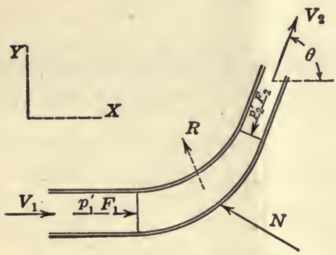


FIG. 139.

If the velocity of the water in a closed passage undergoes a change in its direction, as in the pipe bend shown in Fig. 139, the procedure would be similar to that in the preceding case. The forces acting on the water in the bend are the pressures p'_1F_1 and p'_2F_2 and the pressure exerted by the walls of the pipe, designated by N. By equation (83) the resultant force acting upon this volume of water will be $R = \frac{W}{g}\Delta V$, but R is the resultant of the three forces just mentioned. Since these are vector quantities not in the same straight line, it will be better to take x and y components. Thus we should write

$$R_x = \frac{W}{g}(V_2 \cos\theta - V_1) = p'_1F_1 - p'_2F_2 \cos\theta - N_x$$

and

$$R_y = \frac{W}{g} V_2 \sin\theta = -p'_2F_2 \sin\theta + N_y.$$

Solving these equation we find that

$$N_x = p'_1F_1 - p'_2F_2 \cos\theta + \frac{W}{g}(V_1 - V_2\cos\theta) \tag{87}$$

and

$$N_y = p'_2F_2 \sin\theta + \frac{W}{g} V_2 \sin\theta. \tag{88}$$

But again N represents the force exerted by the pipe bend upon

the water. The force exerted by the water upon the bend will be equal and opposite to this.

It may be seen that these forces tend to move the portion of pipe considered. Hence a pipe should be "anchored" where such changes in velocity occur.

EXAMPLES

1. On the end of a 6-in. pipe is a nozzle which discharges a jet 2 in. in diameter. The pressure in the pipe is 55 lb. per sq. in. and the pipe velocity is 10 ft. per sec. The jet is discharged into the air. (*a*) What is the resultant force acting on the water within the nozzle? (*b*) What is the axial component of the force exerted on the nozzle?

Ans. (*a*) 304 lb. (*b*) 1,250 lb.

2. Water under a pressure of 40 lb. per sq. in. flows with a velocity of 8 ft. per sec. through a right-angle bend having a uniform diameter of 12 in. (*a*) What is the resultant force acting on the water? (*b*) What is the total force exerted on the bend?

Ans. (*a*) 137.8 lb. (*b*) 6,530 lb.

106. Theory of Pitot Tube.—The Pitot tube has been briefly described in Art. 69 and illustrated in Fig. 94. We shall now

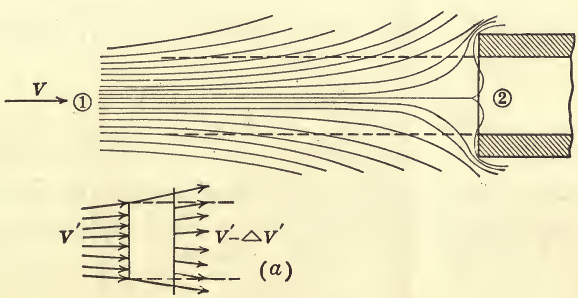


FIG. 140.

consider the dynamic action of the water upon it. In Fig. 140 a Pitot tube is placed with its opening facing upstream, the velocity of the water being denoted by V. The dotted lines in the figure are intended to represent an imaginary cylinder of cross-section area F equal to that of the mouth of the tube and extending to point (1) as far up stream as the influence of the tube is felt.[1]

[1] The method of derivation of the Pitot tube formula given here, as well as some interesting experimental results, will be found in a paper by L. F. Moody, "Measurement of the Velocity of Flowing Water," *Proc.* of the Engineers' Soc. of West. Penn., vol. 30, page 279 (1914).

At point (1) the water within this cylinder has a velocity V and, as it approaches the Pitot tube, its velocity continually decreases until it becomes zero at point (2). But if water flows into this cylinder, bounded by the dotted lines, it must also flow out. It does this along the sides, for since the area F is constant while the velocity is a decreasing quantity, it follows that q (within the cylinder) must become less as the Pitot tube is approached. The conditions for a certain mass of water are therefore as shown in Fig. 140(a). As the velocity of the water decreases the cross-section area must increase for the same value of q. Referring to Fig. 140(a), consider that water flows into this portion of the stream with a velocity V' and leaves it with a velocity $V' - \Delta V'$. If the cross-section area of the stream entering the section is F, we have $W = wq = wFV'$. If the two faces of this volume be taken at an infinitesimal distance apart the velocity will decrease by an amount dV', hence the dynamic force exerted upon this small mass of flowing water will be

$$dR = -\frac{W}{g}\,dV' = -\frac{wF}{g}\,V'dV'.$$

The value of V' varies from V at point (1) to 0 at point (2). Since F is constant we may write

$$R = -\frac{wF}{g}\int_V^0 V'dV' = \frac{wF}{g}\,\frac{V^2}{2}.$$

The dynamic force exerted by the flowing water upon the body of still water within the Pitot tube is equal and opposite to R. If the force be represented by P, we have

$$P = wF\frac{V^2}{2g} \qquad (89)$$

This is the value of the total force distributed over the area F. The intensity of pressure is $p' = wV^2/2g$, or dividing by w we have intensity of pressure in feet of water so that

$$h = \frac{V^2}{2g}. \qquad (90)$$

That this is true has been amply demonstrated by experimental evidence. If the water is under pressure, the Pitot tube will read the sum of the pressure head and the dynamic head, given by equation (90). It is therefore necessary to determine the

pressure head separately or else use a differential manometer, one side of which shall be connected to the Pitot tube and the other to a piezometer tube. The chief source of error in the use of the Pitot tube lies in the measurement of the pressure.

If the Pitot tube is used in a pipe, the pressure reading should be taken by a piezometer tube which does not project within the walls of the pipe and which is at right angles to it, as in the second tube of Fig. 94. If it is necessary, for some reason, to have the tube project into the stream, a correct reading may be obtained if the piezometer orifice is made in a flat plate, the plane of which may be parallel to the stream lines, as is shown by the third tube in Fig. 94. Or the orifice may be made in the side of a smooth tube, whose axis is parallel to the stream lines and whose closed upstream end is pointed so as to diminish eddy disturbances.[1]

107. Water Hammer and Surges in Unsteady Flow.—In all the rest of this book the treatment is restricted to cases of steady flow, but in the present article a brief description will be

[1] The Pitot tube formula has often been derived by an incorrect application of the principles of Art. 104. If a jet of water with cross-section area F impinges normally upon a flat plate, the dynamic force will be

$$P = \frac{W}{g}\Delta V = \frac{wFV}{g}V = wF\frac{V^2}{g}.$$

This is twice the value given by equation (89), and dividing this by the area of the Pitot tube orifice, which is also assumed equal to F, the intensity of pressure in feet of water is apparently $h = \frac{V^2}{g}$. But this reasoning is incorrect; for a flat plate of an area the same as that of a jet would not be able to deflect all the water through an angle of 90°. Experiment shows that the dynamic pressure exerted by a circular jet is distributed over a circular area whose diameter is at least twice that of the jet. Therefore if the entire stream of water is to be deflected through an angle of 90° the area of the plate must be at least four times that of the jet. Dividing P by $4F$ we should have the average intensity of pressure to be $p' = wV^2/4g$. It is found experimentally that the maximum intensity of pressure at the center of the plate in feet of water is $V^2/2g$, and that this pressure diminishes in intensity as the outer margins of the area in question are approached, as shown in Fig. 140(b).

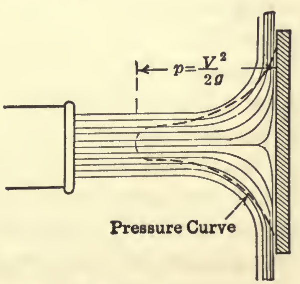


FIG. 140(b).

See "Pitot Tube Formulas—Facts and Fallacies" by B. F. Groat, *Proc. of Engineers' Soc. of West. Penn.*, vol. 30, page 324 (1914).

given of the problems of unsteady flow that are of the most practical importance. An adequate mathematical treatment of unsteady flow would occupy too much space to warrant its inclusion here and no attempt will be made to do more than record some accepted results.

In the event of a valve at C in Fig. 141(a) being rapidly closed in a short interval of time Δt, the velocity of the water in the pipe will be abruptly reduced to zero. But in so doing there will be a considerable rise in pressure within the pipe, which may be much greater than any static pressure that could possibly exist in the given pipe. This high pressure lasts for an instant only, and then follows a periodic fluctuation of pressure which finally dies out, if the pipe does not burst in the meantime. This is known as water hammer.

What happens is that the lamina of water next to the valve at C is brought to rest and is then compressed by the rest of the column of water flowing up against it. At the same time

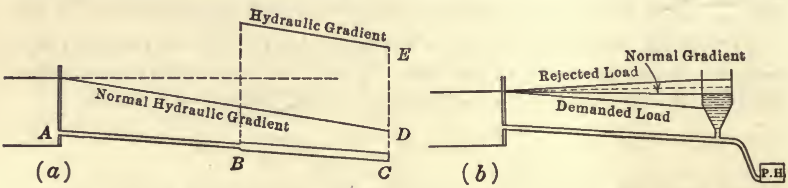


Fig. 141.

the walls of the pipe surrounding this lamina will be stretched by the excess pressure. The next lamina of water will be brought to rest by the first and so on. It is seen that the volume of water in the pipe does not behave as a rigid body but that the phenomena is affected by the elasticity of the water and the pipe. Thus the cessation of flow and the increase of pressure progresses along the pipe as a wave action. After a short interval of time the volume of water BC will have been brought to rest, while the water in the length AB will still be flowing with its initial velocity, and with its initial pressure. But the volume of water in BC will be under a much higher pressure due to the compression it is under and the pipe walls will be stretched. The excess pressure DE is the same for all portions of the pipe and is independent of the length of the pipe.

Finally the pressure wave will have reached the reservoir and the entire volume of water will be at rest. But, owing to its

compression as well as the tension of the pipe, the flow will tend to start toward the reservoir. Thus a wave of rarefaction proceeds from A to C, and so on until the waves die out. If the valve is alternately opened and closed at just the proper intervals of time it is possible to add one pressure wave on top of another, so that there is no limit to the maximum pressure that might be attained.

The velocity with which this pressure wave progresses along the pipe will be given by the following formula:[1]

$$V_\omega = 4700 \sqrt{\frac{E}{E + 300{,}000 \frac{d}{t}}} \tag{91}$$

where V_ω = velocity of pressure wave in feet per second, E = modulus of elasticity in tension of the material composing the pipe in pounds per square inch, and d/t is the ratio of the diameter of the pipe to the thickness of the walls, which means that both d and t must be in the same units. The values of E for steel, cast iron, and wood are about 30,000,000 lb. per sq. in., 15,000,000 lb. per sq. in., and 1,500,000 lb. per sq. in. respectively. For pipes of ordinary dimensions the velocity of this pressure wave will be about 3,300 ft. per sec. In any event it will be less than 4,700 ft. per sec., which is the velocity of sound in water or the velocity with which a pressure wave would be propagated in water in a rigid pipe. See equation (91).

The time required for a pressure wave to travel the length of the pipe, or the time that it takes for the entire mass of water to be brought to rest will be

$$T = l/V_\omega \tag{92}$$

The total force exerted may be determined by applying the principle that force equals mass times acceleration. Since the volume of water is a non-rigid body we must deal with the acceleration of the mass center. The pressure wave travels at a uniform rate; hence the velocity of the mass center is uniformly retarded. Therefore the acceleration may be determined by dividing the change in velocity by the time required for the change to occur. The velocity of all the water, and

[1] The complete expression involves the volume modulus of elasticity of the water, the density of the water, and the value of g. Using average values of these quantities the above is obtained.

hence that of the mass center, decreases from V to 0 in the time T. Thus we may write

$$P = \frac{wFl}{g}\frac{V}{T} = \frac{wFl}{g}\frac{VV_\omega}{l} = wF\frac{VV_\omega}{g}$$

Dividing by the area F and also by w we obtain intensity of pressure in feet of water. If this excess pressure, due to water hammer, be denoted by p_m, we have

$$p_m = \frac{VV_\omega}{g} \tag{93}$$

It will be noted that this pressure increase is independent of the length of the pipe line.[1]

The length of the pipe line enters into the problem in this way. The time required for a pressure wave to make the round trip from the gate to the reservoir and back is twice the value given by equation (92). It has been found that the pressure created is independent of the time of closure of the gate provided that it is closed in less time than it takes for a pressure wave to make the round trip. That is the gate must be closed in less time that $2l/V_\omega$. If the time is greater than this the pressure is reduced in proportion as follows:

$$p = p_m\frac{T_r}{T'} \tag{94}$$

where T_r is the time for a round trip of the pressure wave, T' is any time greater than this and p is the pressure that will be attained in such a case.

It is seen that in a short pipe line the value of T_r is so small that it is nearly impossible to close the gate quickly enough to produce water hammer of maximum intensity. In a long pipe line it is necessary to close the gate slowly in order to prevent this and the longer the pipe line the slower the gate must be closed.

For the sake of clearness in explanation it has been assumed in the preceding discussion that the velocity of the water has

[1] The subject of water hammer has been experimentally investigated by Joukovsky of Moscow on pipes of 2-, 4-, 6-, and 24-in. diameter and with lengths ranging from 1,050 ft. to 7,007 ft. He found the results to agree with the formulas given. For a résumé of his work see "Water Hammer" by Simin, *Trans.* Amer. W. W. Ass'n, 1904.

been reduced to zero. But the results are true for any reduction in velocity, it being simply necessary to substitute ΔV for V.

Water hammer may be prevented by the use of slow closing valves, or its effects diminished by the use of automatic relief valves which permit water to escape when the pressure exceeds a certain value. Also air chambers of suitable size provide cushions which absorb a great portion of the shock. But for water power plants a standpipe or surge chamber such as is shown in Fig. 141(*b*) has certain marked advantages.

In the event of a sudden decrease in load on a water power plant it would be necessary for the governors to rapidly reduce the amount of water supplied to the wheels, if the speed of the latter is to be maintained constant. A surge chamber provides a place into which this excess water may flow and thus avoids water hammer in the supply pipe. The inertia of the mass of water flowing down this supply pipe may be such as to carry the water level above the static level and produce an ascending hydraulic gradient. But this excess pressure acts as a retarding force on the mass of water in the pipe line and thus reduces its velocity. In any event the temporary water level in this surge chamber will be higher than the normal value and hence it will reduce the velocity of flow too much. The result will be that there will be fluctuations of velocity in the pipe line accompanied by "surges" of the water level in the chamber until a condition of equilibrium is finally reached. The phenomenon is very similar to that of water hammer as there are periodic alternations in pressure and velocity, but the pressure variations are much less severe.

The surge chamber fulfills another valuable function in that it not only takes care of excess water in case of a sudden reduction of flow but it also provides a source of water supply in the event of a sudden demand. When the load on the plant increases it is necessary to supply more water to the wheels at once. If the pipe line is long it may take some time to accelerate the entire mass of water and in the meantime the head at the plant has dropped considerably in order to provide an accelerating force. But the surge chamber permits a certain amount of water to flow out during that period. To be sure enough flows out so that the hydraulic gradient drops below its normal level for the new load, but the effect is not as serious as if the surge chamber were absent.

In Fig. 195 is shown a surge chamber of large size. It is at the end of a pressure tunnel which is approximately 7.76 miles in length, with an average cross-section area of 100 sq. ft. and in which is a maximum velocity of flow of 10 ft. per sec.[1]

EXAMPLES

1. A cast-iron pipe line is 24 in. in diameter and the metal is 0.75 in. thick. If the velocity of water in it is 6 ft. per sec., find the pressure that would be created by the instantaneous closure of a valve.

Ans. 296.5 lb. per sq. in.

2. If the pipe line in problem (1) were 500 ft. long, within what length of time must the valve be closed to produce the same pressure as an instantaneous closure? What would the length of time be if it were 5,000 ft. long?

Ans. $T = 0.27$ sec., 2.7 sec.

3. If the pipe line in problem (1) were 7,000 ft. long what would be the time of closing the valve so that the pressure produced were only one-third of that in case of instantaneous closure?

Ans. 11.43 sec.

108. Relation between Absolute and Relative Velocities.—In much of the work that follows it will be necessary to deal with

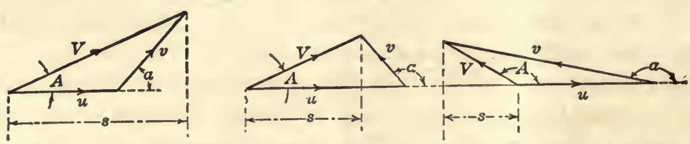


FIG. 142.—Relation between absolute and relative velocities.

both absolute and relative velocities of the water. The absolute velocity of a body is its velocity relative to the earth. The relative velocity of a body is its velocity relative to some second body which may in turn be in motion relative to the earth. The absolute velocity of the first body is the vector sum of its velocity relative to the second body and the absolute velocity of the latter. The relation between the three is shown in Fig. 142.[2]

[1] W. F. Durand, "Control of Surges in Water Conduits," *Journal*, A. S. M. E., June, 1911.

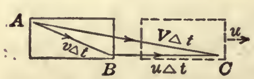


FIG. 143.

See also, "The Differential Surge Tank" by R. D. Johnson, *Trans.* A. S. C. E., vol. 78, page 760 (1915).

[2] A clearer idea of this relationship may be obtained from the illustration in Fig. 143. Suppose a raft is moving downstream with a uniform velocity u. A man on the

109. Dynamic Action upon Moving Body.—The dynamic force exerted by a stream upon a moving object can be determined by a direct application of equation (83). The principal difference between action upon a stationary and upon a moving object is that in the latter case we need to deal with both absolute and relative velocities, and the determination of ΔV may be more difficult.

Let us assume a stream of cross-section area F_1 and absolute velocity V_1 to flow upon a moving object. The rate of discharge will be F_1V_1 so that $W = wF_1V_1$. But this may not be the amount of water that strikes the object per second. For instance, if the body is moving as rapidly as the stream and in the same direction it is clear that none of the water will strike it. The amount of water which will flow over any object is proportional to the velocity of the water relative to the object itself. If we denote by W' the pounds of water striking the moving body per second, then $W' = wf_1v_1$, where f_1 is the cross-section area normal to v_1.

As a *special* case to illustrate the above let us consider a jet from a nozzle acting upon a body moving in the same direction as the jet with a velocity u. In this case, since u and V_1 are in the same direction, $F_1 = f_1$ and $v_1 = V_1 - u$. But in general we should have a vector relationship as shown in Fig. 142. However, for this particular case $W' = wF_1(V_1 - u)$. The reason that less water strikes the body per second than issues from the nozzle per second is that the body is moving away from the latter and there is an increasing volume of water between the two. If we consider an impulse wheel with a number of vanes around its circumference, the above is true for one vane only. But the wheel as a whole does not move away from the nozzle and hence the amount of water striking the wheel may equal that issuing from the nozzle. The explanation is that two or more vanes are acted upon by the water at the same time as is shown in Fig. 202.

In order to find ΔV it is necessary to determine the magnitude

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raft at A walks over to the diagonally opposite corner at a uniform rate. But by the time he reaches B the latter point on the raft will have moved downstream to point C. Thus the path of the man relative to the raft is AB but relative to the earth it is AC. Since the velocities are all uniform they are all proportional to the distances traversed in this interval of time.

and direction of the absolute velocity at outflow, assuming the V_1 to be known. The direction of the relative velocity at outflow is tangent to the surface of the body at that point. The angle between v_2 and the positive direction of u is denoted by a_2. Assuming u and a_2 to be known we may proceed as follows. Solve the vector triangle (Fig. 142) for v_1 if V_1, u, and A_1 are known. In flow over the object there may be a loss of energy due to friction such that v_2 is less than v_1 and hence we may write $v_2 = nv_1$, where n is less than unity. Having now the values of v_2, u, and a_2 we may solve the vector triangle (Fig. 142) for V_2 and A_2. This enables us to find the value of ΔV.

Fig. 144.

As a *special* case to illustrate the procedure let us consider Fig. 144 where the jet strikes an object moving with a uniform velocity in the *same* direction as the jet. Hence $A_1 = 0°$. Let us assume the x axis parallel to the jet and use equations (84) and (85).

$$P_x = -\frac{W'}{g}\Delta V_x = \frac{W'}{g}(V_1 \cos A_1 - V_2 \cos A_2)$$

$$\begin{aligned} V_1 \cos A_1 &= V_1 \\ V_2 \cos A_2 &= u + v_2 \cos a_2 \text{ from Fig. 142.} \\ &= u + nv_1 \cos a_2 \end{aligned}$$

Since $A_1 = 0°$, $v_1 = V_1 - u$ and hence for this special case

$$V_2 \cos A_2 = u + n(V_1 - u) \cos a_2.$$

Substituting this in the above and reducing we have

$$P_x = \frac{W'}{g}(1 - n \cos a_2)(V_1 - u) \tag{95}$$

$$= \frac{wF_1}{g}(1 - n \cos a_2)(V_1 - u)^2 \tag{96}$$

The value of P_y may be determined in a similar manner, noting that in this case $V_2 \sin A_2 = v_2 \sin a_2$. It must be borne in mind that these equations are true only for the special case considered and that they apply to a single moving object. The general case would differ from the above only in the fact that v_1 is then the vector and not the algebraic difference between V_1 and u.

It may be seen that the magnitude of the force exerted by a jet depends both upon the shape and the velocity of the object struck. In fact the same value of ΔV might be had with either a stationary or a moving object or with moving objects having different velocities provided only that their shapes, which in this case means their values of a_2, were suitable.

As another illustration of the foregoing let us consider the dynamic force exerted by a jet of water upon the moving body from which it issues. When a stream of water issues from any device, such as the vessel shown in Fig. 145, a force is required to accelerate the water and impart to it the velocity it has upon leaving. This force is exerted upon the particles of water flowing out the orifice by adjacent particles of water and ultimately by the walls of the vessel. By the law of action and reaction an equal and opposite force will be exerted upon the vessel. It is impossible to analyze this reaction in detail but we know that its total value will be given by an application of equation (83).

Let us assume that the vessel in Fig. 145 moves to the left with a uniform velocity u, and that the orifice is so small compared to the size of the vessel that the relative velocity of the water in the latter may be neglected as may also the change in h. Then $V_1 = u$. If the jet issues from the orifice with a velocity v_2 the absolute velocity of the jet will be $V_2 = u - v_2$. Hence $\Delta V = V_1 - V_2 = u - (u - v_2) = v_2$. Therefore

$$P = \frac{W}{g} v_2 = \frac{wF}{g} v_2^2 \tag{97}$$

This might have been determined more directly if another proposition had been previously established. That is that for any case whatever $\Delta V_x = \Delta v_x + \Delta u_x$, where the subscript x merely denotes a component along any axis. In this case, since u is constant, it may be seen that $\Delta V = \Delta v$ and is independent of the velocity of the vessel.

Since $v_2 = c_v\sqrt{2gh}$, we may write equation (97) as

$$P = 2c_v^2 wFh \tag{98}$$

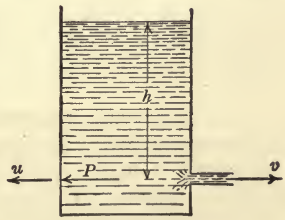


Fig. 145.

If losses of energy be neglected in both cases, it may be seen that the reaction of the jet in Fig. 145 is equal to the force of impact upon

a flat plate, normal to the jet, providing the area of the plate be large enough to deflect the water through 90°.[1]

EXAMPLES

1. A jet of water 3 in. in diameter has a velocity of 120 ft. per sec. It strikes a vane with an angle $a_2 = 90°$ which moves in the same direction as the jet with a velocity u. Assume that the loss in flow over this vane is such that $n = 0.9$. When u has values of 0, 40, 60, 80, 100, and 120 ft. per sec., find values of: (*a*) W', (*b*) $V_2 \cos A_2$, (*c*) P_x.

2. If the jet in the preceding problem strikes a vane for which $a_2 = 180°$, all other data remaining the same, find values of: (*a*) W', (*b*) v_2, (*c*) V_2, (*d*) P.

110. Impulse and Reaction of a Jet.—When a stream of water strikes any object, the dynamic force exerted, due to the impact, is often termed the *impulse* of the jet. The dynamic force exerted by the jet upon the vessel from which it issues is often called the *reaction* of the jet. But in both cases the force is due to the change that is produced in the velocity of the water.

111. Distinction between Impulse and Reaction Turbine.—The distinction between these two fundamental types of turbines according to the action of the water as defined in the preceding article was proper in primitive wheels. But in modern turbines the so-called impulse at entrance and reaction at exit may both be effective in either type. A better classification is as to "pressureless" and "pressure" turbines.

Thus the water within the impulse wheel is not confined but is open to the air, while in the reaction turbine the wheel passages must be completely filled with water. In the former the pressure remains unchanged in flowing over the buckets, while with the latter the pressure decreases during flow through the runner. The energy delivered to the impulse turbine is all kinetic, while that delivered to the reaction turbine is partly kinetic and partly "pressure energy."

[1] The hydrostatic pressure on an area equal to that of the jet F at a depth h is given by wFh. The fact that this is only half the dynamic pressure considered is of no significance. As has already been pointed out, the dynamic pressure on a plate is distributed over an area much larger than that of the jet and we have not increased the intensity of pressure in either case.

But it is well to bear in mind that in both types the essential thing is that the velocity of the water must be altered in order that a dynamic force may be exerted upon the wheel. And in both types it is necessary if high efficiency is attained that the absolute velocity of the water as it leaves the wheel be low, since this velocity represents so much kinetic energy that is not utilized.

112. Theorem of Angular Momentum.—In Fig. 146 we will suppose a particle of mass dm to be located at a point whose coordinates are x and y and to be moving with a velocity V. The momentum of this particle will be $dm.V$. The moment of momentum is called angular momentum. For this particle the angular momentum is $dm.V \times r \cos A$. Since the moment of

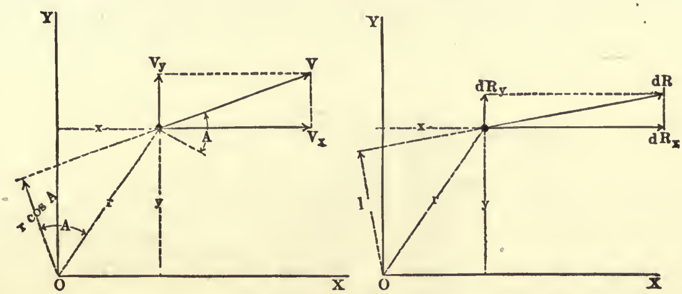


FIG. 146.

any quantity is the algebraic sum of the moments of its components, we may write

$$dm.rV \cos A = dm.V_y x - dm.V_x y = dm\left(\frac{dy}{dt}x - \frac{dx}{dt}y\right).$$

Differentiating the above with respect to time we obtain

$$dm\frac{d(rV \cos A)}{dt} = dm\left(\frac{dy}{dt}\cdot\frac{dx}{dt} + x\frac{d^2y}{dt^2} - \frac{dx}{dt}\cdot\frac{dy}{dt} - y\frac{d^2x}{dt^2}\right)$$
$$= dm(a_y x - a_x y)$$

where a denotes acceleration, with a_x and a_y as its axial components. ($V_x = dx/dt$, $a_x = dV_x/dt = d^2x/dt^2$, etc.). If the resultant force acting on the particle be denoted by dR, $dma_y = dR_y$, and $dma_x = dR_x$. Thus

$$dmd(rV \cos A)/dt = dR_y x - dR_x y.$$

The torque exerted upon the particle with respect to point O is seen to be $dR \times l$. By the principle of moments

$$dR \times l = dR_y x - dR_x y$$

Thus, if T denotes torque so that $dR \times l = dT$,

$$dT = dm d(rV \cos A)/dt \qquad (99)$$

That is, the time rate of change of the angular momentum of any particle with respect to an axis is equal to the torque of the resultant force on the particle with respect to the same axis.

113. Torque Exerted upon Turbine by Water.—When a stream flows through a turbine runner in such a way that its distance from the axis of rotation remains unchanged, the dynamic force can be computed by the principles of Art. 104. But when the

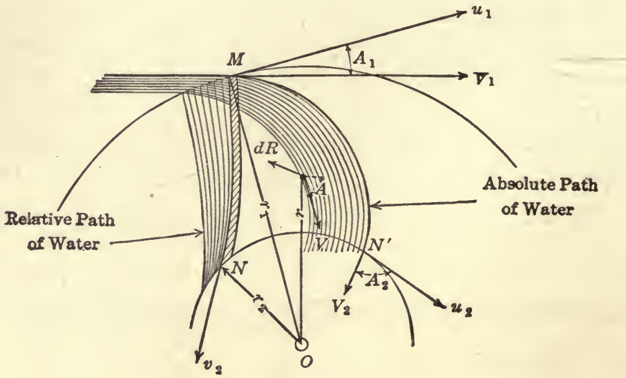


Fig. 147.—Hydraulic turbine.

radius to the stream varies it is not feasible to compute a single resultant force. It is necessary to find the total torque exerted by summing up the elementary torques produced by all the elementary forces.

In Fig. 147 let MN represent a vane of a wheel which may rotate about an axis O perpendicular to the plane of the paper. Water enters the wheel at M and since the wheel is in motion, by the time the water arrives at N on the vane that point of the vane will have reached position N'. Thus the absolute path of the water is really MN'.

Let us consider an elementary volume of water forming a hollow cylinder, or a portion thereof, concentric with O. Let the

time rate of mass flow be dm/dt. Then in an interval of time dt, there will flow across any section the mass $(dm/dt)dt$. Let this be the mass of the elementary volume of water we are to consider. Substituting this value in equation (99) we have

$$dT = \left(\frac{dm}{dt}\cdot dt\right)\frac{d(rV\cos A)}{dt} = \left(\frac{dm}{dt}\right)\cdot d(rV\cos A)$$

The above procedure is similar to that in Art. 104, the only difference being that here we are dealing with the moment of a force. In the case of steady flow $(dm/dt) = (W/g)$ and thus

$$T = \frac{W}{g}\int_1^2 d(rV\cos A)$$

Integrating between limits we have the value of the torque exerted by the wheel upon the water, or by changing signs, the value of the torque exerted by the water upon the wheel. Therefore the torque exerted upon the wheel by the water is

$$T = \frac{W}{g}(r_1V_1\cos A_1 - r_2V_2\cos A_2) \tag{100}$$

It may be seen that $V\cos A$ is the tangential component of velocity. It is convenient to represent this by a single letter s and so

$$T = \frac{W}{g}(r_1s_1 - r_2s_2) \tag{101}$$

It is immaterial in the application of this formula whether the water flows radially inward, as in Fig. 147, radially outward, or remains at a constant distance from O. In any case r_1 is the radius at entrance and r_2 is that at exit.

114. Torque Exerted upon Water by Centrifugal Pump.—The derivation of an expression for the torque exerted by a pump impeller is exactly the same as in the preceding article except for the substitution of limits. As turbines are universally constructed there are certain guide vanes surrounding the runner which give the water its initial direction A_1. Thus any angular momentum which the water has as it flows into the runner is imparted by the guides. In some centrifugal pumps there are guide vanes within the "eye" of the impeller, which give the water a definite direction as it flows into the latter. For such pumps we should merely reverse equation (101), since we desire the torque exerted upon the water and not by it.

But for the usual type of centrifugal pump there is nothing at

entrance to the impeller to give the water any definite direction. In fact the water enters a given impeller at different angles depending upon conditions of operation, and, while the ordinary pump is designed for a "radial entrance" this can be had only for the normal rate of discharge. But any angular velocity with which the water enters the impeller has been really derived from the impeller and so should be credited to the latter. Hence we should take as our lower limit of integration not (1) where the water enters the impeller but some point back in the suction pipe, where the angular momentum is zero.[1] Thus we should have

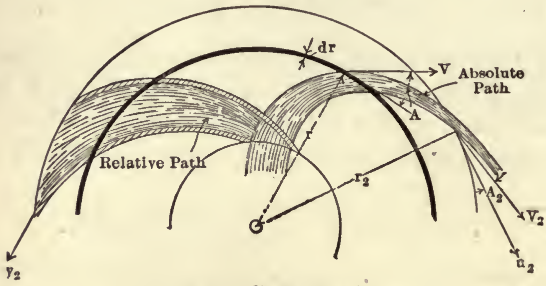


FIG. 148.—Centrifugal pump.

for the ordinary centrifugal pump under all conditions of operation,

$$T = \frac{W}{g} r_2 s_2 \tag{102}$$

115. Power.—If torque be multiplied by angular velocity the product represents power. Angular velocity must be expressed in radians per second, hence, if N = r. p. m., the horsepower will be

$$\text{Hp.} = \frac{T \times 2\pi N}{33{,}000} \tag{103}$$

If T has the value given by equation (101), the power will be less than that supplied to the turbine by the water, the difference being the power lost in hydraulic friction within the turbine case, runner, and draft tube. It is greater than the power delivered by the turbine by an amount equal to the losses in mechanical friction. It is the power that is actually delivered to the shaft

[1] This point is fully discussed by the author in "Centrifugal Pumps," page 61.

from the water, and is analogous to the indicated power of a steam engine.

If T has the value given by equation (102) the power given by equation (103) will be less than that required to run the pump by an amount equal to the mechanical losses and it will be greater than the power delivered in the water by the amount of the hydraulic losses. It represents the power actually expended by the impeller on the water and is analogous to the indicated power of a reciprocating pump.

While equations (101) and (102) are true, they are of little real service because the proper values to use in them are often not known with exactness. The precise values of velocities and directions of stream lines are difficult matters to determine. Since water does not fulfill the ideal conditions assumed, it will be found that these equations often yield numerical results that are considerably in error.[1]

It should be noted that power can be expressed in the following forms; as well as by equation (103).

$$\text{Hp.} = \frac{Pu}{550} \tag{104}$$

$$\text{Hp.} = \frac{WH}{550} = \frac{wqH}{550} = \frac{qH}{8.81} \tag{105}$$

In the last expression H may represent any head for which the corresponding power is desired.

116. Definitions of Heads.—In turbine and pump practice we find the word "head" used to express several different physical quantities. The head h under which a turbine or pump actually operates is explained in Arts. 88 and 89. But there is energy lost in hydraulic friction within the runner or impeller and thus there is head lost. We shall designate this by h'. And in the turbine a portion of the energy of the water is delivered in the form of mechanical work and the head thus utilized by the runner we shall denote by h''. In the centrifugal pump h'' will represent the head actually imparted to the water by the impeller.

Thus for the turbine we shall have $h'' = h - h'$ and for the pump $h = h'' - h'$.

117. Definitions of Turbine Efficiencies.—The word "efficiency" without any qualifying adjective is always understood

[1] See "Hydraulic Turbines," page 83 and "Centrifugal Pumps," pages 76, 81, 82, and 84.

to mean gross or total efficiency. It is the ratio of the developed or brake horsepower to the power delivered in the water to the turbine. That is

$$e = \text{b.hp.}/\text{w.hp.} \tag{106}$$

Mechanical efficiency is the ratio between the power delivered by the machine and the power delivered to its shaft by the water. If q represents the total turbine discharge while q' equals the amount of leakage through the clearance spaces, the actual amount of water doing work is $q - q'$. Hence

$$e_m = \text{b.hp.}/\frac{w(q - q')h''}{550} \tag{107}$$

Hydraulic efficiency is the ratio of the power actually delivered to the shaft to that supplied in the useful water. That is

$$e_h = w(q - q')h''/w(q - q')h = h''/h \tag{108}$$

Volumetric efficiency is the ratio of the water actually used by the runner to total amount discharged. Thus,

$$e_v = (q - q')/q \tag{109}$$

The total efficiency is the product of these three separate factors. That is,

$$e = e_m \times e_h \times e_v \tag{110}$$

118. Definitions of Pump Efficiencies.—The various pump efficiencies are similar to those for the turbine. The total efficiency is

$$e = \text{w.hp}/\text{b.hp.} \tag{111}$$

The mechanical efficiency is

$$e_m = \frac{w(q + q')h''}{550}/\text{b.hp.} \tag{112}$$

The hydraulic efficiency is

$$e_h = w(q + q')h/w(q + q')h'' = h/h'' \tag{113}$$

The volumetric efficiency is

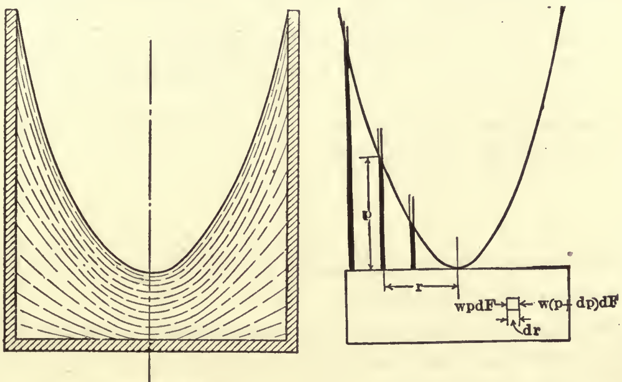
$$e_v = q/(q + q') \tag{114}$$

As in equation (110) the total efficiency is the product of these three.

3 **119. Centrifugal Action or Forced Vortex.**—If a vessel containing a liquid is rotated about its axis, the liquid will tend to

rotate at the same speed. If the vessel is open the free surface of the water will assume the curve shown in Fig. 149*a*. If the water is confined within a closed vessel, which it completely fills so that it cannot change its position, the pressure along a horizontal line will vary in the same way as in the preceding case. In fact if piezometer tubes were connected to the vessel the water would rise in them as shown in Fig. 149*b*. Since the hydraulic gradient represents the free surface that corresponds to the actual pressure conditions, it is seen that the two cases are equivalent. Such a rotation is sometimes described as a forced vortex because the water is forced to rotate by external forces.

The variation in pressure in such a body of water may be



(*a*) OPEN VESSEL (*b*) CLOSED VESSEL

FIG. 149.—Forced vortex.

found in the following manner. If we take an elementary volume in Fig. 149 whose length along the radius is dr and whose area normal to the radius is dF, we have an elementary mass $wdFdr/g$ moving in a circular path. This mass has an acceleration u^2/r or $\omega^2 r$, directed toward the axis of rotation. Consequently the accelerating or resultant force is $(wdFdr/g)\omega^2 r$ directed toward the axis. The intensity of pressure on the two faces of the elementary volume differs by $dp' = wdp_r$. The value of the resultant force is therefore $wdp_r dF$. Consequently,

$$wdp_r dF = (wdFdr/g)\omega^2 r$$
$$dp_r = (\omega^2/g)rdr.$$

But this expression shows only the difference of pressure along

the radius and in the same horizontal plane. If we move along a path parallel to the vertical axis of rotation so that the radius is constant, the pressure decreases directly as the elevation increases. Thus,

$$dp_z = -\,dz.$$

The variation of the intensity of pressure in *any* direction may be found by combining the two preceding equations. Thus, in general, when both r and z vary,

$$dp = -\,dz + (\omega^2/g)rdr \tag{115}$$

To find the equation of the free surface or any surface of equal pressure we need only place dp equal to zero. We then have

$$\int dz = (\omega^2/g)\int rdr$$
$$z = r^2\omega^2/2g + \text{constant}.$$

To determine the constant we may assume $z = 0$ when $r = 0$. Thus the constant $= 0$ so that

$$z = r^2\omega^2/2g \tag{116}$$

From this it may be seen that the free surface or any surface of equal pressure is a paraboloid.

To find the variation of pressure in the same horizontal plane we need only assume $dz = 0$, and integrating between limits we obtain

$$p_2 - p_1 = (r_2^2 - r_1^2)\omega^2/2g = (u_2^2 - u_1^2)/2g \tag{117}$$

For the difference in pressure between any two points we must integrate equation (115) which gives

$$p_2 - p_1 = z_1 - z_2 + u_2^2/2g - u_1^2/2g \tag{118}$$

$z_1 + p_1 + \frac{u_2^2}{2g} = z_2 + p_2 + \frac{u_1^2}{}$

120. Free Vortex.—Where external forces are applied to the water, as in the preceding case, we have a *forced* vortex. Where no external forces are applied but the water rotates by virtue of its own angular momentum, previously derived from some source, we have a *free* vortex.

A free *circular* vortex consists of a body of water in rotation without any appreciable flow so that the stream lines are con-concentric circles. Since no torque is exerted on the water, neglecting friction, it follows that there can be no change in angular momentum. Since angular momentum is proportional to

$rV \cos A$, it is apparent that $V \cos A$ varies as $1/r$, as the angular momentum is constant. Since the stream lines are circles $V \cos A$ is the value of V itself. Since no energy is imparted to the water, we will have, if friction is neglected,

$$H = p + z + \frac{V^2}{2g} = \text{constant.}$$

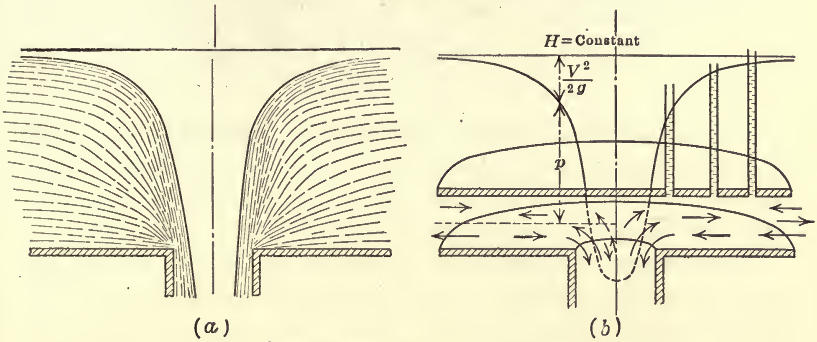


FIG. 150.—Free vortex.

The free surface of such a vortex is shown in Fig. 150a. A familiar example of such a surface is when water entering a vertical pipe sets up a rotation and sucks air down the center, though of course that velocity then has a radial component. Since $V \times r$ = constant, it is seen that when $r = 0$, the value of V is infinity and $p + z = -$ infinity. Since this is impossible, we never have the free vortex exist with extremely small values of r. (If p is constant or equal to zero as in the case of the free surface, values of z will give the elevation of the surface. If z is constant values of p will give the hydraulic gradient, which is the same curve.)

Considering a pure *radial* flow between two circular plates, either inward or outward, as in Fig. 150b, and letting b equal the distance between the two plates we have $q = 2\pi rbV$. For steady flow q is constant and hence rbV is constant. And if the plates are parallel rV is constant. Thus V varies as $1/r$, as in the preceding case.

A free *spiral* vortex is a combination of radial flow and circular flow. The velocities in the two cases above are then merely components of the velocity in the latter case. Since each component varies as $1/r$ it follows that the velocity in a spiral flow

also varies as $1/r$. Also since both components vary at the same rate, the angle A, which the velocity makes with the tangent to the circle, remains constant. Thus the free stream line is the equi-angular or logarithmic spiral. Since the total head is also constant here, neglecting friction, the free surface or the hydraulic gradient, as the case may be, is the same as shown in Fig. 150.

Since H is constant we may write, considering the two points to be at the same elevation

$$H = p_1 + V_1^2/2g = p_2 + V_2^2/2g$$

$$p_2 - p_1 = [1 - (r_1/r_2)^2]V_1^2/2g \tag{119}$$

Of course the effect of friction is always to make p_2 smaller than would be given by the above, since H is not constant. (Note that the flow may be either inward or outward.) It may be seen that as r increases V decreases and p approaches H as a limit.

The principal application of Arts. 119 and 120 is in the case of the centrifugal pump. In Fig. 221 may be seen a forced vortex in the impeller from (1) to (2) and a free vortex in the casing between (2) and (3). It may be added that the foregoing treatment may readily be extended to the case where the width b is variable.

120a. Flow Through Rotating Channel.—We shall now extend the treatment of the forced vortex (Art. 119) to the more general case where the water flows through the rotating vessel. It has been seen that with the free vortex the hydraulic gradient or the resulting equation (119) is the same whether the water merely rotates in concentric circles or flows in spiral paths, but with the forced vortex the equation will be found to be somewhat different when flow occurs. The reason is that in the free vortex no energy, save that lost by friction, is imparted to or taken from the water; but in case water flows through a rotating vessel, energy is delivered either to it or by it.

The torque exerted by the water on a moving object is given by equation (100). When multiplied by the angular velocity, $\omega = u/r = u_1/r_1 = u_2/r_2$, this reduces to

$$T\omega = (W/g)(u_1V_1 \cos A_1 - u_2V_2 \cos A_2)$$

But torque times angular velocity is power or $T\omega = Wh''$. Hence

$$h'' = \frac{u_1V_1 \cos A_1 - u_2V_2 \cos A_2}{g} \tag{119a}$$

This h'' is the head given up by the water and converted into mechanical work. But if h'' is found to have a negative value it signifies that energy is being delivered to the water by the vessel instead of being abstracted from it. In practice the former action takes place in a turbine and the latter in a centrifugal pump.

The general equation of energy (21) may be applied to this case as well as any other provided that in addition to the head lost in hydraulic friction we consider that lost (or gained) in mechanical work. Hence we may write

$$H_1 - H_2 = h' + h''$$

where h' represents the head lost in hydraulic friction. This may be expanded by substituting $p + z + V^2/2g$ for H and the value given by equation (119*a*) for h''. Noting that by trigonometry $V^2 = v^2 + u^2 + 2uv \cos a$ and $V \cos A = u + v \cos a$, we may replace the absolute velocities by the relative velocities so that the above readily becomes

$$\left(p_1 + z_1 + \frac{v_1^2 - u_1^2}{2g}\right) - \left(p_2 + z_2 + \frac{v_2^2 - u_2^2}{2g}\right) = h' \qquad (119b)$$

If there is no flow, both v_1 and v_2 become zero and the equation reduces to that of the forced vortex, equation (118). If there is no rotation, both u_1 and u_2 become zero, the relative velocity v becomes the same as the absolute velocity V and we have the general equation of energy in its usual form.

The head lost in hydraulic friction is proportional to the square of the velocity of flow and is commonly taken as

$$h' = k\frac{v_2^2}{2g}. \qquad (119c)$$

The equation of relative velocities (119*b*) is chiefly used in turbine and centrifugal pump theory to fix the relation between conditions at inflow to and outflow from the runner. With the impulse turbine $p_1 = p_2$ (usually $z_1 = z_2$ also), and the equation is then used to determine the relation between v_1 and v_2. In the reaction turbine the streams fill the runner passages, hence the areas f_1 and f_2 may be assumed to be known and the relation between the velocities may then be found by the equation of continuity, since $q = f_1v_1 = f_2v_2$. The sole use of equation (119*b*) would then be to find the drop (or gain) in pressure, $p_1 - p_2$.

121. PROBLEMS

1. Find the horsepower of a jet of water with a cross-section area of 3 sq. in. if it has a velocity of 100 ft. per sec.

Ans. 36.8 hp.

2. Suppose this jet in problem (1) were to strike a wheel with curved vanes. Assume that $A_1 = 0°$, $r_1 = r_2$, and that the vanes reversed the relative velocity of the water through 180° without friction loss. Find values of the force exerted when the peripheral speeds of the vanes are 0, 30, 50, 80, and 100 ft. per sec. (For an entire wheel we use W and not W'.)

Ans. 808, 566, 404, 161.5, and 0 lb., respectively.

3. Find the horsepower for the five speeds given in problem (2).

Ans. 0, 30.8, 36.8, 23.5, and 0 hp., respectively.

4. What are the efficiencies of the wheel in problem (2) at the various speeds given? When the power of the wheel is less than that of the jet, what becomes of the difference?

5. Suppose that the wheel in problem (2) were equipped with vanes for which $a_2 = 90°$. How would the values of force and power compare with the values when $a_2 = 180°$?

6. Suppose that the wheel in problem (2) were equipped with vanes for which $a_2 = 160°$, and that the loss in flow over the vanes were such that $v_2 = 0.8\ v_1$. Find the values of force exerted, power, and efficiency for the five speeds given.

7. What would be the force of the reaction of the jet in problem (1)?

8. The absolute velocity of water entering a turbine runner is 60 ft. per sec. and that leaving is 15 ft. per sec. $A_1 = 20°$, $A_2 = 80°$, $r_1 = 2.5$ ft., $r_2 = 4.0$ ft. (*a*) If $W = 600$ lb. per sec., find the torque on the wheel. (*b*) If $u_1 = 50$ ft. per sec., find the power delivered to the wheel.

Ans. (*a*) 2,430 ft.-lb. (*b*) 88.5 hp.

9. If the radius = 3 ft., find the torque exerted on the wheel in problem (2). Find it by using equation (101) and compare with values obtained by multiplying P and r.

10. An open cylindrical vessel is rotated about its axis, which is vertical. If the vessel is partially filled with water, what speed would be necessary to cause the water surface at a radius of 1.5 ft. to be 4.0 ft. higher than the surface at the center of rotation?

11. A closed vessel completely filled with water is rotated about its axis at a speed of 2,000 r.p.m. If the pressure at the center of rotation is 2 ft. of water, what will it be at a radius of 6 in.?

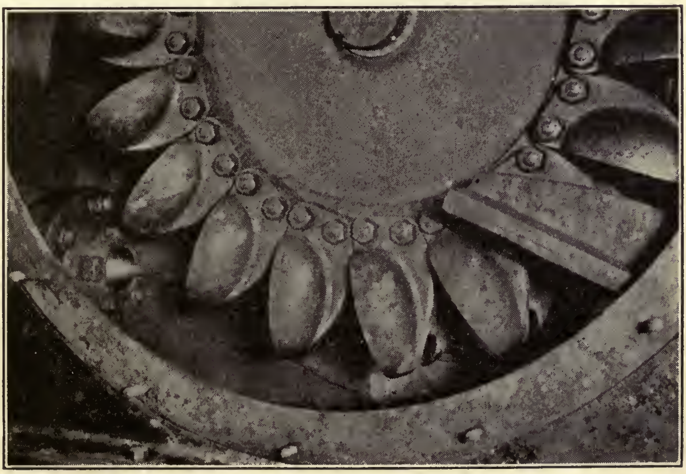
Ans. 172 ft.

12. If the inner and outer radii of the "whirlpool chamber" in Fig. 150 are 8 in. and 16 in. respectively, what will be the values of V and A at the outer diameter when water enters the inner diameter with a velocity of 80 ft. per sec. at an angle of 15° with the tangent? What will be the gain of pressure, neglecting losses?

CHAPTER X

DESCRIPTION OF THE IMPULSE WHEEL

122. The Impulse Wheel.—There have been several types of impulse turbines produced, but the only one that has survived in this country is of the kind shown in Fig. 151. This is the impulse wheel or the Pelton wheel, so called in honor of L. A. Pelton who contributed to its early development. It may be also designated by the name of the tangential waterwheel, from the fact that the center line of the jet is tangent to the path of the center of the buckets.



From a photograph by the author.
FIG. 151.—Impulse water wheel with needle nozzle. (Needle is drawn back and nozzle is wide open.)

The wheel in Fig. 151 is operated by a jet of water from the nozzle at the left. This same wheel in action may be seen in Figs. 206 to 210. A view of another wheel showing the relation of the nozzle to the buckets is shown in Fig. 152. The jet strikes the dividing ridge, or "splitter," of the buckets, is divided into two parts, flows over the face of the bucket, and is finally discharged at both sides of the latter.

In Figs. 153 and 154 we see views of an assembled wheel with the "chain type" of construction. That is, each bolt is instrumental in holding two buckets, so that the latter are fastened together as a chain. This permits of a compact construction and enables the buckets to be placed closer together than in the type shown in Fig. 151.

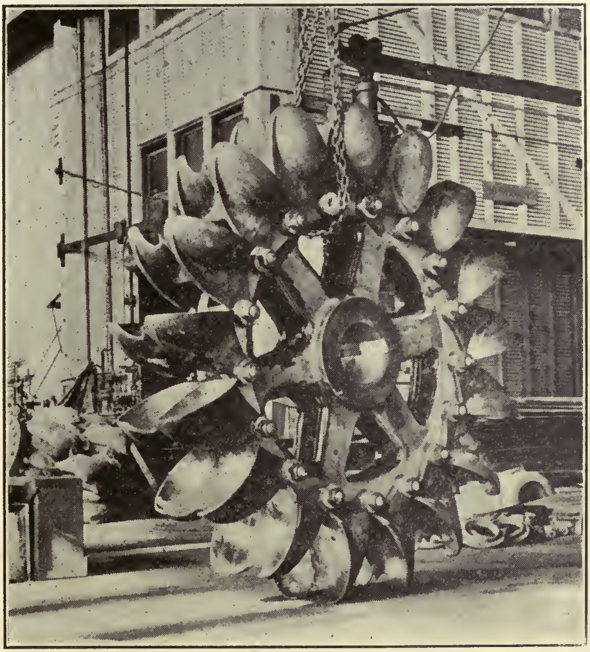
From a photograph by the author.

FIG. 152.—Impulse wheel viewed from below. (Nozzle closed by needle.) $D = 84''$; $h = 134'$; $N = 124$; $Hp = 280$.

The device shown at the right in Fig. 151 is the "stripper," its function being to prevent water being carried around with the wheel and thus adding to the windage losses. The buckets pass through an opening in this with a clearance of about 0.5 in.

123. Buckets.—Typical styles of buckets now in use for impulse wheels are seen in Figs. 155 and 156. The theory shows that the face of the bucket should be a surface of double

curvature, and it is also found that the shape of the back of the bucket may be as important as that of the face. The reason for this is that the back of the bucket may interfere with the water which is acting upon the bucket ahead, for when a bucket swings down into the jet it merely cuts off the jet from the preceding bucket and leaves a "slug" of water to complete its work on the one ahead. If the back of the bucket is not properly shaped it may not leave sufficient clearance for the water. The "notch"



From a photograph by the author.

FIG. 153.—Pelton–Doble wheel, in shop of Pelton Water Wheel Co. $D = 76''$; $h = 540'$; $N = 257$; $Hp = 2100$.

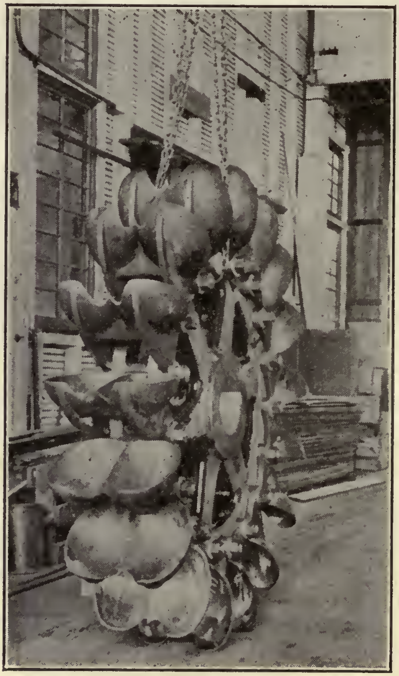
is cut out of the Pelton-Doble bucket so that it may reach a position where its path is more nearly tangent to that of the jet before the latter strikes it.[1]

For service under moderate heads these buckets may be made

[1] For impulse wheels of high specific speeds there are other reasons for this construction which space forbids taking up here in detail. In brief, it is so that every bit of water may complete its work upon the bucket before the latter leaves the line of action of the jet, in which event some of the water would not be utilized. (See Fig. 202.)

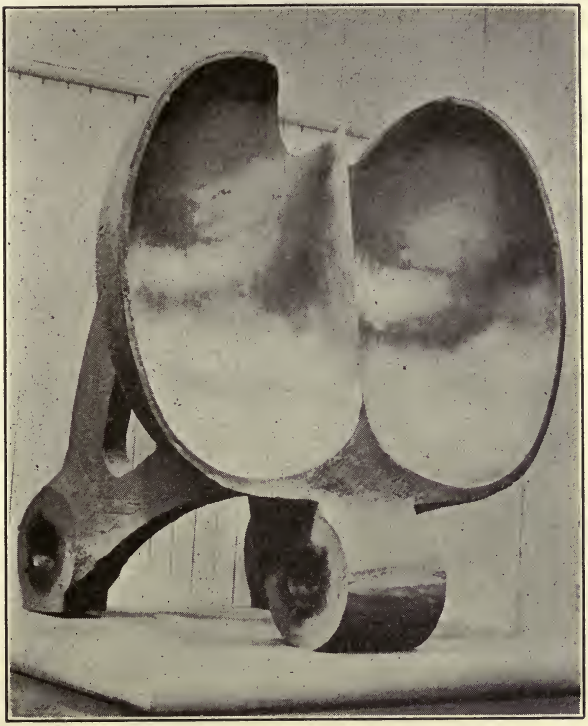
of cast iron, though the better ones are of bronze or steel. For very high heads only the latter may be employed. The working face of the bucket should be smoothed up or polished and the dividing edge, or "splitter," ground to a knife edge in order to reduce hydraulic friction losses.

For high efficiency it is desirable that the bucket reverse the relative velocity of the jet as nearly as is feasible. But a com-



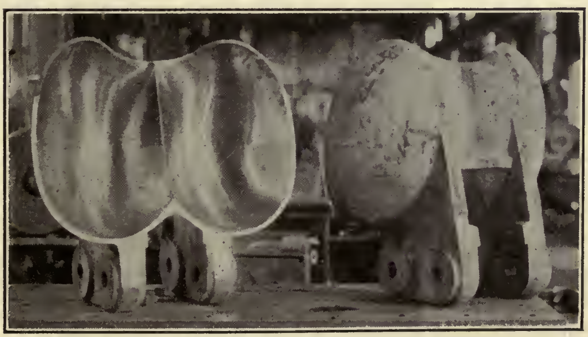
From a photograph by the author.
FIG. 154.—Pelton-Doble wheel.

plete reversal of 180° is not permissible, as the water must be thrown to one side so as to clear the following bucket. An angle of about 165° is usually employed, though even 170° may frequently be used. Due to surface tension the actual direction of the water will always be somewhat less than the bucket angle, the difference between the two decreasing as higher heads are used. For good efficiency the width of the bucket should be at least three times the diameter of the jet, and the diameter of the wheel should be at least nine times that of the jet. (The usual



From a photograph by the author.

FIG. 155.—Pelton-Doble ellipsoidal bucket.

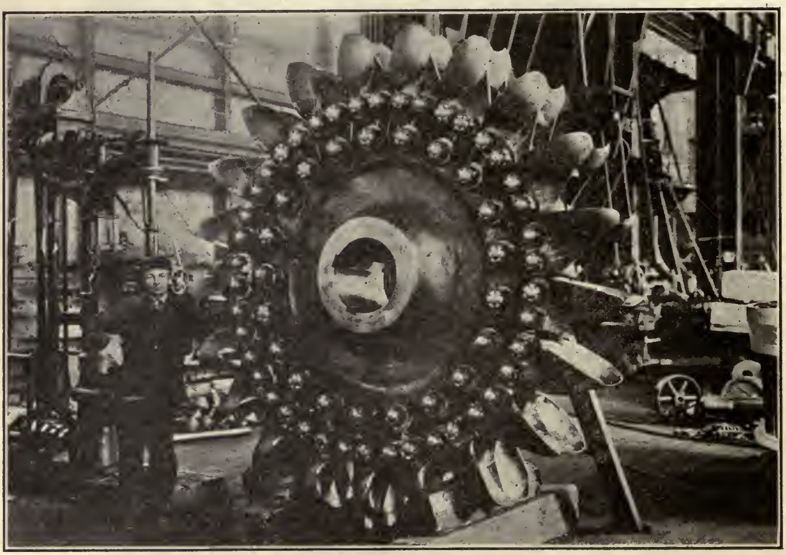


Courtesy Allis-Chalmers Mfg. Co.

FIG. 156.—Allis-Chalmers buckets.

ratio is 12 in the latter case.) Since jets 10 in., or more in diameter are in use, buckets of at least 30 in. in width are sometimes seen.

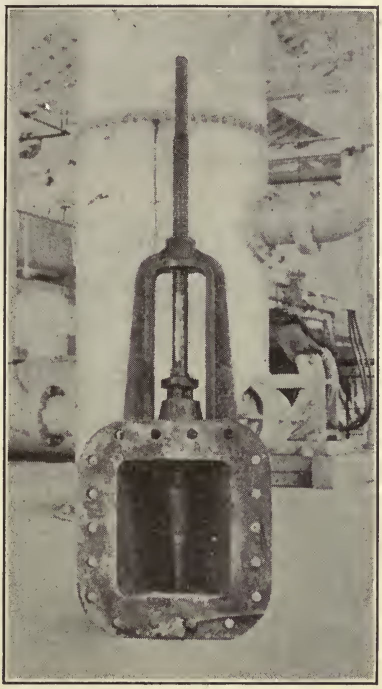
124. Nozzles and Governing.—The jets used in impulse wheels are almost always furnished by needle nozzles, of which the earliest type is shown in Fig. 158. The needle of the style used today is shown in Fig. 159. As it is moved back and forth in the nozzle it varies the size of the nozzle opening and hence varies the amount of water discharged. But fortunately it does not involve any serious loss of head until the nozzle is nearly closed. The efficiency of a needle nozzle when it is wide



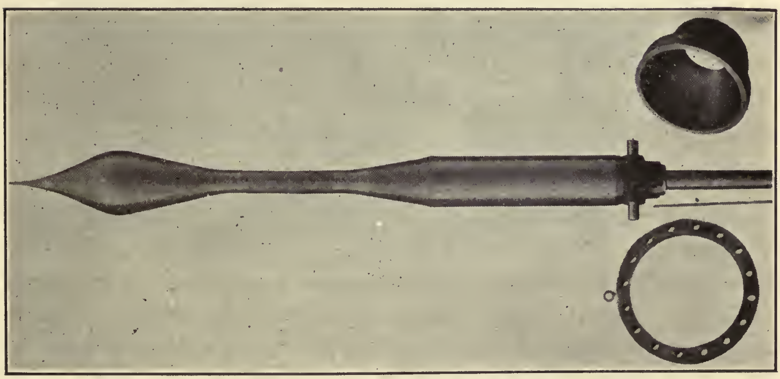
Courtesy Allis-Chalmers Mfg. Co.
FIG. 157.—Allis-Chalmers impulse wheel for Pacific Light & Power Co. $D = 94''$; $h = 1860'$; $N = 375$; $Hp = 10{,}000$.

open may be about 97 or 98 per cent., the velocity coefficient being about 0.99 or a little less. The nozzle efficiency would not fall below 90 per cent. until the needle was closed so far that about half the maximum amount of water was being discharged. Thus it is a very efficient regulating device.

In order to keep the speed of a wheel constant under different loads it is necessary to vary the amount of water so that the power supplied to the turbine shall be proportional to the power



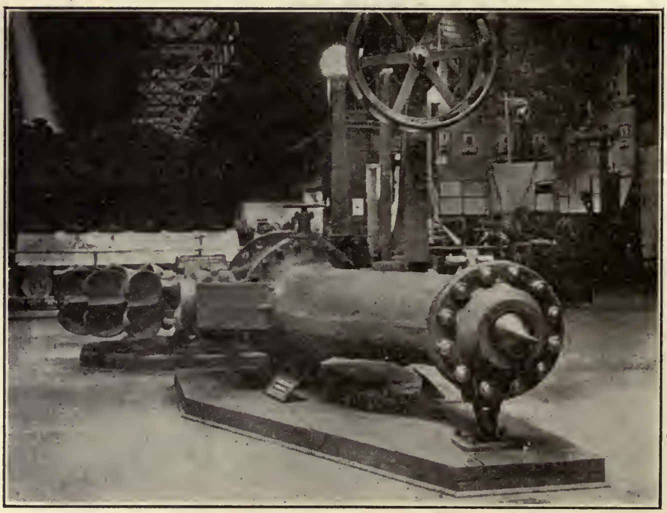
From a photograph by the author.
FIG. 158.—The original needle nozzle.



Courtesy of Pelton Water Wheel Co.
FIG. 159.—Pelton-Doble needle and nozzle tip.

demanded. This can sometimes be done by changing the position of the needle in accordance with the power the wheel must deliver. Under certain conditions the governor may control the position of the needle for this purpose. But if the changes of load are rapid and the pipe line is long, this procedure would involve serious water hammer, if close speed regulation were attempted.

In order to secure close speed regulation and yet be free from the danger of water hammer, the deflecting nozzle is often used. The entire nozzle is movable about a ball and socket joint near the base and swings on trunnions. In case of a sudden drop of

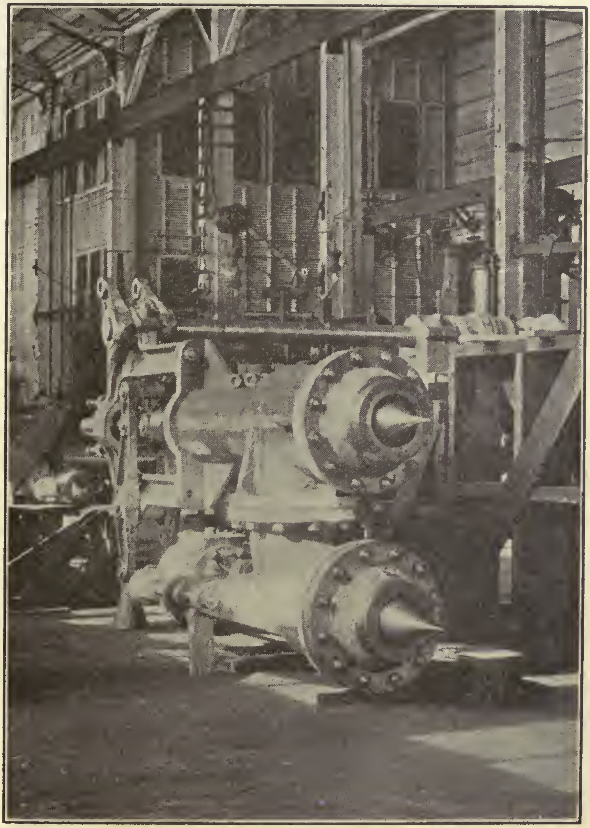


From a photograph by the author.

Fig. 160.—Deflecting needle nozzle for a 10,000 h.p. jet.

load on the machine the governor could lower the end of the nozzle so that only a small part of the jet struck the buckets, the rest of the water being wasted. As the load increased the nozzle could be raised so that a larger amount of water would strike the wheel. As this would be wasteful of water, such nozzles are almost always equipped with needles as well, which can be set by the station attendant in accordance with the load the wheel carries. Thus water would be wasted for a short time only, but the needle would be closed so slowly that no damage would be done to the pipe line. But the nozzle may be deflected

with any degree of rapidity so that close speed regulation may be secured. Of course in case of an increase in load it would be necessary for the operator to open the nozzle, as the governor is powerless there. But the experience is that increases of load come on gradually enough for this to be done. The chief function of the governor is to prevent racing in cases of abrupt de-



From a photograph by the author.

FIG. 161.—The needle nozzle with auxiliary relief.

creases in load. Occasionally the nozzle is so made that the governor deflects it first and then slowly closes the needle.

The needle nozzle with an auxiliary relief, as shown in Fig. 161, is frequently used. In this type the jet from the upper nozzle strikes the wheel while that from the lower nozzle goes below it.

Courtesy of Allis-Chalmers Mfg. Co.

FIG. 162.—Needle nozzle with deflecting tip.

From a photograph by F. H. Fowler.

FIG. 163.—DeSabla power plant in normal operation. Under head of 1531 ft.

It is so arranged that when the governor closes the upper nozzle it opens the lower one. Thus there is no abrupt change in flow in the pipe line as the surplus water simply flows out through another place. But in order to prevent waste of water, the connection between the governor and the auxiliary nozzle is a dash-pot arrangement which permits the needle to be moved only when the governor movement is rapid, and when the relief has been opened, this arrangement permits it to be gradually

From a photograph by F. H. Fowler.

Fig. 164.—DeSabla power plant with nozzles deflected.

closed again. Thus we have accomplished close speed regulation and have also secured economy in the use of water.

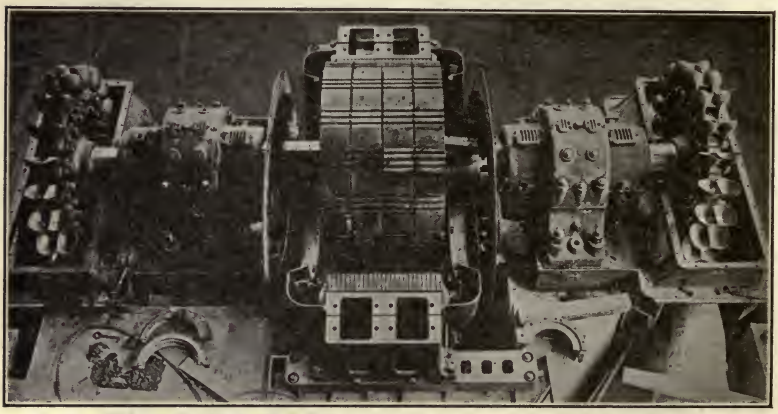
The nozzle shown in Fig. 162 is similar in principle to the deflecting nozzle in that the jet is deflected below the wheel. But it is so constructed that only the tip of the nozzle has to be moved rather than the entire nozzle. This has certain advantages.

It will be noted that all of these devices may prevent rapid changes in flow in the pipe line in the case of decreasing loads.

But only a surge chamber located near the wheels will be able to supply water in the case of a sudden demand.

125. Conditions of Service.—The impulse wheel is well adapted for service under high heads, though it may also be employed under low heads if the power is small. In fact the choice of the type of turbine is a function of power as well as head.

The highest head that has ever been developed is in Switzerland where 15,000 hp. is generated under a head of 5,412 ft. The power of each wheel in the plant is 3,000 hp., its diameter



Courtesy of Allis-Chalmers Mfg. Co.

Fig. 165.—Double overhung Allis-Chalmers wheels for Pacific Light and Power Co. $D = 94''$; $h = 1860'$; $N = 375$; $Hp = 20{,}000$ (for unit).

is 11.5 ft. and it runs at 500 r.p.m. The diameter of the jet is 1.5 in.

In this country the highest head that has been used is 2,100 ft., and heads ranging from 1,000 ft. to 2,000 ft. are not uncommon. A plant under 2,100 ft. static head is shown in Fig. 198.

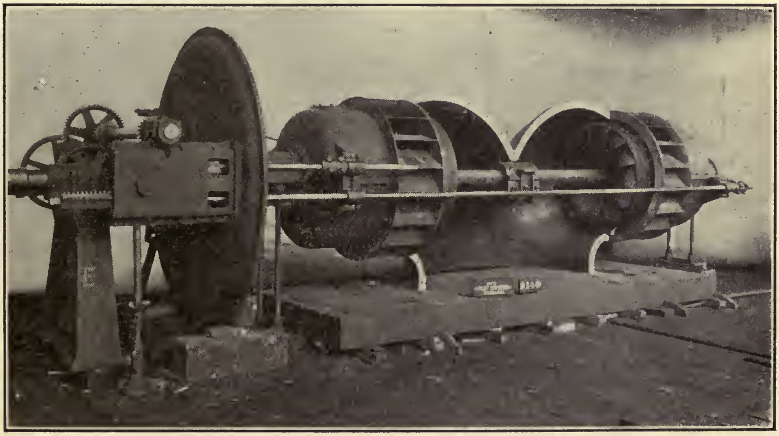
The jets used upon impulse wheels are of all sizes up to about 10 in. or a little over. Ordinarily only one jet is used with a single wheel, but occasionally two or more nozzles may be employed, though at a slight sacrifice of efficiency. In order to increase the power of a single unit two separate wheels are often used on the same shaft, as in Fig. 165.

The largest power developed by a single impulse wheel with one jet upon it is 10,000 hp. A wheel of that capacity is shown in Fig. 157, though there are several cases where the power of a single wheel has approached such a value.

CHAPTER XI

DESCRIPTION OF THE REACTION TURBINE

126. The Reaction Turbine.—The inward-flow type of reaction turbine is the only one that is of any importance at the present time, all others having been eliminated because of certain relative disadvantages. Modern inward-flow turbines are commonly known as Francis turbines in honor of James B. Francis, who built the first successful one in 1849. However, the wheels of today differ considerably from his, which was a purely radial-flow turbine. See page 197.



Courtesy of Platt Iron Wks. Co.

Fig. 166.—Turbine with cylinder gates for open flume.

By radial flow is meant that a particle of water, during its flow through the rotating runner, remains in a plane which is normal to the axis of rotation, so that its position changes only with respect to its distance from the axis of rotation. In the evolution of the modern turbine it became desirable to have the water enter the runner with a "radial" flow and then to turn and flow in such a manner that a component of its velocity might be parallel to the shaft. In fact some of the particles of water, at

least, before they reached the discharge edge of the bucket or vane might be following paths which lay on the surfaces of cylinders concentric with the axis. This is known as mixed flow, and such a type of turbine is sometimes called the American turbine, though the name Francis is generally extended to cover all inward-flow wheels. In Figs. 167 and 168 may be seen the nearest approach in present practice to a radial-flow runner, while in Figs. 171 and 172 may be seen the mixed-flow type.

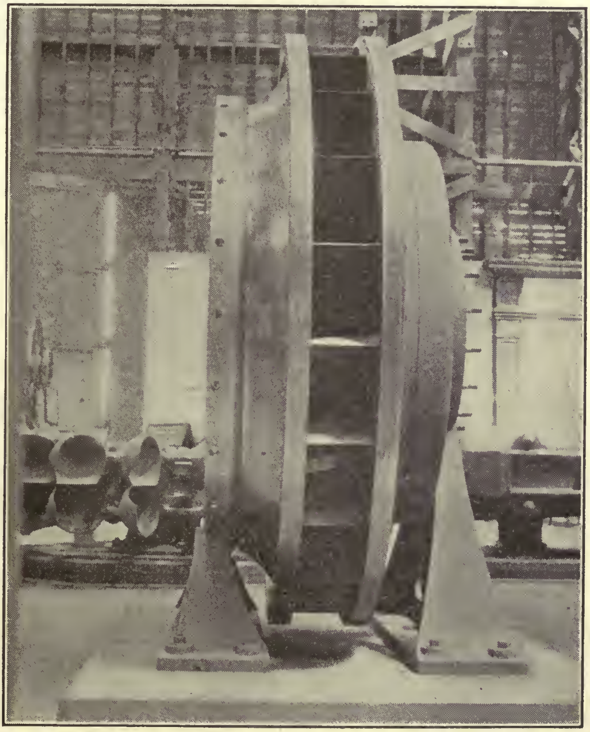
The general arrangement of a reaction turbine may be seen in Fig. 166. This particular one is of the open-flume type and is

Courtesy of Pelton Water Wheel Co

FIG. 167.—Low-speed turbine runner $D = 74''$; $h = 487'$; $N = 360$; $Hp = 20{,}000$

set so as to be completely surrounded by water in a manner similar to the vertical-shaft turbine shown in Fig. 181. The water flows through the stationary guide vanes and enters the runner, which is in the center. During flow through the runner, the velocity of the water suffers a change in both direction and magnitude and thereby exerts a dynamic force. In Fig. 166 there are two runners set on the same shaft and discharging into a common draft chest, from which the water flows down to the tail race through a draft tube.

127. Runners.—The part of the turbine upon which the water does its work is called the runner. Runners may be built up of separate pieces of metal which are welded together but they are usually cast in one piece. Occasionally they are built in sections and the sections bolted together. For large sizes and low heads cast iron is employed. Better runners are made of bronze and occasionally cast steel is used for high heads.



From a photograph by the author.

FIG. 168.—Low speed turbine runner for Pacific Gas and Electric Co.

Runners differ considerably in their proportions and appearances. One extreme is shown in Figs. 167 and 168 while the other extreme is shown in Figs. 170, 172, and 173. It may be noted that the runners in Figs. 167 and 172 develop the same amount of power though differing widely in size. This is due to the fact that the smaller runner operates under a much higher head and consequently needs to discharge less water for the same amount of power. And the largest runner in the world,

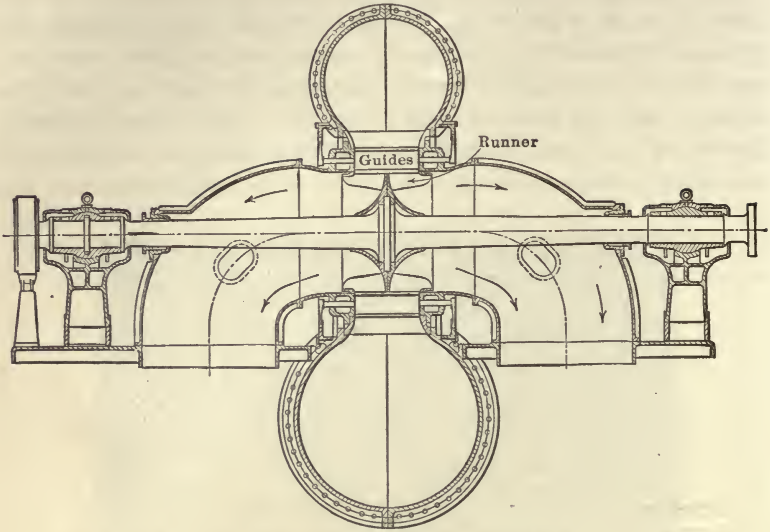


FIG. 169.—Double discharge turbine.

From a photograph by the author.

FIG. 170.—The largest turbine runner in the world. For Cedars Rapids Mfg. and Power Co. $D = 143''$; $h = 30'$; $N = 55.6$; $Hp = 10{,}800$.

shown in Fig. 170, develops less power than either of the others because it is under a still lower head.

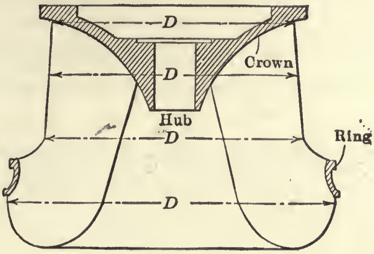


FIG. 171.

Courtesy of I. P. Morris Co.

FIG. 172.—High-speed turbine runner. $D = 102''$; $h = 76'$; $N = 120$; $Hp = 20{,}000$.

It may be noted that the width of the runner parallel to the shaft in Fig. 168 is a very much smaller proportion of the diameter of the runner than in the type shown in Figs. 170 and 173.

Sometimes runners are of the double discharge type as in Fig. 169 which is equivalent to placing two single discharge runners back to back. Such a turbine must have two separate draft elbows.

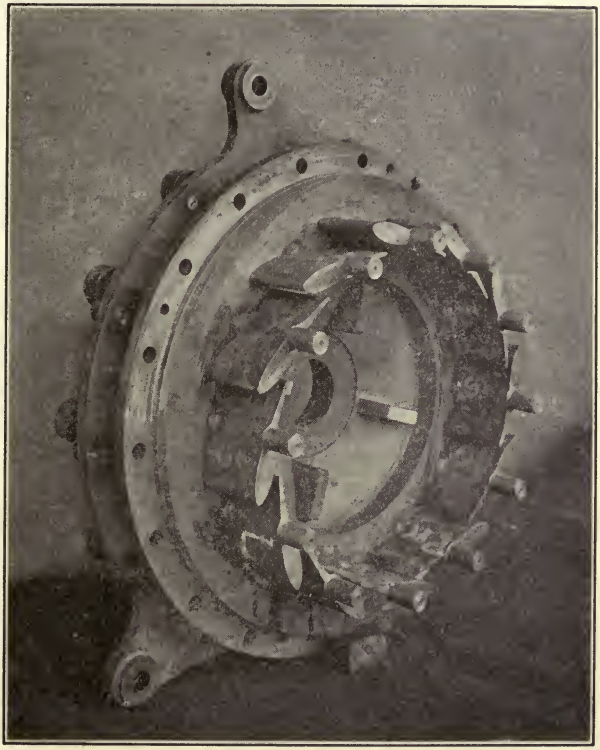
As shown in Fig. 171 there may be several places at which the diameter of a turbine runner may be measured and practice differs in this respect. The custom that is generally followed is to give the mean diameter at entrance to the runner. This is the dimension that will be found in Figs. 170 and 172. The

Courtesy of I. P. Morris Co.
Fig. 173.—Turbine runner for Laurentide Co.

maximum diameters in these two cases are 17 ft. 7 in. and 12 ft. 7 in. respectively.

128. Gates and Governing.—The quantity of water passed through the turbine is regulated by means of gates, of which there are several kinds. In Fig. 166 we find the cylinder gate used. In that class of turbine the guide vanes surrounding the runner are absolutely fixed. Between the ends of these vanes and the runner is a metal cylinder which may slide along parallel to the shaft: If moved in one direction it admits water to the runner and may be so far withdrawn as to offer no obstruction whatever

between the guides and the wheel. And if it is moved in the other direction it is possible to shut off the water altogether. This style of regulation causes the turbine to have a poor efficiency on "part gate," which is the term used when the turbine is running under less than full load. But such a style of gate permits a turbine to be constructed at less cost.

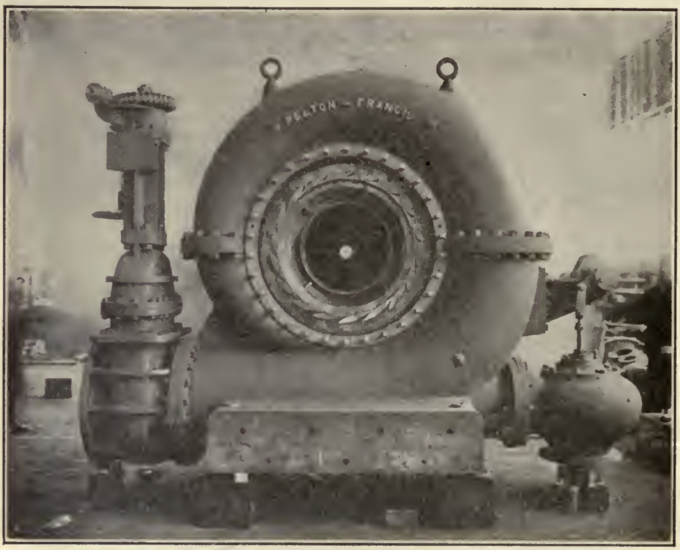


Courtesy of Pelton Water Wheel Co.

FIG. 174.—Wicket gates or swing gates.

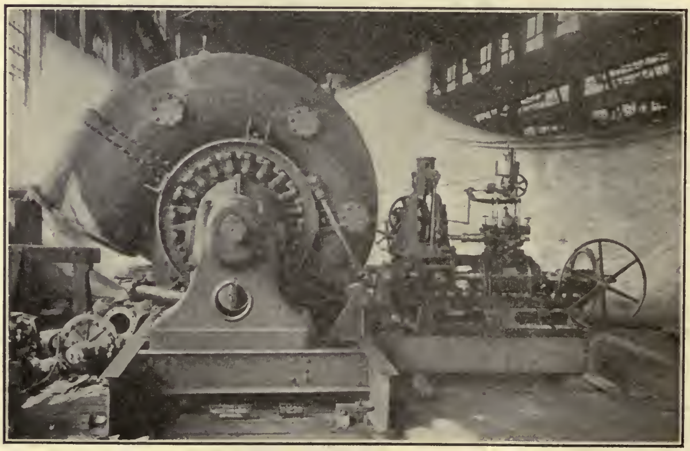
The better type of gate so far as efficiency is concerned is the kind shown in Fig. 174. Here the guide vanes themselves are movable and by rotating about their axes they may vary the size of the area through which water may flow. This means that the angle A_1 changes. These gates are known either as swing gates, wicket gates, or pivoted guide vanes. They involve more expensive construction than the cylinder gate but are vastly better if economy of water is any object.

In Fig. 175 may also be seen some movable gates as they are installed in the turbine. The runner is to go into the space in the center.



Courtesy of Pelton Water Wheel Co.

FIG. 175.—Spiral-case turbine showing swing gates.



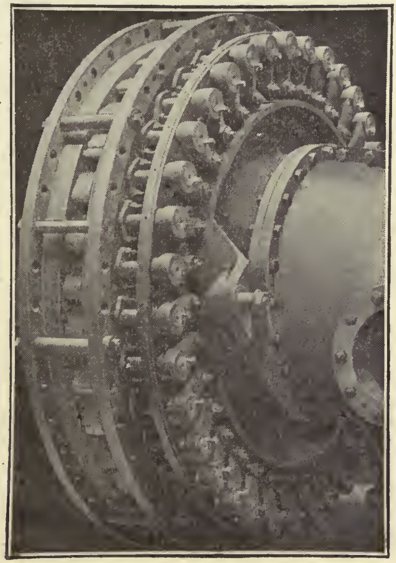
Courtesy of Platt Iron Wks. Co.

FIG. 176.—Shifting ring for operating gates.

The swing gates are operated by moving a "shifting ring" to which each gate is attached by links. In Fig. 176 may be seen

the rods from the governor connected to this ring so that, when it is moved slightly with the turbine shaft as a center of rotation, each gate will be turned through some angle. The links which connect the gates to this ring can be seen more clearly in Fig. 177.

The problem of governing a reaction turbine is similar to that of the impulse wheel. When the governor closes the gates and thus reduces the discharge through the turbine it is necessary to provide some bypass for the water in order to prevent water hammer in the pipe line. The usual practice is to use a relief



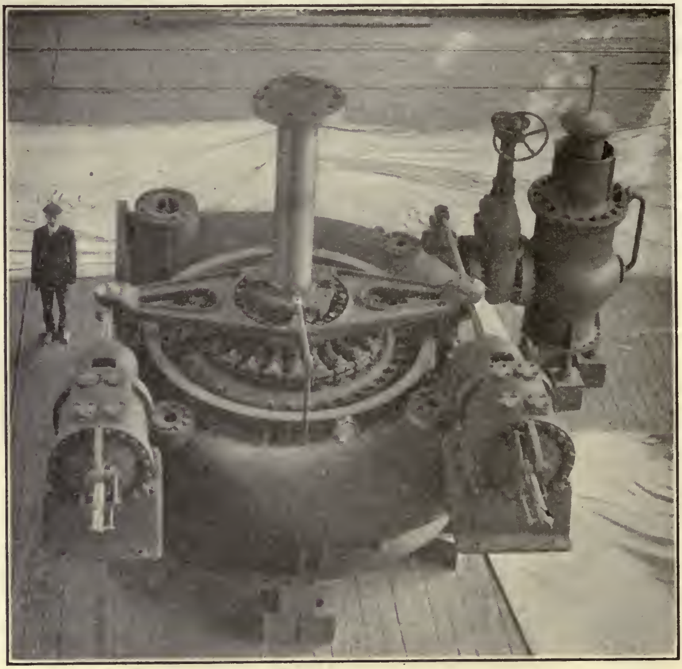
Courtesy of Allis-Chalmers Mfg. Co.
FIG. 177.—Swing gates.

valve such as that shown over at the right in Fig. 178. When the governor closes the gates it opens the relief valve at the same time, and the water coming down the pipe is then discharged through this into the tail race alongside the draft tube. The action of such a relief valve may be seen in Figs. 179 and 180.

The connection between the governor and the relief valve is usually not a rigid connection, in order that the relief valve may slowly close and prevent the waste of water.

129. The Draft Tube.—The water is conducted from the turbine to the tail race through a draft tube, which may be con-

structured of riveted steel plates as in Fig. 179 or may be molded in concrete as in Fig. 182. The draft tube should be made air-tight so that a partial vacuum can exist within it and thus there may be a "suction" produced on the discharge side of the runner which shall compensate for the elevation of the latter above the tail water level. By the use of the draft tube it is possible to set



Courtesy of S. Morgan Smith Co.

FIG. 178.—Tallulah Falls turbine showing gate mechanism and relief valve $H = 580'$; $N = 514$; $Hp = 19{,}000$.

the turbine at a convenient distance above the water level without losing any head thereby.

But this is not the sole function of the draft tube. The velocity with which the water is discharged from the runner represents kinetic energy that is not utilized and such a loss cuts down the efficiency of the wheel. If the draft tube is made to diverge the velocity at its mouth will be much less than that with which the water enters it from the runner and hence the kinetic energy finally lost may be much reduced. With some types of turbine runners it is necessary to allow the water to be discharged with

a relatively high velocity and such wheels would not possess favorable efficiencies if it were not for the use of suitable draft tubes. The usual rate of diffusion provided for is such that a circular tube will be made a frustum of a cone the vertex angle of which is 8°. Some experiments by the author indicate that a larger angle than this might be permissible. For a given rate of diffusion the longer the tube the greater the reduction of the

From a photograph by the author.

FIG. 179.—Small discharge from relief valve near draft tube in Cornell University power plant.

kinetic energy of the water. Therefore, in some cases it is desirable to have a long draft tube even though the runner might be set very near the water level.

On account of the function which is fulfilled by the draft tube it is properly regarded as an integral part of the turbine. Considering the turbine and the draft tube as a unit, it may be seen that the less the kinetic energy lost from the mouth of the tube the higher the efficiency of the wheel, thus justifying the state-

ment of the preceding paragraph. But it is not yet clear just how this saving in the draft tube enables the turbine itself to deliver more power until we consider the effect of the draft tube upon the pressure at the exit from the runner. The less the losses within the draft tube and the less the discharge loss from the draft tube the less the pressure may be at this point.

130. Cases and Settings.—The turbine, draft tube, and all parts intimately connected with it comprise what is called the

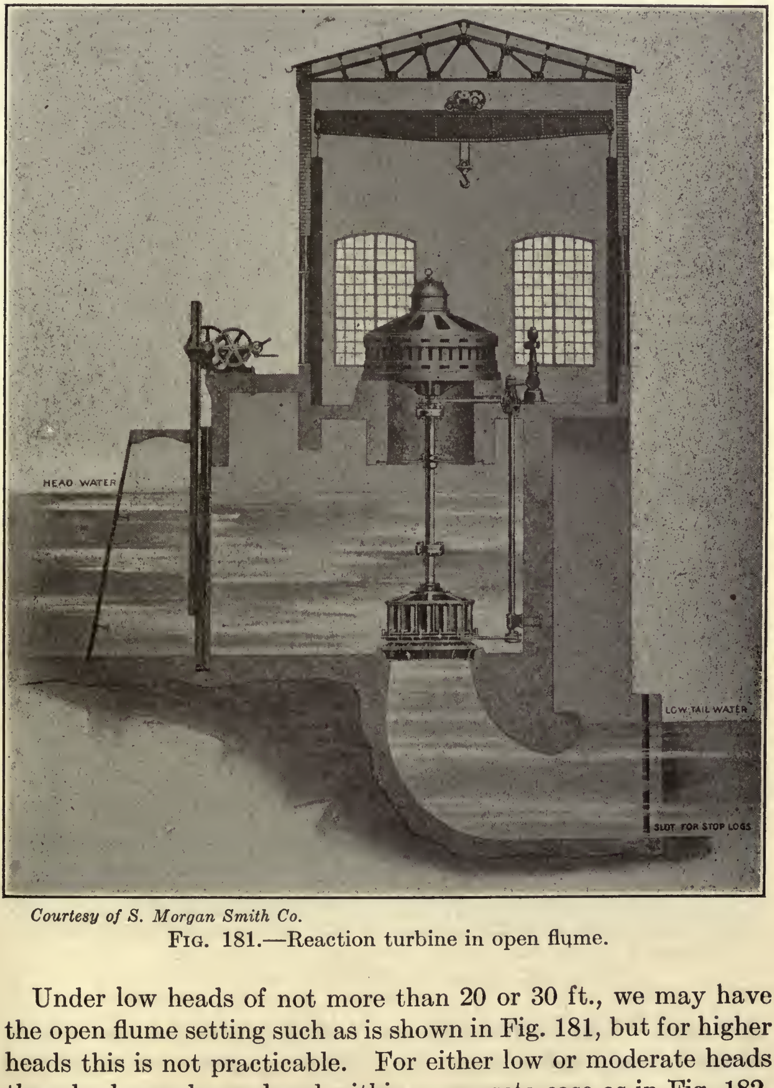
From a photograph by the author.

FIG. 180.—Discharge from relief valve when opened.

setting. Impulse wheels are almost always set with horizontal shafts, but reaction turbines may have either horizontal or vertical shafts. For large units under low heads the vertical shaft is the most recent practice, as it permits several desirable features to be attained.[1] Occasionally several runners may be

[1] H. B. Taylor, "Present Practice in Design and Construction of Hydraulic Turbines," Canadian Soc. of C. E., Jan. 15, 1914.

mounted on the same shaft but the tendency is to eliminate such construction and have larger runners and fewer of them, and two on the same shaft as in Fig. 166, is as many as are desirable. As in the case of the impulse wheel, we may have two separate turbines connected to a single generator.

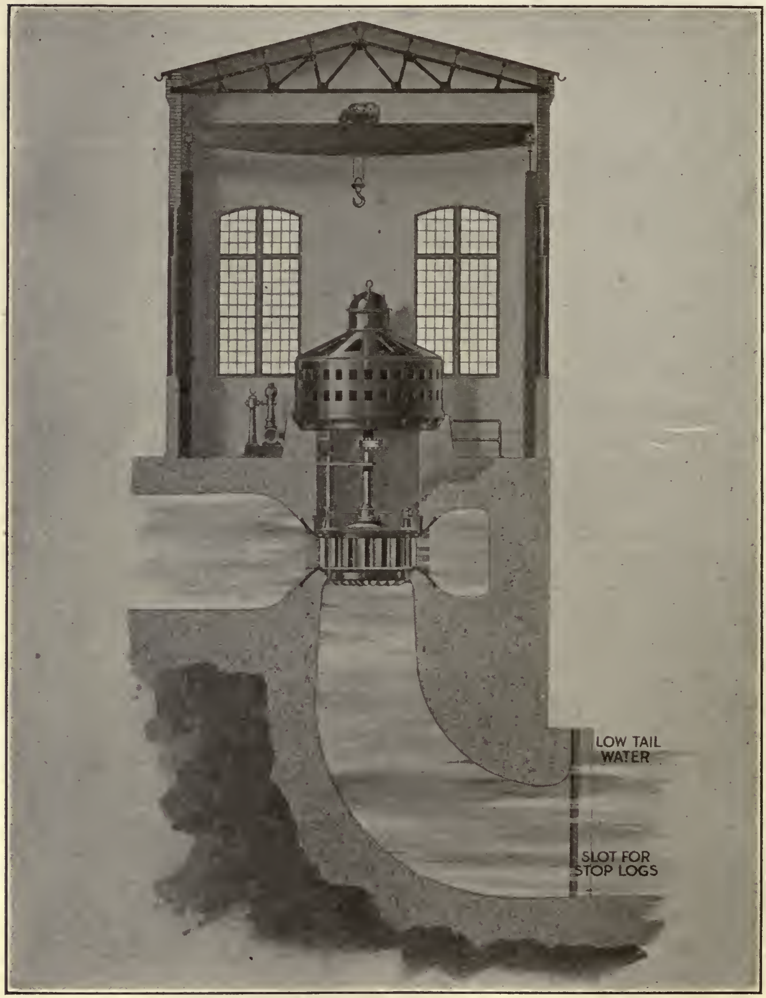


Courtesy of S. Morgan Smith Co.

FIG. 181.—Reaction turbine in open flume.

Under low heads of not more than 20 or 30 ft., we may have the open flume setting such as is shown in Fig. 181, but for higher heads this is not practicable. For either low or moderate heads the wheel may be enclosed within a concrete case as in Fig. 182,

but the action of the turbine is no different from that in the preceding case. The water has no free surface immediately above the turbine but it is under practically the same pressure as if it

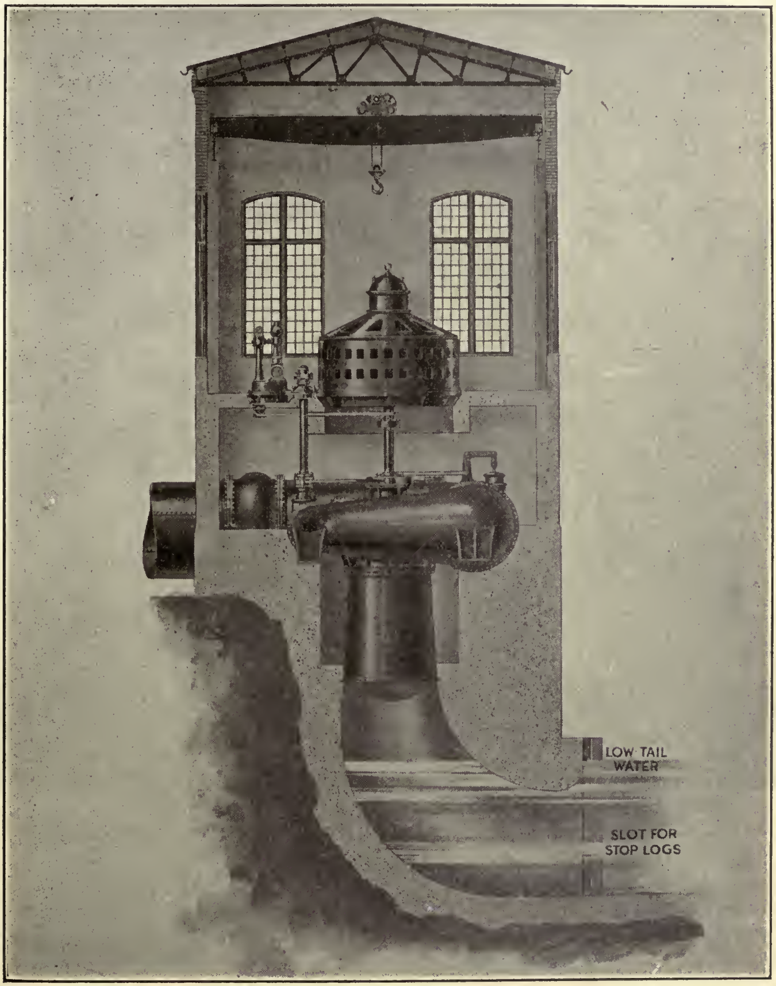


Courtesy of S. Morgan Smith Co.

FIG. 182.—Reaction turbine in concrete case.

did have. The only difference is that, since the area of the water passage is less than before, the velocity with which the water approaches the turbine will be somewhat higher, and thus

there will be a lesser rate of acceleration as the water enters the guide vanes. For still higher heads a concrete case would be unsuitable and then the guide vanes are surrounded by a metal



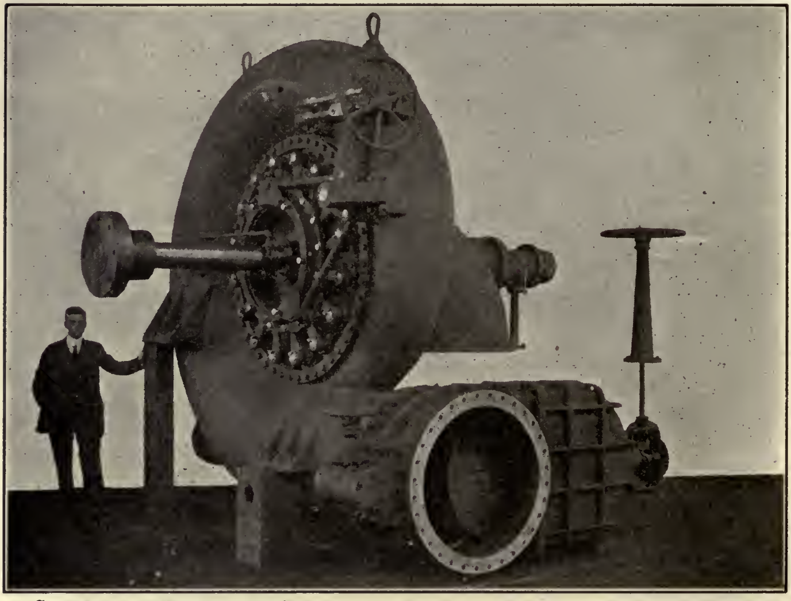
Courtesy of S. Morgan Smith Co.

Fig. 183.—Reaction turbine in metal case.

case as shown in Fig. 183. The only difference between this and the other two cases so far as the hydraulics is concerned is largely one of appearances, save that the velocity of the water as

it approaches the guides may be somewhat higher owing to the smaller area.

In order that the water may have the same velocity of approach to the guides all around the circumference, the spiral case is frequently used. Cases of this type are illustrated in Figs. 175, 183, 184, and 185. In Fig. 184 may also be seen the main gate valve which may be used to shut off the water more completely than is possible with the wicket gates, and on the right-hand side may be seen a portion of the draft elbow. Very large

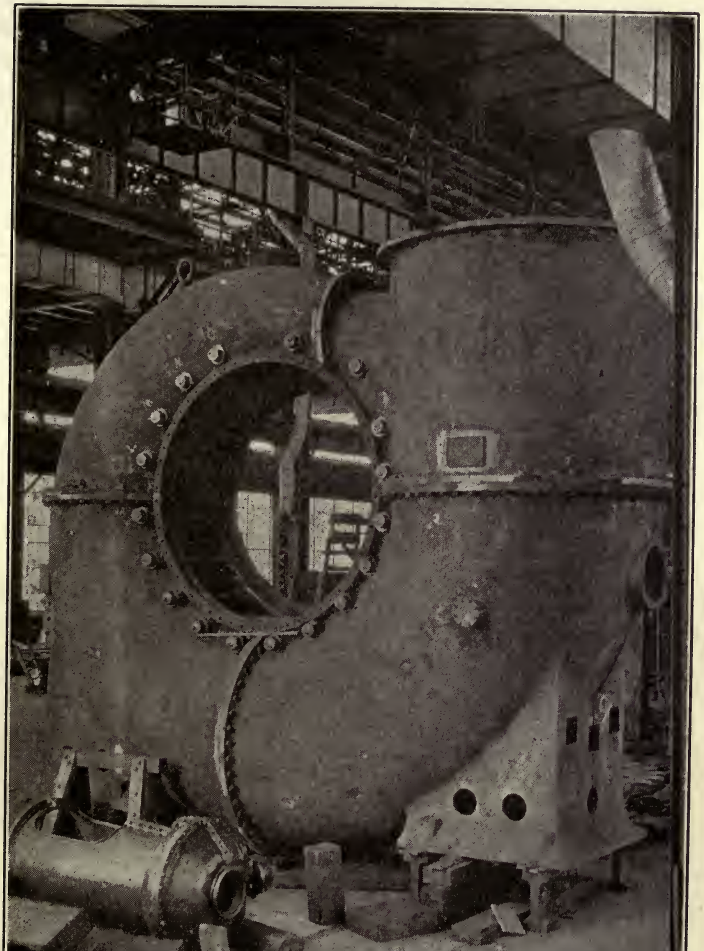


Courtesy of S. Morgan Smith Co.

FIG. 184.—Spiral case turbine showing main gate valve, shifting ring and links for guide vanes, and draft elbow.

cases are built in sections as shown in Fig. 185. The spiral case is considered the most desirable type though other less expensive ones are sometimes used.

In Fig. 186 is shown a type of turbine that might be set as in Fig. 181, while in Fig. 187 we get a glimpse into the intake of a large turbine set as in Fig. 182. In such a setting the runner and guide vanes may be surrounded by a "speed ring" such as shown in Fig. 188. The columns which support the upper crown plate and its load are made of a shape similar to guide vanes so as to



From a photograph by the author

FIG. 185.—Large spiral case for Canadian Light and Power Co. in shop of I. P. Morris Co.

reduce eddy losses and also to give the water the proper direction as it enters the real guide vanes. In Fig. 189 we see a vertical shaft turbine for a higher head.

131. Conditions of Service.—The reaction turbine is well adapted for service under low heads especially for large powers. They may also be used very satisfactorily for heads of several hundred feet. The highest head that has ever been employed

Courtesy of Allis-Chalmers Mfg. Co.

FIG. 186.—Vertical open-flume turbine for Eastern Michigan Edison Co. $h = 14'$; $N = 100$; $Hp. = 575$.

for a reaction turbine is 670 ft. for two 6,000-hp. units installed by the I. P. Morris Co. in Mexico. There are several cases where heads of over 500 ft. have been used.

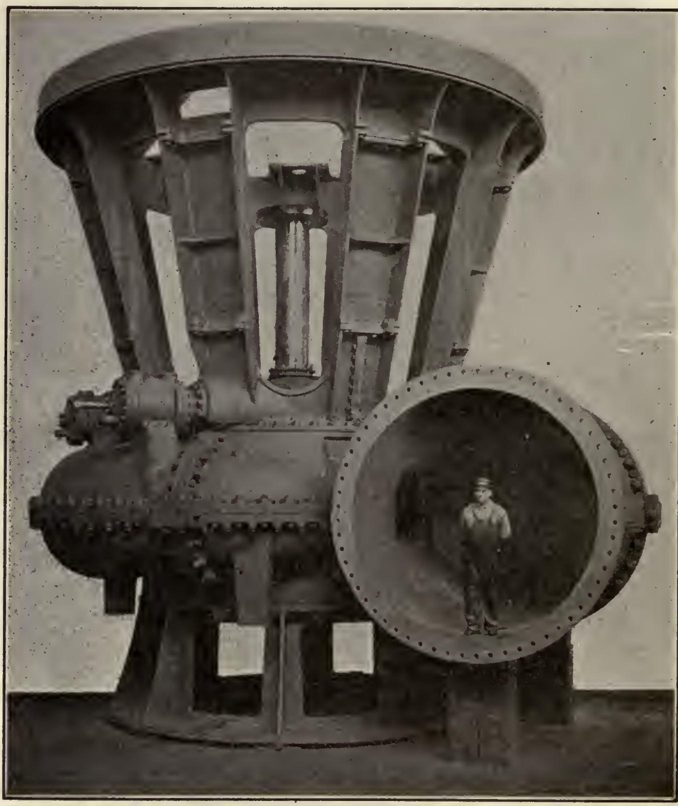
The most powerful turbine unit yet built is shown in Fig. 190, the power of the two wheels combined being 25,000 hp. The greatest power developed in a single runner is 22,500 hp. in a

Courtesy of Mississippi River Power Co.
FIG. 187.—Intake for turbine at Keokuk.

From a photograph by the author.
FIG. 188.—Speed ring for Canadian Light & Power Co. in shop of I. P. Morris Co.

wheel built by Allis-Chalmers for service under a head of 480 ft. It is a double-discharge runner, however. The greatest power ever developed in a single-discharge runner is 20,000 hp. in the runners shown in Figs. 167 and 172.

But the power of a turbine depends not only upon its size but also upon the head under which it operates. Thus the most

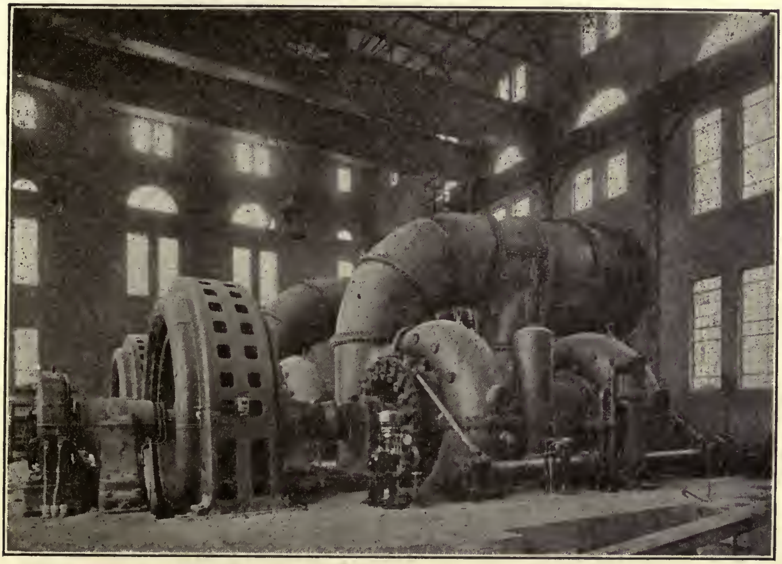


Courtesy of S. Morgan Smith Co.

FIG. 189.—Vertical shaft spiral case turbine for Great Falls, Mont. $h = 150'$; $N = 200$; $Hp = 15{,}000$.

powerful turbines may not be as big in size as others which develop less power because they run under lower heads. The largest runners in the world in size are those at Cedars Rapids, Canada, one of which is shown in Fig. 170. They slightly exceed in size those at Keokuk.

131a. Historical Note.—The inward flow turbine was proposed by Poncelet in 1826. Howd patented and built the first one in 1838 and a number of his wheels were installed in the mills of New England. In 1849 Francis constructed a pair of pure radial inward flow turbines from the Howd patent but his wheels were of much better design and mechanical construction. About 1860 Swain produced runners in which the flow was



Courtesy of I. P. Morris Co.
FIG. 190.—Washington Water Power Co. Two 22,500 hp. units at 200 r.p.m. under head of 168 ft.

mixed, somewhat like that shown in Fig. 167, and in 1876 McCormick built the first of the modern high capacity runners, somewhat like that shown in Fig. 171. All modern turbine runners more nearly resemble either the Swain or McCormick types than they do the Howd-Francis type. The use of the name Francis is in part a misnomer and in part due to the fact that the mixed flow runner was really a natural development from the original Francis turbine.

CHAPTER XII

WATER POWER PLANTS

132. Elements of a Water Power Plant.—A complete water power development may comprise a great deal of construction and equipment aside from the power house and contents, so

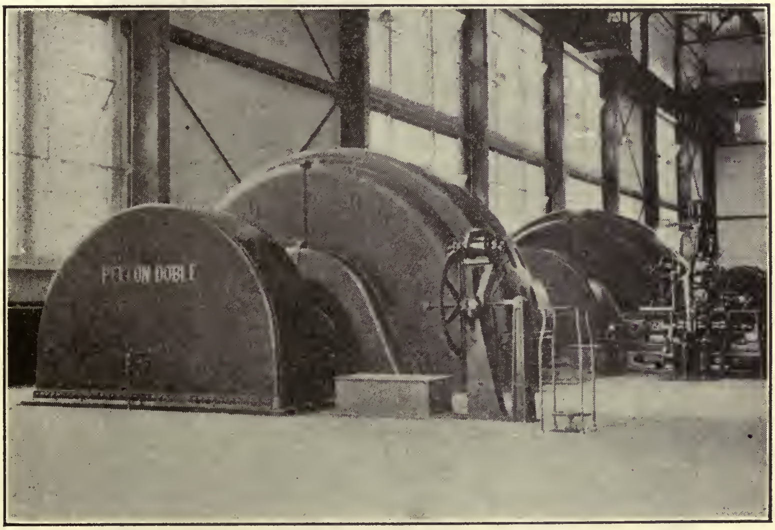
From a photograph by the author.

FIG. 191.—Penstock leading to Drum power house of Pacific Gas & Elec. Co. under 1375 ft. head.

much so that the cost of the latter is often a small proportion of the total investment. For a complete plant some or all of the following details may be required according to the physical situation.

A dam of some sort is usually essential. It may be nothing more than a wing wall extending a short way into the river to divert a small portion of the flow, or it may extend clear across the stream. In the latter case the water level will be raised above its former height and also a certain amount of water will be stored up by it. If the contour of the land permits, a dam may create an artificial lake or storage reservoir. In some cases the power plant draws water directly from this body and in other cases it would be used merely as a "feeder."

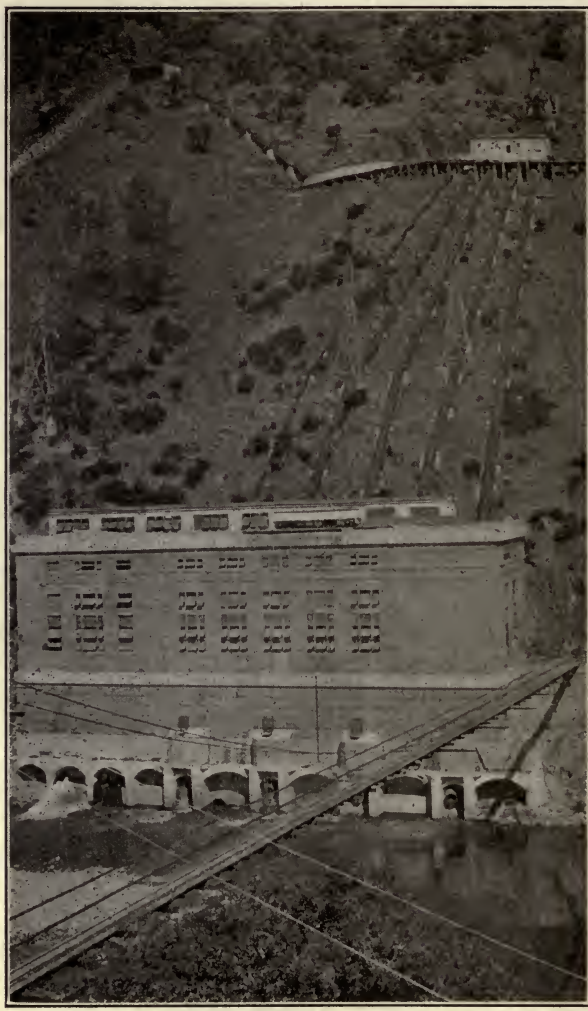
The water is conducted to the power house through canals, flumes, pressure tunnels, or pipe lines, as the case may be. It is



From a photograph by the author.

FIG. 192.—Pelton-Doble impulse wheels in Drum power house of Pacific Gas & Elec. Co. $h = 1375'$ static or $1300'$ under normal load; $N = 360$; $Hp = 8{,}500$ per wheel.

not uncommon for the water to be carried from 5 to 10 miles or more in order to permit the utilization of a higher fall than could be obtained near the intake. It is desirable that the water be kept at as high an elevation as possible during the first portion of its course as this permits the use of open channels or low-pressure pipes, which is cheaper than if the water had to be carried under high pressure all the way. This portion of the conduit is often called the "flow-line" from the fact that its main function is to deliver water and not to transmit pressure.

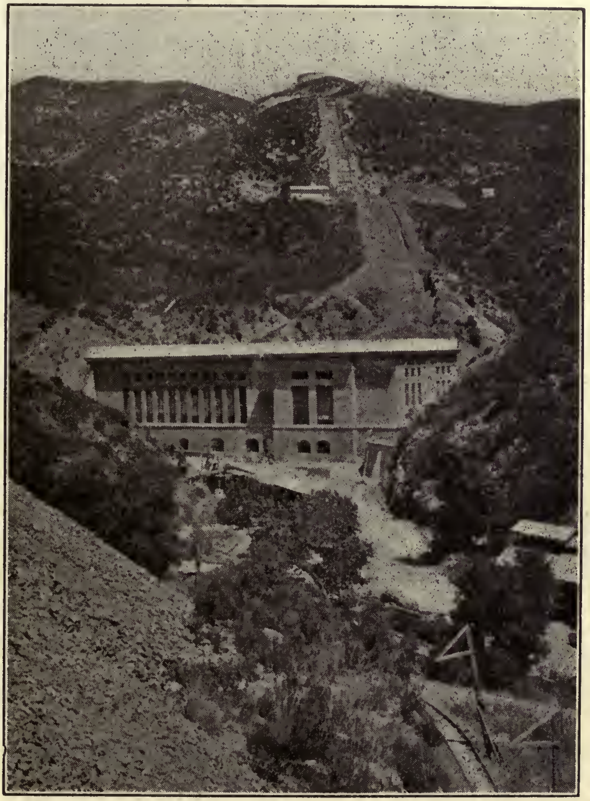


From a photograph by F. H. Fowler.

FIG. 193.—Las Plumas plant at Big Bend on the Feather River in California, containing six reaction turbines of 18,000 hp. each under a head of 465 ft.

At the end of such a flow-line the water will be abruptly dropped down the hillside as shown in Fig. 191. This portion of the pipe line is the penstock.

Where the distance from the intake to the power plant is a number of miles, it is desirable that there be some break in the continuity of flow, on account of speed regulation. If conditions



From a photograph by the author.

FIG. 194.—San Francisquito Power Plant No. 1 on the Los Angeles Aqueduct. Static head from maximum water level in surge chamber on creast of hill to the nozzles is 941 ft.

permit, a forebay may be constructed at the head of the penstock. The forebay is a reservoir of limited capacity whose function is to equalize the flow. Into it the water may be delivered at a uniform rate, while from it the water may be drawn by the penstock at varying rates according to the demands upon the tur-

bines. Thus the fluctuations in the flow of water through the turbines need not extend back all the way to the source.

Where a forebay is impossible or not really necessary, it is desirable to provide surge chambers or other means of relieving the abnormal conditions attendant upon changes of flow. In the upper left-hand corner of Fig. 193 is seen a small surge chamber, and an overflow. The five penstocks receive water from a pressure tunnel 3 miles in length. In case of a sudden decrease in discharge through the turbines, the excess water could surge up the large pipe line running up the hillside and if the surge was

From a photograph by the author.

Fig. 195.—Surge chamber designed by W. F. Durand. It is 100 ft. in diameter at the top and the maximum water level is 150 ft. above the pressure tunnel. Only 35 ft. projects above the ground.

great enough some water would overflow, thus preventing any excessive increases in pressure.

In Fig. 194 is shown a power plant with a large surge chamber at the end of a pressure tunnel which is 7.76 miles in length and in Fig. 195 is seen as much of the surge chamber as is visible above the ground. This is also provided with a spillway so that it may overflow if the surge is violent.

The power plant shown in Fig. 196 receives water through a conduit 1,711 ft. in length and is equipped with a large air chamber within the power house to absorb shocks.

The water from a power plant may be discharged directly into

some natural stream or it may be necessary to construct an artificial channel for a tail race, as in Fig. 200. In other cases, as with some of the plants at Niagara Falls, the tail race may be a long tunnel.

133. High-head Plants.—It is impossible to establish any definite number of feet which is required to differentiate a high-

From a photograph by the author.

FIG. 196.—Cornell University hydro-electric plant. Head = 140 ft., 1–550 hp. turbine, 2–280 hp. impulse wheels, 2–50 hp. impulse wheels.

head from a medium- or a low-head plant. A high-head is one of several hundred feet or more, while a low-head plant would doubtless be under 50 ft. But one type shades very gradually into the other.

In Fig. 197 is shown a high-head development, where a fall of 4,000 ft. is divided between two power houses in series. In this one view may be seen a complete plant with many of the features that have been described, except that a forebay is not required. The mountain ranges, which rise to a height of 11,000 ft., provide a water shed, the runoff from which is gathered by a lake about 4 miles long, and with an elevation of 6,000 ft. The lake is created by the erection of three dams which can be seen placed in gaps in the hills. From the lake the water flows down the penstocks to the first plant. The discharge from this supple-

Courtesy of Stone and Webster.

Fig. 197.—Big Creek development of Pacific Light and Power Corporation. The fall from the lake to the first power house is 2100 ft., and from that to the second power house is 1900 ft.

mented by some water from a little stream then flows through a tunnel for a way until it takes another drop to the second power house which can be seen in the lower part of the picture, a little to the left of the center. In Figs. 198 and 199 are closer views of these two plants. At the upper right-hand corner of Fig. 198 can be seen two standpipes, the water level in which will be nearly as high as that in the lake so that the entire 2,100 ft. drop is shown here. The standpipes for the second plant can barely be seen on the crest of the hill in the upper left-hand corner of Fig. 199.

A high-head plant requires but little water for a given amount of power and it is usually so situated that a storage reservoir is

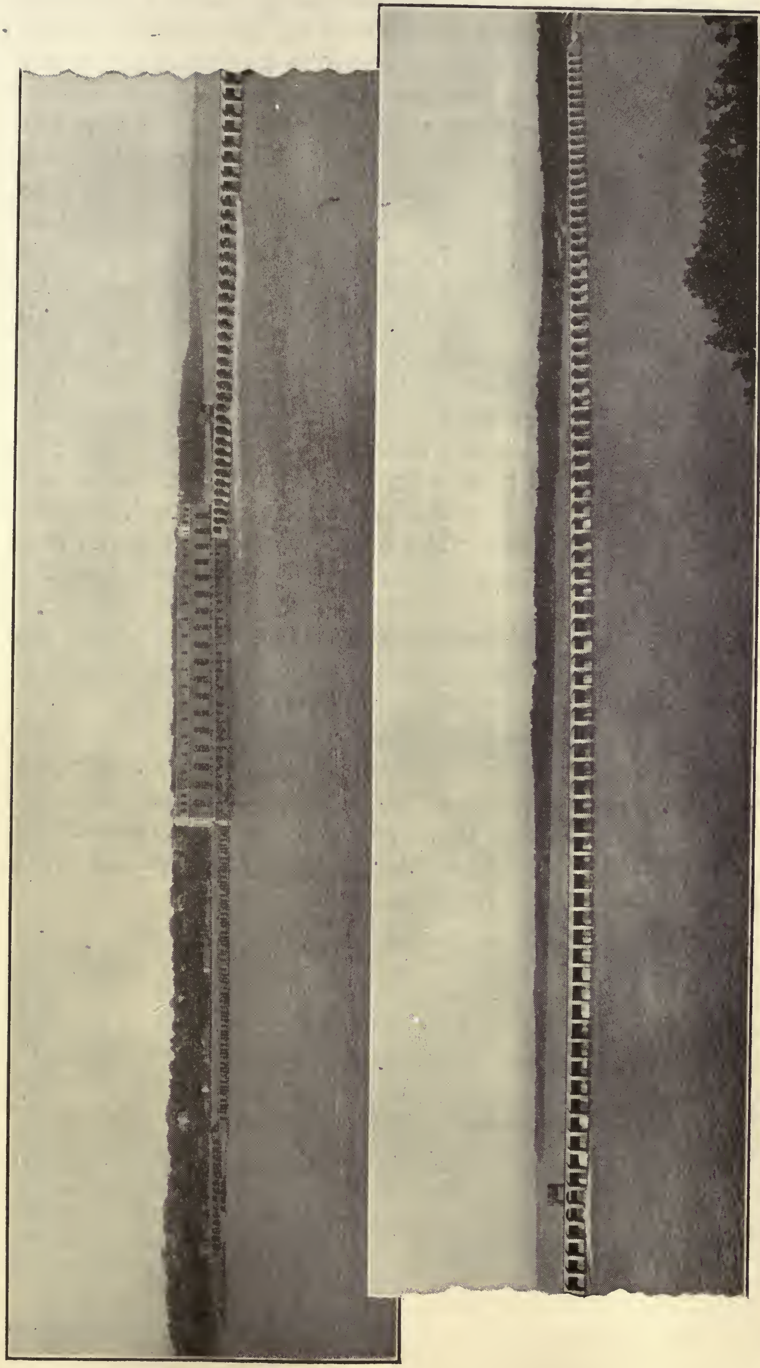
Courtesy of Stone and Webster.
FIG. 198.—Power Plant No. 1 at Big Creek near Fresno, Calif. Static head = 2100 ft; Hp = 40,000.

Courtesy of Stone and Webster.

FIG. 199.—Power Plant No. 2 at Big Creek. Static head = 1900 ft.; $Hp. = 40{,}000$.

Courtesy of I. P. Morris Co.

FIG. 200.—Appalachin Power Co. development No. 2. Head = 49 ft., four turbines of 6000 hp. each at 116 r.p.m,



Courtesy of Mississippi River Power Co.

FIG. 201.—The Mississippi River Power Co. at Keokuk, Ia. Head = 32 ft., capacity is 15 units of 10,000 hp. each at 57.7 r.p.m. Maximum capacity of plant is 200,000 hp.

to be had. Consequently it may be able to run for a long time merely on the water that is conserved by the creation of such reservoirs. It is always necessary to have a penstock and many of the other details that have been enumerated.

134. Low-head Plants.—A typical low-head plant is shown in Fig. 200. The head, under which the turbines operate, has been practically created by the erection of a dam. There are no pipe lines and the body of water produced by the dam now becomes the forebay. The turbines in such a plant may have any one of the three types of settings shown in Figs. 181, 182, and 183.

It may be seen that fluctuations in the flow of the river, with consequent changes in water level, cause variations in the head under which the turbines operate. This is something that scarcely exists in a high-head plant. Also low heads are usually found in fairly flat countries, where the nature of the topography renders it impractical to store up large quantities of water and furthermore under a low head a large amount of water is required to develop a given power. This makes it impossible to run very long on storage and hence the plant is dependent upon a regular stream flow.

The differences between the high- and low-head plants are such as to require turbines of different characteristics in order to meet the conditions most satisfactorily.[1]

Another typical low-head plant is shown in Fig. 201. The length of the dam across the river is nearly a mile. While a low-head plant is often free from many of the items that are required in a high-head development, it must be remembered that it must be built to handle large volumes of water and much massive construction is required.

[1] R. L. Daugherty, "Hydraulic Turbines," Chap. XII.

CHAPTER XIII

THEORY OF THE IMPULSE WHEEL

135. Action of the Water.—The impulse wheel is more accurately described as a tangential waterwheel from the fact that the center line of the jet is tangent to the path described by the center of the buckets. The latter is called the "impulse circle" and computations are based upon the linear velocity of the wheel at this radius. For an impulse wheel the nominal value of D is the diameter of this circle.

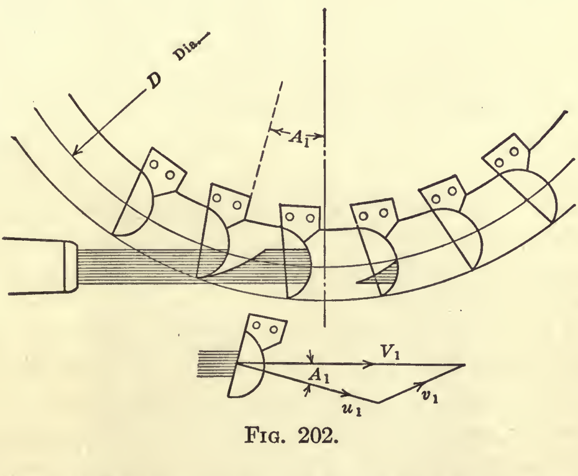


Fig. 202.

It is often stated that the jet at impact is also tangent to this circle, but this is not a true representation of the facts as Fig. 202 will show. The jet strikes the buckets before they arrive at a point directly under the center of rotation and hence the angle A_1 is not zero. Observation of various Pelton wheels in action has convinced the author that average values of A_1 may vary from 5° to 20°, according to the design of the wheel. The value of A_1 must be an average value for a given wheel from the fact that the bucket moves through a certain angle from the time it first enters the jet until the last drop of water has struck it.

The illustration also shows that when a bucket first enters the jet it cuts off the water from the preceding bucket and leaves a "slug" of water to catch up with the latter and to complete its work upon it. Thus the water may be acting upon several buckets at the same time. This explains why there is a difference between the W' of Art. 109 and W. The former is the amount of water acting upon a single bucket, the latter is the total acting upon all the buckets. It is not necessary to know how many buckets are in action at a time for, since the wheel does not move away from the nozzle, it follows that all of the water discharged by the nozzle may act upon it.

But it is not necessarily true that the wheel utilizes all of the water under all circumstances. Suppose for instance that the buckets were to move as fast as the jet; it would then be seen that none of the water could overtake them, but that all would go right on through. And for speeds somewhat less than this a portion of the water would deliver its energy to the buckets and the latter portions of the intercepted "slugs" would not be able to overtake the buckets before they had swung up above the line of action of the jet. The problem is to so design the buckets and the wheel that all of the water in the jet will be able to do its work upon the wheel, when running at the proper speed. For speeds much above the normal speed a certain amount of water must necessarily go right on through without having had a chance to do work.

By the proper speed or normal speed is meant the one that the wheel should have for the jet velocity in question. A high jet velocity would require a high wheel speed and *vice versa.* In fact we are concerned with the relation between the various velocities rather than with their actual values, and hence it is desirable to introduce factors which shall express this relationship and be independent of the head. Thus if the jet velocity be denoted by V_1 and the linear velocity of the bucket at the impulse circle by u_1, we may use c_v and ϕ such that

$$V_1 = c_v\sqrt{2gh} \qquad (120)$$

$$u_1 = \phi\sqrt{2gh} \qquad (121)$$

It may be seen that c_v is the velocity coefficient of the nozzle, the value of which is constant for any setting of the needle. Thus for any given value of ϕ the relation between V_1 and u_1 is known at once regardless of the value of h.

$\frac{u_1}{V_1} = \frac{c_v}{\phi}.$

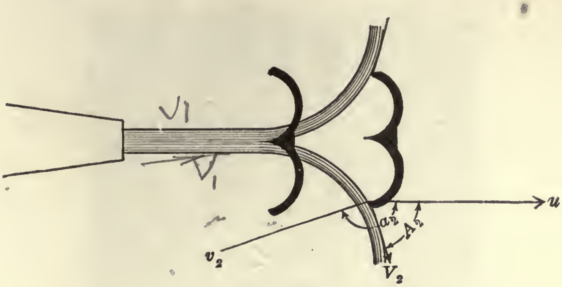


FIG. 203.

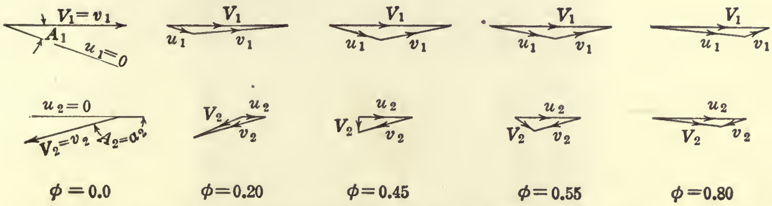
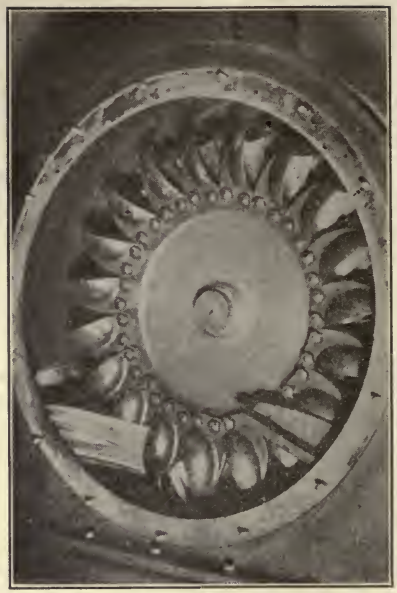


FIG. 204.—Velocity diagrams for different speeds.

From a photograph by the author.

FIG. 205.—A 42-in. Pelton-Doble impulse wheel.



From a photograph by the author.

FIG. 206.—Showing discharge from buckets when wheel is at rest, or $\varphi = 0.0$.

From a photograph by the author.

FIG. 207.—Wheel running at slow speed. $\varphi = 0.20$.

From a photograph by the author.

FIG. 208.—Wheel running at normal speed. $\varphi = 0.45$.

From a photograph by the author.

FIG. 209.—Wheel running at higher speed. $\varphi = 0.55$.

The absolute path of the water and the velocity vectors at discharge from the buckets may be seen in Fig. 203. For different wheel speeds under the same head, which means different values of ϕ, we should have such diagrams as are shown in Fig. 204. As the speed of the wheel increases from zero, the angle of deflection of the jet continually decreases. It may also be seen from the diagrams that the value of V_2 is relatively high when the wheel is at rest, that it becomes a minimum at such a

From a photograph by the author.

FIG. 210.—Wheel at run-away speed. $\varphi = 0.80$.

speed that A_2 is approximately equal to 90°, and then increases again.

The action of the water as just described is illustrated by some rather unusual photographs taken of a 42-in. wheel in action. The side of the casing was removed for the purpose. The needle was withdrawn as far as possible so that the maximum size jet which the design permitted is shown in the photographs. In Fig. 206 the wheel was prevented from rotating by applying a sufficient torque to the shaft. The jet cannot be seen, but the water leaving the bucket is shown. In Fig. 208 the wheel is running at its most efficient speed. The water

leaving the buckets drops down into the tailrace with most of its energy abstracted. In Fig. 210 the wheel is shown at run-away speed, all load having been removed save its own friction and windage and that of the generator to which it is direct-connected.

136. Force Exerted by Jet.—The tangential waterwheel, Pelton wheel, or impulse wheel, as it is variously called, is really an impulse turbine with approximately "axial" flow. By this is meant that $r_1 = r_2$. The latter is not strictly true but is sufficiently close for all practical purposes.[1]

The force desired is really the tangential component of the resultant force. This may be obtained by computing the tangential component of the ΔV in equation (83), or, since r_1 and r_2 are equal, it may be obtained from equation (100). The results are identical. The desired component is

$$P = \frac{W}{g}(V_1 \cos A_1 - V_2 \cos A_2). \qquad (122)$$

The values of V_2 and A_2 depend upon the jet velocity and the speed of the wheel, and are therefore variable and unknown. It is desired to replace them in terms of V_1 and u_1 and wheel dimensions which may be supposed to be known. It is thus necessary to find some relation between the velocities at entrance and those at discharge. The equation of flow between these two points will be found on page 164c, and for the impulse turbine it becomes

$$\frac{v_1^2}{2g} - \frac{v_2^2}{2g} + \frac{u_2^2}{2g} - \frac{u_1^2}{2g} = k\frac{v_2^2}{2g} \qquad (123)$$

where k is a coefficient of loss in flow over the buckets, such that the head lost is $kv_2^2/2g$. Since we are assuming that $u_1 = u_2$, this gives us for the Pelton wheel the special relation

$$v_2 = \frac{v_1}{\sqrt{1+k}}.$$

$V_2 = n V_1 \quad n = \frac{1}{\sqrt{1+K}}$

In a numerical case the value of v_1 can be computed by trigonometry according to whichever one of the methods given on page 264 is deemed to be more convenient or more accurate. Having v_1 the value of v_2 may be found by the equation just given. The velocity diagram at outflow is now determined since v_2, u_2, and a_2 are known.

[1] For further details see "Theory of the Tangential Waterwheel," by R. L. Daugherty, *Cornell Civil Engineer*, vol. 22, p. 164 (1914). Also see "Hydraulic Turbines," Chap. VII.

From the vector diagram (Fig. 142) it may be seen that

$$V_2 \cos A_2 = u_2 + v_2 \cos a_2.$$

The numerical value of P can be readily calculated after inserting the latter quantity in equation (122).

For an algebraic solution we may write the above as

$$V_2 \cos A_2 = u_1 + \frac{\cos a_2}{\sqrt{1+k}} v_1 \tag{124}$$

and in turn v_1 may be replaced by a trigonometric expression involving V_1 and u_1.

The resulting expression may be greatly simplified by the following assumption, though our result will not be precisely correct. While it is not true that $A_1 = 0°$, yet it is small enough so that its cosine differs from unity by only a few per cent. Assuming that $\cos A_1 = 1$, the expression for v_1 reduces to $v_1 = V_1 - u_1$ and thus we have as an approximation

$$V_2 \cos A_2 = u_1 + \frac{\cos a_2}{\sqrt{1+k}}(V_1 - u_1).$$

Inserting this value in the expression for P we have

$$P = \frac{W}{g}\left(1 - \frac{\cos a_2}{\sqrt{1+k}}\right)(V_1 - u_1) \tag{125}$$

The equation just derived shows that P decreases as the speed of the wheel is allowed to increase. This is what we should expect, since the value of ΔV decreases as shown by Figs. 206 to 210. Equation (125) is apparently the equation of a straight line between P and u_1. However, the factor k is not strictly constant and, as has been shown, the value of W will decrease for speeds much above the normal. Likewise the correct equation involves A_1 and this also changes somewhat with the speed.[1]

The torque exerted by the water upon the wheel may be obtained by multiplying P by r_1, the radius of the impulse circle. The torque which the wheel can deliver is somewhat less than this because of bearing friction and windage.

Fig. 211 shows the performance of a certain wheel at different

[1] The exact equation for all impulse turbines is

$$P = \frac{W}{g}\left[V_1 \cos A_1 - x^2 u_1 - \frac{x \cos a_2}{\sqrt{1+k}}\sqrt{V_1^2 + x^2 u_1^2 - 2V_1 u_1 \cos A_1}\right]$$

where $x = r_2/r_1$. For the Pelton wheel or axial flow turbine $x = 1$; for the outward or inward flow Girard impulse turbines it is more than or less than unity respectively.

speeds under a constant head. The variation in the force at zero speed is due to changes in the angle A_1 and r_2/r_1 with different positions of the buckets. This is shown for one nozzle setting only, though it exists in all.

137. Power of Wheel.—Since power is the product of P and u_1 or T and ω, it may be seen that it is zero when the wheel is at rest, though the torque is then a maximum, and it is also zero when the speed is a maximum for the torque is then zero.

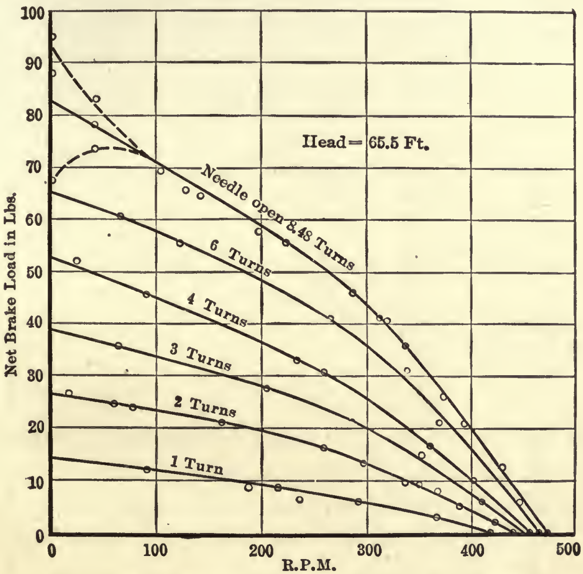


FIG. 211.—Relation between torque and speed.[1]

The maximum power will be obtained for some speed between these two extremes, as shown by Fig. 212.

Since for a given head and nozzle opening the power input is constant regardless of the speed of the wheel, it follows that the efficiency is directly proportional to the power developed. But it should be noted that the power delivered in the water increases with the nozzle opening so that the needle setting that gives the largest power is not necessarily the most efficient.

138. Speed.—From equation (125) we should conclude that P would become zero when $u_1 = V_1$ or when $\phi = c_v$, the value of which would be about 0.98. Also if we should multiply this

[1] From the test of a 24-in. wheel by F. G. Switzer and the author.

equation by u_1 the power would be seen to be a maximum when $u_1 = 0.5V_1$. But equation (125) is only an approximate representation of the actual facts. Because of the large amount of water that is not utilized at high values of ϕ, and also because

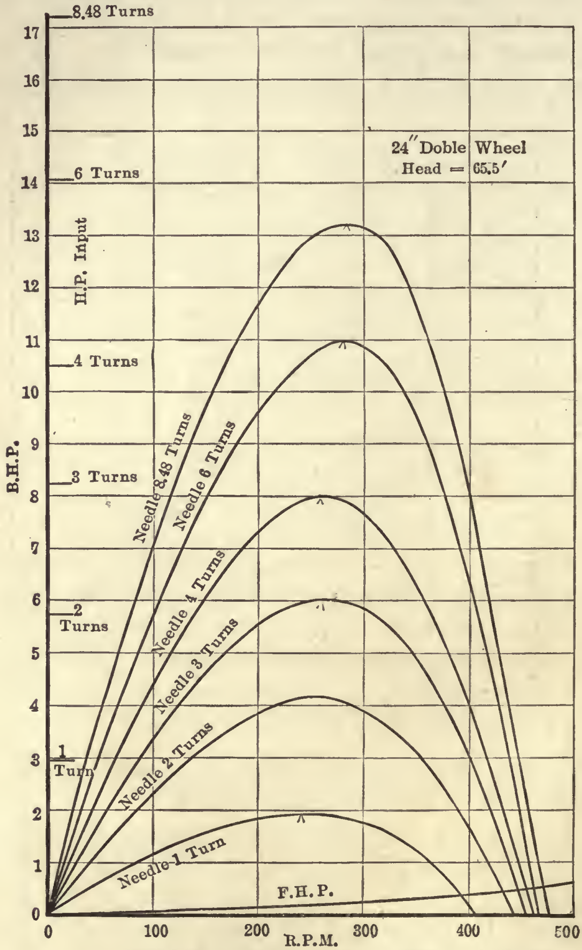


FIG. 212.—Relation between power and speed.

the bearing friction and windage prevent the torque from ever being reduced to zero, the actual maximum speed attained by the tangential waterwheel is such that ϕ is approximately equal to 0.80.

In like manner the maximum power, and hence the maximum

efficiency for a given nozzle opening, is also attained when the wheel speed is something less than $0.5V_1$. Thus in actual practice we have for the best efficiency

$$\phi_e = 0.43 \text{ to } 0.48.$$

In practical applications we are usually interested in the performance of a wheel at a constant speed under a constant head. Values for this may be obtained from Figs. 211 and 212 by following along any vertical line. Generally the vertical line should be the one for the speed at which the maximum efficiency is found. The resulting curves for the impulse wheel would be very similar to those for the reaction turbine shown in Fig. 216.

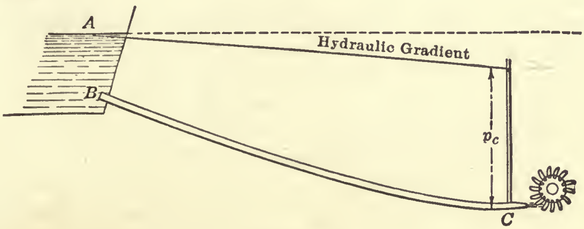


FIG. 213.

139. Head on Impulse Wheel.—The nozzle is considered an integral part of the impulse wheel and hence the head under which the wheel is said to operate must include it. If C in Fig. 213 indicates a point at the base of the nozzle,

$$h = H_c = p_c + \frac{V_c^2}{2g} \tag{126}$$

It is this value of h that should be used in determining the efficiency of the wheel.

This value of h is the total fall from headwater to nozzle minus the head lost in the pipe line. The energy supplied at this point is expended in four ways. A small amount is lost in flow through the nozzle, a portion is expended in hydraulic friction and eddy losses within the buckets, kinetic energy is carried away in the water discharged from the buckets, while the rest is delivered to the wheel to do useful work and overcome mechanical friction and windage losses. Calling h'' the head delivered to the buckets we may write

$$h = \left(\frac{1}{c_v^2} - 1\right)\frac{V_1^2}{2g} + k\frac{v_2^2}{2g} + \frac{V_2^2}{2g} + h'',$$

It may also be noted that $Wh'' = Pu_1$, hence h'' can be obtained from equation (122) by merely substituting u_1 for W.

See also Art. 120*a*.

140. PROBLEMS

1. A nozzle having a velocity coefficient of 0.98 discharges a jet 6 in. in diameter under a head of 800 ft. This jet acts upon a wheel with the following dimensions: diameter 6 ft., $A_1 = 10°$, $a_2 = 165°$, and it is assumed $k = 0.70$. Find the force exerted upon the buckets when $\phi = 0.45$.

2. Solve problem (1), assuming that $A_1 = 0°$.

3. Find the power developed upon the buckets of the wheel in problem (1), assuming $A_1 = 0°$. What is the hydraulic efficiency of the wheel?

4. If the mechanical efficiency of the wheel is 0.97, what is the gross efficiency in problem (3)?

5. Assuming $A_1 = 0°$ in problem (1), find the power lost in hydraulic friction within the buckets. Find the value of V_2 and determine the power carried away in the water discharged from the turbine.

6. Is the hydraulic efficiency of an impulse wheel dependent upon the head under which the wheel is run? What equation would express the value of the hydraulic efficiency for any tangential waterwheel?

7. What would be the proper r.p.m. of the wheel in problem (1)?

8. A good proportion between jet and wheel is that the diameter of the wheel in feet should equal the diameter of the jet in inches. Using this ratio, what size wheel would be required to deliver 5,000 hp. under a head of 1,400 ft., assuming an efficiency of 82 per cent.? What would be the speed of the wheel?

9. The maximum speed attained by the wheel of Fig. 211 was 475 r.p.m. under a head of 65.5 ft. What was the value of ϕ?

10. The best efficiency for the wheel whose curves are shown in Fig. 212 was found with the needle open 6 turns. The speed was 275 r.p.m. and the head 65.5 ft. What was the value of ϕ_e?

CHAPTER XIV

THEORY OF THE REACTION TURBINE

141. Introductory Illustration.—The reaction turbine is so called because an important factor in its operation is the reaction of the streams of water discharged from the runner. It is well to bear in mind, however, that the total dynamic effort is due to the entire change in the momentum of the water just as in the impulse turbine.

As an illustration, consider the vessel ABC of Fig. 214 into which water enters across AB with a velocity V_1 and is discharged at C with a velocity V_2. Now the reaction of the jet alone could be determined by an application of Art. 110. But the total force is due not only to this reaction but also to the impulse of the water entering at AB. It is not feasible to separate the effect of impulse from that of reaction, neither is it necessary to do so. The horizontal component of the total dynamic force is obtained directly by

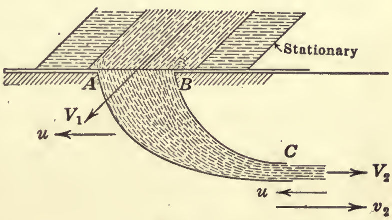


FIG. 214.

$$P = \frac{W}{g}(V_1 \cos A_1 + V_2).$$

Suppose now that this vessel moves to the left with a uniform translation u. Assume that in some way the water is still supplied to it with a velocity V_1. This might be the case if the vessel passed under a series of stationary passages each of which in turn was permitted to discharge water into it. The value of the absolute velocity of discharge is now

$$V_2 = v_2 - u.$$

Inserting this value above we have

$$P = \frac{W}{g}(V_1 \cos A_1 + v_2 - u)$$

This equation indicates that P decreases as the speed increases, just as in the case of the impulse turbine. Also the water entering the vessel at AB is under pressure and is not a free jet. Therefore V_1 must be less than $\sqrt{2gh}$. Since all the passages are completely filled with water the equation of continuity can be applied, and it will show that V_1 and v_2 are inversely proportional to the areas of their respective streams. But the value of v_2 depends upon the losses of head, and in a real turbine these

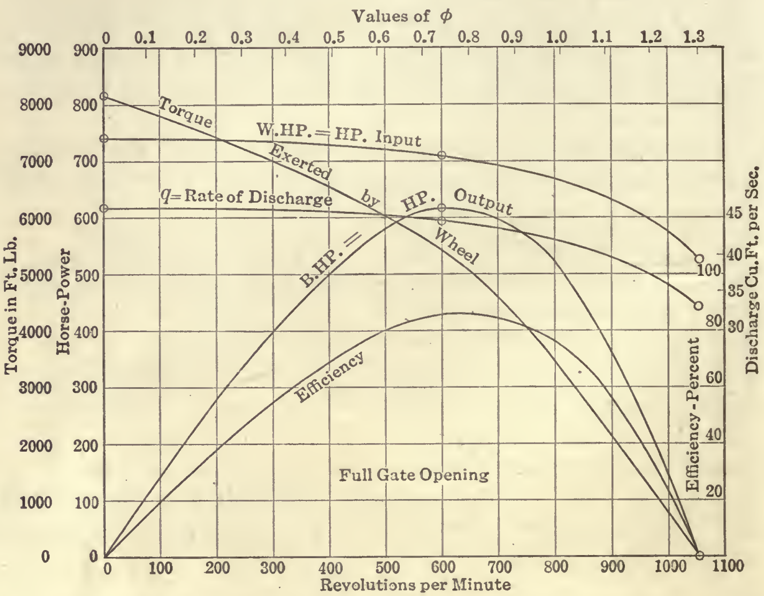


FIG. 215.—Test of 27 in. I. P. Morris turbine. Head and gate opening constant. Speed variable.[1]

hydraulic losses vary with the speed. Since V_1 is proportional to v_2, it follows that V_1 varies with the speed of the wheel.

Thus some fundamental differences between impulse and reaction turbines are that in the former $V_1 = c_v\sqrt{2gh}$, where c_v is a velocity coefficient nearly equal to unity. This velocity, and hence the amount of water discharged by the nozzle, is

[1] Figs. 215 and 216 are from the test of a reaction turbine in the Cornell University power plant. See "Investigation of the Performance of a Reaction Turbine" by R. L. Daugherty, *Trans.* A. S. C. E., vol. 78, p. 1270 (1915).

entirely independent of the design of the wheel and its operation. But for the reaction turbine

$$V_1 = c\sqrt{2gh} \tag{127}$$

where c is not a velocity coefficient but a factor whose value varies from about 0.6 to 0.8 for ordinary designs. The value of c depends upon the design of the wheel and the speed at which it is run under a given head. This means that c is also a function of ϕ, where ϕ has the meaning given by equation (121).

With the radial-flow type of turbine centrifugal force also causes the value of c to vary with the speed of the wheel, the head remaining constant. The centrifugal force opposes the flow of water in the case of the inward-flow turbine so that, as the speed increases under a constant head, the discharge tends to decrease as shown in Fig. 215. But there are other influences at work also, so that for some inward-flow turbines the value of q actually increases somewhat as the speed increases above zero, but after a certain speed is exceeded the rate of discharge falls off again.

142. Torque Exerted.—The preceding article merely illustrates a few fundamental points regarding the reaction turbine. Since with the real machine the radii of the water at inflow and outflow differ materially, it is not feasible to compute the force exerted by the water and we must get the torque instead. Before proceeding any further with the theory it should be noted that, while our equations are rational, they must assume that all particles of water move in similar paths with equal velocities. Actually we have to deal with average values. But we do not know these average values with any precision. For example, we have no assurance that the angles A_1 and a_2 are the same as the angles of the guide vanes and the runner vanes respectively. In fact we have some evidence to indicate that they differ by as much as 5° or 10°. The same condition exists with regard to the areas of the streams and all other dimensions used. Thus the numerical results of such computations cannot be expected to agree precisely with actual facts.

Despite this the theory has its value. It serves to explain the principles of operation of such machines, to indicate the nature of their actual characteristics, and to account for numerous observed facts. In design the theory shows what proportions are desirable and what the effect of certain changes of dimensions

would be. Thus if we have some actual test data to work from, the theory would enable us to alter existing designs with some degree of assurance.

In order to compute the torque exerted upon the runner by the water we should take the fundamental formula of Art. 113,

$$T = \frac{W}{g}(r_1V_1 \cos A_1 - r_2V_2 \cos A_2).$$

Just as in the case of the impulse turbine in Art. 136, the values of V_2 and A_2 are variable and unknown, and it is necessary to replace them in terms of known quantities. It is assumed that all the dimensions of the wheel and the values of V_1 and u_1 are known.

From the vector diagram it may be seen that

$$V_2 \cos A_2 = u_2 + v_2 \cos a_2.$$

But $u_2 = (r_2/r_1)u_1$, and, since the passages are completely filled with water in the reaction turbine, the equation of continuity gives $q = F_1V_1 = f_2v_2$, or

$$v_2 = (F_1/f_2)V_1 \tag{128}$$

(Contrast this procedure with that for the impulse turbine in Art. 136 and note that equation (122) does not apply here.) Making the proper substitutions we easily derive

$$T = \frac{W}{g}r_1\left[\left(\cos A_1 - \frac{r_2}{r_1}\cdot\frac{F_1}{f_2}\cos a_2\right)V_1 - \left(\frac{r_2}{r_1}\right)^2 u_1\right] \tag{129}$$

In the use of equation (129) we should have to determine the value of W for any value of u_1, either by experiment or by computing the rate of discharge by theory.

143. Power.—The power developed by the water is determined by multiplying T by the angular velocity. The torque actually exerted *by* the shaft and the power delivered are obtained by multiplying these values by the mechanical efficiency.

The hydraulic efficiency of the turbine is obtained by

$$e_h = \frac{T\omega}{Wh} = \frac{Wh''}{Wh} = \frac{h''}{h}.$$

It is difficult to obtain the hydraulic efficiency by test as it is necessary to determine the bearing friction and also the disk friction due to the drag of the runner through the water

in the clearance spaces. But these losses may be allowed for and the hydraulic efficiency then secured approximately.

Because of the necessary defects of the theory, the hydraulic efficiency may be assumed with less error than is usually involved in computing T. For turbines of rational design and running at their proper speeds the value of the hydraulic efficiency may range from 0.80 to 0.95. The higher values are found only in large turbines and with favorable proportions. Only experience can enable one to select the proper value be-

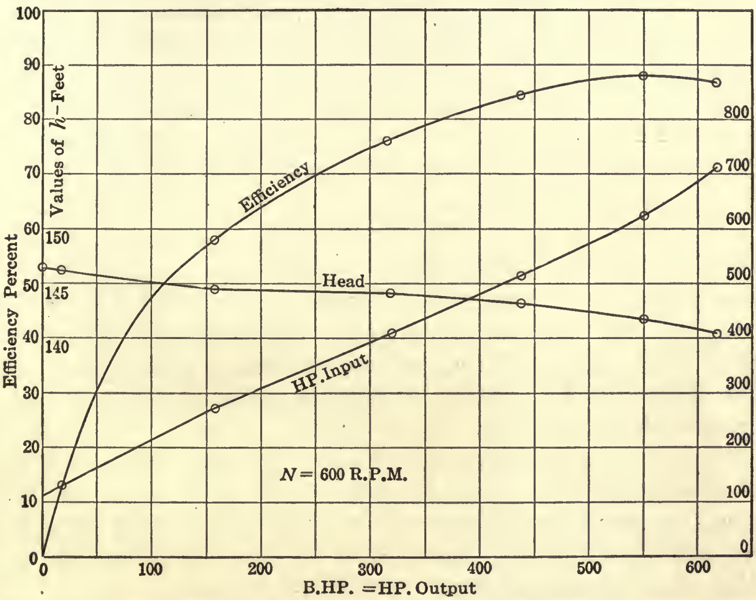


FIG. 216.—Test of 27 in. I. P. Morris turbine. Head and φ approximately constant. Gate opening variable.

tween these two extremes, which are not necessarily limits. For improper speeds and incorrect designs no values can be assigned.

The curves of Fig. 215 show the characteristics of a reaction turbine with a fixed gate opening and the speed variable. These are similar to those of the impulse wheel except that q is not a constant. Hence maximum power developed and the maximum efficiency do not necessarily occur at the same speed. The characteristics of the same turbine at a constant speed are

shown in Fig. 216. The maximum efficiency is 88 per cent. at 550 hp. under a head of 141.8 ft.

144. Speed.—Although the water flowing through the runner of a reaction turbine is entirely confined, the velocity undergoes changes similar to that in the impulse turbine, except that for a fixed gate opening the angle A_1 is constant. Hence the values of V_2 and A_2 vary in just the same way as is shown by Figs. 204 to 210.

The speed at which the efficiency is the highest will be somewhere in the neighborhood of the one for which the discharge loss is the least. The value of V_2 will be a minimum in such a case as Fig. 208. For this condition it will be found that approximately $A_2 = 90°$ or that $u_2 = v_2$. Note that these two conditions are not identical but they differ but little It is customary to assume one or the other according to convenience.

In the case of the reaction turbine V_1 is less than for an impulse turbine under the same head. But the water at entrance is under pressure and, as it flows through the runner, this is converted into velocity. Hence at discharge v_2 may easily be as large as in the case of the impulse wheel. And if $u_2 = v_2$, it may be seen that u_2 will be about the same in either type. But with the inward flow reaction turbine u_1 is greater than u_2, and therefore the peripheral velocity of the reaction turbine is greater than that of the impulse wheel.

Not only is the peripheral speed higher for maximum efficiency but also the runaway speed is higher. The maximum value of ϕ for the reaction turbine is about 1.30, though with some it may easily exceed this value. And for the normal speeds at which the maximum efficiency is obtained we have

$$\phi_e = 0.60 \text{ to } 0.90,$$

the exact value for a given wheel depending upon its design.

145. Values of c_e and ϕ_e for Maximum Efficiency.—The turbine should run normally at such a speed under any head that the maximum efficiency will be attained. It will be assumed that this speed is such that the discharge is "radial" or $A_2 = 90°$. The angle which the runner vane at entrance makes with u_1 will be denoted by a'_1. As the turbine is ordinarily designed, the value of the vane angle would be such that it would agree with a_1 as determined by the vector diagram for this same speed. But at any other speed the value of a_1 would be different

from a'_1, hence there would be an abrupt change in the direction of the water entering the runner giving rise to what is known as "shock loss."

The following expressions therefore apply *only* to the *special* case where $A_2 = 90°$ and $a'_1 = a_1$. From the vector diagram of velocities we have

$$V_1 \sin A_1 = v_1 \sin a_1 = v_1 \sin a'_1$$

$$V_1 \cos A_1 = u_1 + v_1 \cos a_1 = u_1 + v_1 \cos a'_1$$

Eliminating v_1 between these two equations we have

$$u_1 = \frac{\sin (a'_1 - A_1)}{\sin a'_1} V_1, \tag{130}$$

as the relation between u_1 and V_1 when there is no loss at entrance to the runner.

The power delivered by the water to the runner may be expressed as

$$T\omega = Wh'' = e_h Wh,$$

where T has the value given by equation (100). Thus

$$Wh'' = \frac{W}{g} (r_1 V_1 \cos A_1 - r_2 V_2 \cos A_2) \frac{u}{r}.$$

If the discharge is "radial," $A_2 = 90°$ and hence $V_2 \cos A_2 = 0$. Therefore

$$h'' = e_h h = \frac{u_1 V_1 \cos A_1}{g}. \tag{131}$$

Solving equations (130) and (131) simultaneously, we have

$$V_1 = \sqrt{\frac{e_h\, 2gh}{2} \frac{\sin a'_1}{\sin (a'_1 - A_1) \cos A_1}}$$

$$u_1 = \sqrt{\frac{e_h\, 2gh}{2} \frac{\sin (a'_1 - A_1)}{\sin a'_1 \cos A_1}}$$

From this it follows that

$$c_e = \sqrt{\frac{e_h \sin a'_1}{2 \sin (a'_1 - A_1) \cos A_1}} \tag{132}$$

$$\phi_e = \sqrt{\frac{e_h \sin (a'_1 - A_1)}{2 \sin a'_1 \cos A_1}} \tag{133}$$

It must be borne in mind that equations (132) and (133) can be

applied only for the special case stated. For any other speed a different procedure would be necessary but it will not be given here.[1]

The speed desired is the one for which the gross efficiency is a maximum and this may not be quite the same as the one for which the hydraulic efficiency is the highest. Hence the true value of ϕ_e may differ slightly from the value given by equation (133).

These equations appear to be independent of conditions at outflow from the runner. But it must be noted that they are to be used only upon the assumption that the dimensions used at exit will be such as to make $A_2 = 90°$, when c_e and ϕ_e have the values given.

An interesting result may be obtained by multiplying equations (132) and (133). This gives

$$c_e \phi_e = \frac{e_h}{2 \cos A_1}. \tag{134}$$

This would indicate that all other things being equal, the higher the value of ϕ_e the smaller the value of c_e. With the impulse turbine, to which these equations apply also, ϕ_e is small but $c = c_v$ and is near unity. With the reaction turbine ϕ_e is larger than for the impulse wheel but c is smaller.

146. Theory of the Draft Tube.—If the draft tube is properly designed its area next to the runner should be such that the velocity in it is equal to V_2, the absolute velocity of discharge, otherwise there will be an abrupt change of velocity involving losses. For Fig. 217 we may write

$$H_2 = p_2 + z_2 + V_2^2/2g, \quad \text{and} \quad H_4 = 0.$$

The losses between points (2) and (4) are made up of the friction losses within the tube, H'_f, and the discharge loss at (3). Applying the general equation between points (2) and (4) we have

$$p_2 = -z_2 - \frac{V_2^2}{2g} + H'_f + \frac{V_3^2}{2g} \tag{135}$$

The larger the diameter of the tube at its mouth the less will be the value of V_3 and hence the less the pressure at the point of discharge from the runner. But if too great a rate of diffusion is provided for in the tube the flow in it will be unstable and

[1] A general relation between c and ϕ for all conditions will be found in the author's "Hydraulic Turbines," Chap. VIII.

the friction loss H'_f will be increased. The pressure at the top of the draft tube should not be made less than about 5 ft. absolute, and the value of z_2 determined accordingly. A "high-speed" turbine runner with a large value of V_2 cannot be set as far above the water level as a "low-speed" turbine with a smaller value of V_2.

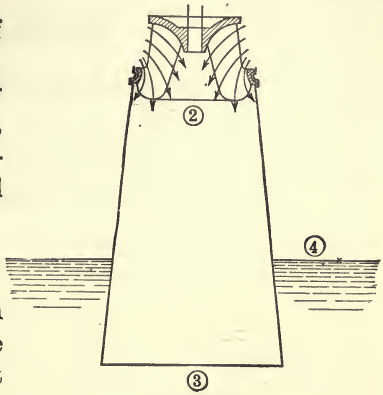


Fig. 217.

If H'_f be assumed equal for both straight and flared tubes, it may be seen that the diverging draft tube increases the head utilized by the turbine by

$$\left[1 - \left(\frac{F_2}{F_3}\right)^2\right]\frac{V_2^2}{2g}.$$

147. Head on Reaction Turbine.—For a reaction turbine the draft tube is an integral part of the machine, hence (Fig. 218) the head under which it operates is given by

$$h = H_c - H_f = p_c + z_c + \frac{V_c^2}{2g}. \tag{136}$$

This is the value of h upon which computations are based, and it is the one to be used in determining the efficiency of the turbine.

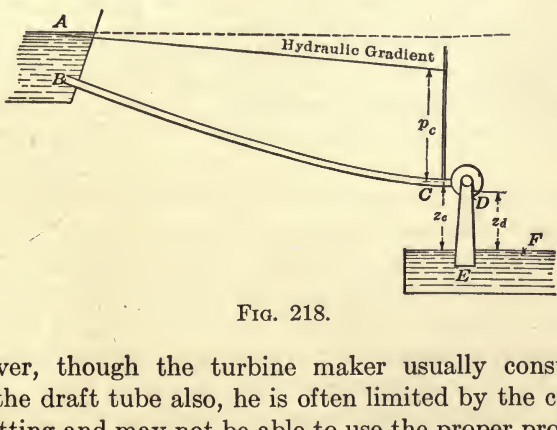


Fig. 218.

However, though the turbine maker usually constructs or designs the draft tube also, he is often limited by the conditions of the setting and may not be able to use the proper proportions. In order to eliminate this defect in the setting, over which he has no control, the velocity head at (E) is sometimes deducted from the value given by equation (136). If it were feasible to

eliminate the friction in the draft tube as well we should then have the efficiency of the runner alone, which is independent of the draft tube. But what we usually desire is the efficiency of the entire unit from the intake of the casing to the tailrace.

148. PROBLEMS

1. A certain reaction turbine was found by actual test to have a hydraulic efficiency of 0.83 when $\phi = 0.670$ and $c = 0.655$. The angles were: $A_1 = 13°$ and $a'_1 = 115°$. Compute the values of ϕ_e and c_e and compare with the actual values. (The slight discrepancy between the two is largely due to the fact that shockless entrance and radial discharge were not obtained at exactly the same speed.)

Ans. $\phi_e = 0.678$, $c_e = 0.628$.

2. For a reaction turbine the dimensions are: $A_1 = 35°$, $a'_1 = 136°$, $e_h = 0.845$. Compute the values of ϕ_e and c_e.

Ans. $\phi_e = 0.85$, $c_e = 0.60$.

3. In the test of the Cornell University turbine the pressure was read by a mercury manometer attached near the intake flange where the diameter was 30 in. At full load when the discharge was 44.5 cu. ft. per sec., the manometer read 9.541 ft. of mercury, the top of the shorter mercury column being 0.500 ft. above the intake. If the elevation of the intake above the water level in the tailrace is 9.230 ft. find the head on the turbine.

Ans. 140.5 ft.

4. In the turbine of problem (3) the diameter of the draft tube at the upper end is 24.5 in. and at the bottom it is 42 in. Find the gain in head due to its use when the discharge is 44.5 cu. ft. per sec.

5. The top of the draft tube in problem (4) is 10.0 ft. above the level of the water in the tailrace. Neglecting the friction in it, but considering the discharge loss at the bottom, find the pressure at its top.

6. A reaction turbine by test was found to discharge 31.8 cu. ft. of water per sec. when running at 600 r.p.m. under a head of 143.1 ft. If $F_1 = 0.535$ sq. ft. and $D = 27$ in., find values of c and ϕ.

CHAPTER XV

TURBINE LAWS AND FACTORS

149. Operation under Different Heads.—In the entire discussion in the two preceding chapters we have assumed that the head remained constant though the other quantities might vary. But a turbine may be installed in a plant where the head changes from time to time, and also a given design of turbine might be used in different plants under a wide range of heads. Thus we desire to investigate this phase.

Let us recall the expression, $u_1 = \phi\sqrt{2gh}$. Suppose now that a turbine is compelled to run at a constant speed while the head varies. It is clear that ϕ also varies then just as it would in the preceding case. But it would be possible under some circumstances to change the speed as well in such a way as to keep ϕ a constant. Hence we need to consider two distinct cases when the head changes; one is where ϕ is constant, and the other is where ϕ also changes.

If ϕ remains constant, the wheel speed must vary as $\sqrt{h}$. But a definite value of ϕ is accompanied by a definite value of c. Hence the rate of discharge must also vary as $\sqrt{h}$, since $V_1 = c\sqrt{2gh}$. Now the power of the water is proportional to the product of q and h. Since q varies as $\sqrt{h}$, it follows that the power varies as $h^{3/2}$. And in similar fashion it may be shown that the torque varies as h.

The hydraulic efficiency is a function of c, ϕ, and the turbine dimensions. As long as ϕ remains constant the hydraulic efficiency remains the same regardless of the head. This must be true because the hydraulic losses may all be shown to vary as $h^{3/2}$, just as the power input. But the friction of the bearings and the windage or the disk friction of the runner do not vary in the same way. It is not possible to formulate an exact law for this but they may be said to vary between N and N^2. Since N varies as $\sqrt{h}$, they must vary between $h^{1/2}$ and h. Hence the mechanical losses become a smaller percentage of the total as

the head increases.[1] But, except for very low heads, the difference in the efficiency is usually a matter of not more than 2 or 3 per cent. at most. See Fig. 219.

Now if the speed remains constant while the head changes, or if it does not vary as $\sqrt{h}$, the value of ϕ will change. Referring to Fig. 215, it may be seen that this means a change of c also. Hence the efficiency will change. Thus none of the simple proportions that have just been stated will be true in such a case. It is impossible to calculate the new results unless

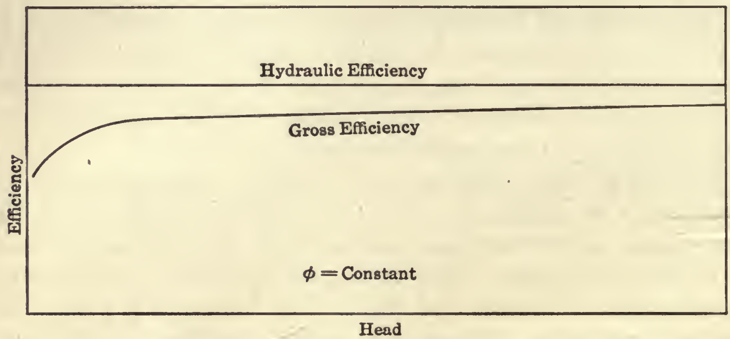


FIG. 219.—Effect of head upon efficiency of a given turbine.

curves, such as those of Fig. 215, are available, or unless we have some complex equations which will give values of all these quantities for any value of ϕ.

150. Different Sizes of Runner.—If a series of runners are all built of the same design with the same angles and proportions so that one is simply an enlargement or reduction of another, they should all have the same values of ϕ_e and c_e. Since their peripheral speeds would all be the same under a given head it follows that their rotative speeds would be inversely proportional to their diameters. And the area F_1 would be proportional to D^2. Hence their capacity and power would vary as D^2. Thus if the performance of one runner is known, that of the rest of the series

[1] An impulse wheel should be set with sufficient space on either side of the buckets at discharge so that the water rebounding from the walls will not strike them. The velocity with which the water rebounds is proportional to V_2 and hence to $\sqrt{h}$. Therefore if this space is not ample for all values of h, a point may be reached where this action would decrease the efficiency.

may be predicted with some assurance, due allowance being made for slight increases in efficiency as the size increased.

We may express these statements algebraically as follows:

$$N_e = \frac{1{,}840\phi_e\sqrt{h}}{D} \tag{137}$$

where 1,840 is a constant which we obtain when we solve for N in terms of peripheral speed, the latter being given by equation (121). For the two types of turbines in common use we have:

Impulse wheel	$\phi_e = 0.43$ to 0.48
Reaction turbine	$\phi_e = 0.60$ to 0.90

according to design. And as to capacity

$$q = K_1 D^2 \sqrt{h} \tag{138}$$

where K_1 has the following range of values:

Impulse wheel	$K_1 = 0.0002$ to 0.0005
Reaction turbine	$K_1 = 0.0014$ to 0.0360

It must be understood that these constants are based upon values corresponding to ϕ_e and that the speed of the wheel must be such that ϕ_e is obtained if they are to apply.

Making a suitable allowance for the efficiency the power delivered by the turbines can be determined when the discharge is known.

It may be seen that the peripheral speed of the reaction turbine is higher than that of an impulse wheel and that it may be varied through a wider range by changes in the design. Also the values of K_1 show that for a given diameter a reaction turbine can discharge more water, and hence develop more power, than an impulse turbine. That means that if they are to deliver the same power the diameter of a reaction turbine will be much less than that of a corresponding impulse wheel. Thus for a given head and power the rotative speed of the reaction turbine will be much higher than that of the impulse wheel due, both to its higher peripheral speed and to its much smaller size.

151. Specific Speed.—A useful factor in turbine work is one that will now be derived. It involves the head, speed, power, and efficiency.

$h_p = K_2 D_2 h^{3/2}$

Since power is proportional to D^2 and to $h^{3/2}$ we may write, b.hp. $= K_2D^2h^{3/2}$. This may be rewritten as

$$\sqrt{K_2}\,D = \frac{\sqrt{\text{b.hp.}}}{h^{3/4}}$$

Inserting the value of D as given by equation (137) we have

$$\sqrt{K_2}\,\frac{1{,}840\phi_e\sqrt{h}}{N_e} = \frac{\sqrt{\text{b.hp.}}}{h^{3/4}}.$$

Rearranging this and letting $\sqrt{K_2}\ 1{,}840\phi_e = N_s$, we have

$$N_s = \frac{N_e\sqrt{\text{b.hp.}}}{h^{5/4}*} \tag{139}$$

While any value of N might be used, the expression has but little meaning unless a particular value is employed. That is generally understood to be N_e, the value of N at which the maximum efficiency under a given head is attained. As to the value of b.hp. it should logically be the one for which the maximum efficiency is obtained under the given head. But in some cases the value of the maximum power at this same speed is used.

The quantity N_s is generally known as *specific speed.* Other names applied to it are unit speed, type characteristic, and characteristic speed. Its value indicates the class to which a turbine belongs. Thus we have seen that for a given head and power the impulse wheel runs at a relatively low r.p.m. Therefore it would have a low value of N_s.

For an impulse wheel under a given head at a given speed the power would increase with the size of the nozzle used. Thus there need not be any lower limit to the value of N_s but the upper limit would be the one for which we had the maximum size jet that could be employed. We find that the efficiency is not appreciably reduced until after we pass a value of about 4.5 for N_s and after a value of 6 the jet is so large for the size of the wheel that the efficiency drops off materially. But any value above 4.5 involves some sacrifice of efficiency.

For the reaction turbine we have limits in both directions as indicated below, though these may be extended in future designs. The values of the specific speed are

* $h^{5/4} = h \times h^{1/4} = h\sqrt{\sqrt{h}}.$

Impulse wheel $N_s = 0$ to 4.5 (6 max.)
Reaction turbine $N_s = 10$ to 100.

For a given turbine the value of N_s is naturally a constant, but it is also practically constant for a whole series of runners of the same design regardless of size. The larger the diameter of a runner the greater its power but the less the value of N for a given head. Hence the product remains constant.

Values of N_s given for the impulse wheel are for a single jet upon a single wheel. When two or more jets are used the power is naturally increased without changing the speed. This enables values between 6 and 10 to be obtained, if necessary. For values above 100 the conditions are impossible. Either the power or the speed of the unit must be decreased.

The specific speed factor shows that the impulse wheel is a low-speed, low-capacity turbine and the reaction turbine is a high-speed, high-capacity turbine. The use of these words is relative rather than absolute. Thus the turbine in Fig. 170 runs at only 55.6 r.p.m. but its specific speed is 82.3, thus indicating that it is a high-speed wheel. For the speed is high as compared with that of other turbines of the same power under that head. For instance the speed of an impulse wheel for similar conditions would be only 4 r.p.m. And the specific speed of the highest head impulse wheels in the world (Art. 125) is only 0.592 though they run at 500 r.p.m. But a slow-speed reaction turbine under the same conditions would run at 8,450 r.p.m. at least, and a high-speed reaction turbine such as those at Cedars Rapids would run at 69,300 r.p.m. Of course these values are absurd and simply demonstrate the fitness of each type for its own field.

152. Uses of Specific Speed.—The values of N_s, as of all other factors in this chapter, are supposed to be obtained from test data, not computed by theory. They serve to classify a turbine and indicate to what type it belongs. They are useful in selecting units for a prospective plant. For such a case the head is known but the size and speed of the units is not. If it is desired to use wheels of a certain type, that fixes the value of N_s between narrow limits, and it is easy to compute the combinations of speed and power that can be produced. Or, if the speed and power be fixed, it may at once be found what type of turbine would be required.

153. Factors Affecting Efficiency.—The efficiency of the impulse wheel is practically independent of the size of the wheel. The author makes this statement after testing sizes from 12 in. to 84 in. in diameter and comparing all the other test data which is accessible. It would seem reasonable that this should be so, for there is no loss in connection with the impulse wheel which would not vary in proportion to the power of the wheel. Aside from questions of design and workmanship the efficiency would appear to be a function of the specific speed. Too low a value of the specific speed would mean a large diameter of wheel for a given power output with a consequently large friction and windage loss. Too high a specific speed would mean that the jet was too large for the wheel and buckets with a consequent lowering of the hydraulic efficiency. The most favorable value

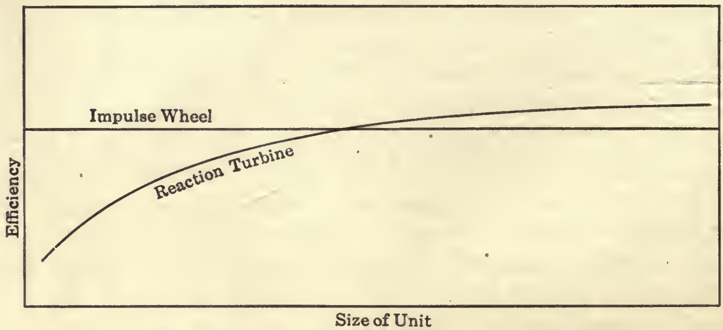


FIG. 220.—Effect of size of turbine upon its efficiency.

of N_s is about 4.0 and the best efficiency that is obtained is about 82.0 per cent. This is slightly exceeded at times and values below it are often obtained.

With the reaction turbine the efficiency is a function of its size. This is partly due to the fact that the hydraulic efficiency increases with the size, but more to the fact that the volumetric efficiency increases. With a reaction turbine there is always a certain amount of leakage between the guides and the runner so that a portion of the water escapes through the clearance spaces and does not pass through the wheel. The area of these clearance rings would naturally be less in proportion to the area of the wheel passages as the size of the wheel increases. Hence a much larger per cent. of the water is made to deliver its power to the runner. Such a condition does not

exist with the impulse wheel. This leads to comparative values for the two types as shown in Fig. 220.

Another distinction between the two types of turbines is that the reaction turbine suffers certain hydraulic losses on part gate that are lacking in the other. Hence, although in some cases the maximum efficiency of a reaction turbine is greater than that of the impulse wheel, the efficiency on a light load might not be as good.

Like the impulse turbine the efficiency of the reaction turbine also depends upon the specific speed, being less at either extreme. The best efficiencies are obtained with values of N_s ranging from 30 to 60. The efficiency of a turbine of good design and workmanship depends upon size, specific speed, and other factors to such an extent that definite values cannot be given, but for fair size units it should range from 80 to 90 per cent. and occasionally more. For small wheels, especially with unfavorable specific speeds, a value of from 60 to 80 per cent. is all that should be expected.

154. PROBLEMS

1. The turbine, whose performance is shown in Fig. 216, developed its maximum efficiency of 88.0 per cent. when delivering 550 hp. at 600 r.p.m. under a head of 141.8 ft. The water consumed was 38.8 cu. ft. per sec. What would be its proper speed under a head of 283.6 ft.? What would then be the rate of discharge and the horsepower?

2. In Fig. 215 the turbine delivered 617 hp. when running at 600 r.p.m. under a head of 140.5 ft., the rate of discharge being 44.5 cu. ft. per sec. and the efficiency 87.0 per cent. If the speed is maintained at 600 r.p.m. when the head is 70.2 ft., find values of discharge, power delivered, and efficiency. (Note: This can be determined only by making use of the curves for this particular turbine. The procedure would be to find the value of ϕ for the new conditions and then take values of q, $hp.$, and e from the curves. These quantities would then have to be reduced to the proper values for the new head.)

3. Compute the factors ϕ and N_s for the runners shown in Figs. 152, 157, 167, 170, and 172. Compare these values with each other.

4. It is desired to develop 6,000 hp. at 514 r.p.m. under a head of 625 ft. Will an impulse or reaction turbine be required? (This can be determined by computing the specific speed.)

5. If only 900 hp. is to be developed for the conditions given in problem (4), what type of turbine will be required?

6. It is desired to use a type of turbine whose specific speed is 30 to deliver 100 hp. under a head of 100 ft. What will be the proper r.p.m. for the unit?

7. An impulse wheel is to be used for 625 hp. under a head of 144 ft.

What will be the maximum rotative speed at which it can be run without material sacrifice of efficiency? What will be the approximate diameter of the wheel?

8. What would be the minimum speed for a reaction turbine for the conditions of problem (7)? If $\phi = 0.60$, what would be the diameter of the runner?

9. What would be the maximum speed for a reaction turbine in problem (7)? Assuming $\phi = 0.85$, what would be the diameter of the runner?

10. The runner in the Cornell University turbine is 27 in. in diameter. The wheel develops 550 hp. when running at 600 r.p.m. under a head of 141.8 ft. What would be the speed and power of a 54-in. runner of the same type under the same head? Would the specific speed of these two be the same?

CHAPTER XVI

THE CENTRIFUGAL PUMP

155. Definition.—Centrifugal pumps are so called because of the fact that centrifugal force or the variation of pressure due to rotation is an important factor in their operation.[1]

In brief, the centrifugal pump consists of an impeller rotating within a case as shown in Fig. 221. Water enters the impeller at the center, flows radially outward, and is discharged around the circumference into the case. During flow through the impeller the water has received energy from the vanes resulting in an increase both in pressure and velocity. Since a large part of the energy of the water at discharge is kinetic, it is necessary to conserve this kinetic energy and transform it into pressure, if the pump is to be efficient.

As a matter of convenience in illustration, the water is represented as entering the impeller in Fig. 221 with a positive pressure. However, the pump is usually set above the level of the water from which it draws its supply, in which case the pressure at this point would be negative. Likewise, the axis of rotation need not be vertical as shown.

156. Classification.—Centrifugal pumps are broadly divided into two classes:

1. Turbine pumps.
2. Volute pumps.

While there are still other types these two are the most fundamental. Also, as we shall see, these may in turn be subdivided in other ways.

The *turbine pump* is one in which the impeller is surrounded by a diffuser containing stationary guide vanes as shown in Fig. 222. These provide gradually enlarging passages whose function it is to reduce the velocity of the water leaving the impeller and thus efficiently transform velocity head into pressure head. The casing surrounding the diffuser may be either circular and con-

[1] For a more complete treatment of this entire subject, either descriptive, theoretical, or practical, see "Centrifugal Pumps," by R. L. Daugherty.

centric with the impeller or it may be spiral like the cases of some reaction turbines.

The *volute pump,* shown in Fig. 223, is one which has no diffusion vanes but, instead, the casing is of a spiral type so

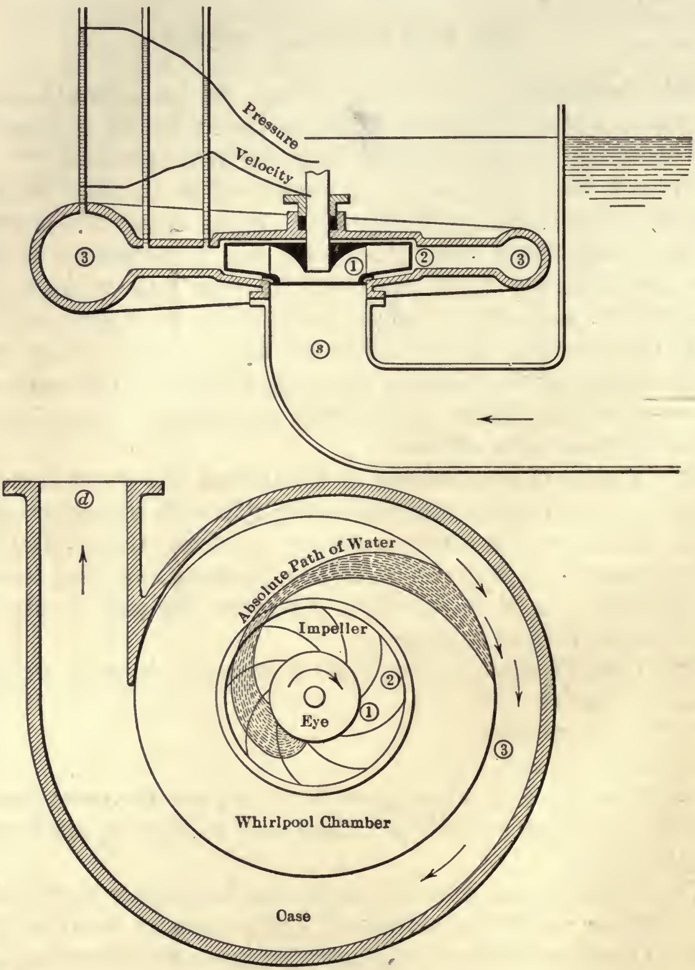


Fig. 221.

proportioned as to produce an equal velocity of flow all around the circumference and also to gradually reduce the velocity of the water as it flows from the impeller to the discharge pipe. Thus the energy transformation is accomplished in a slightly

different way. This spiral is often called a volute, whence the name of the pump.

Occasionally pumps have been built with a whirlpool chamber as shown in Fig. 221. This produces a free spiral vortex, the nature of which has been shown in Art. 120.

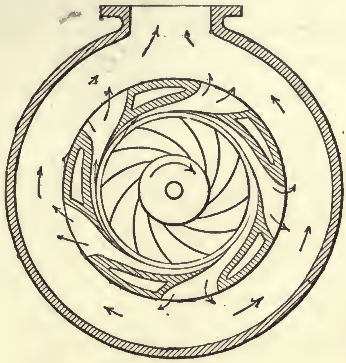


FIG. 222.—Turbine pump.

157. Description of the Centrifugal Pump.—The centrifugal pump is similar to the reaction turbine both in its construction and in its theory. However, one is not the reverse of the other, and their differences are as striking as their similarities.

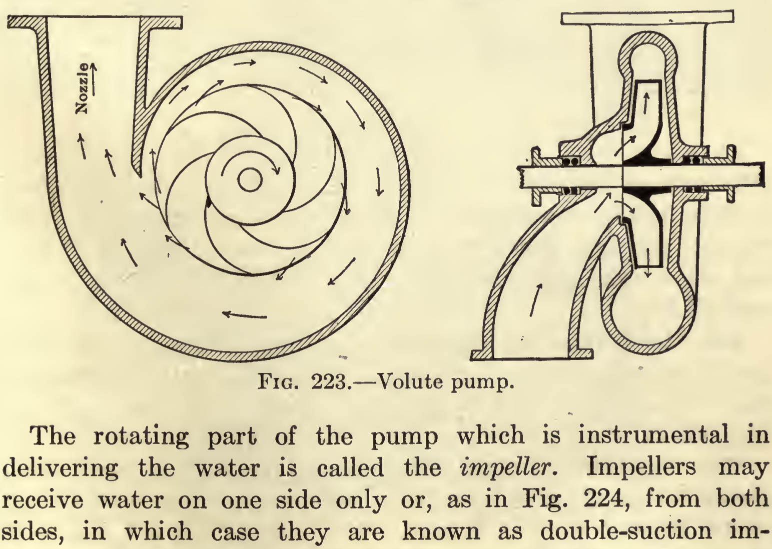
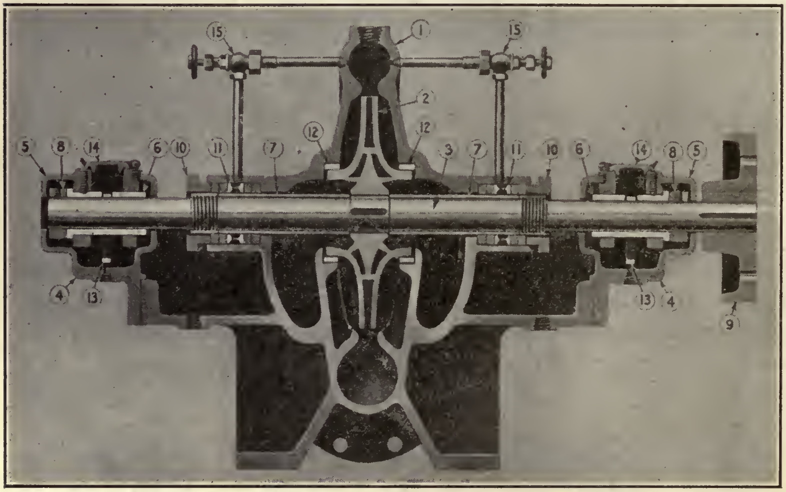


FIG. 223.—Volute pump.

The rotating part of the pump which is instrumental in delivering the water is called the *impeller*. Impellers may receive water on one side only or, as in Fig. 224, from both sides, in which case they are known as double-suction impellers. Fig. 225 gives a view of the pump whose section is

seen in Fig. 224. It may be seen that this impeller is relatively narrow as compared with its diameter, while the opposite type is shown in Fig. 226. For the same rotative speed the latter will discharge more water than the former but at a lower head.

For high heads it becomes desirable to place impellers in series, in which case we have the multi-stage pump, such as is shown in Fig. 227. Multi-stage pumps may be either of the turbine or the volute type. The former may be seen in Fig. 227 and the latter in Fig. 228. The addition of guide vanes so as to produce a turbine pump results in a much more complex con-



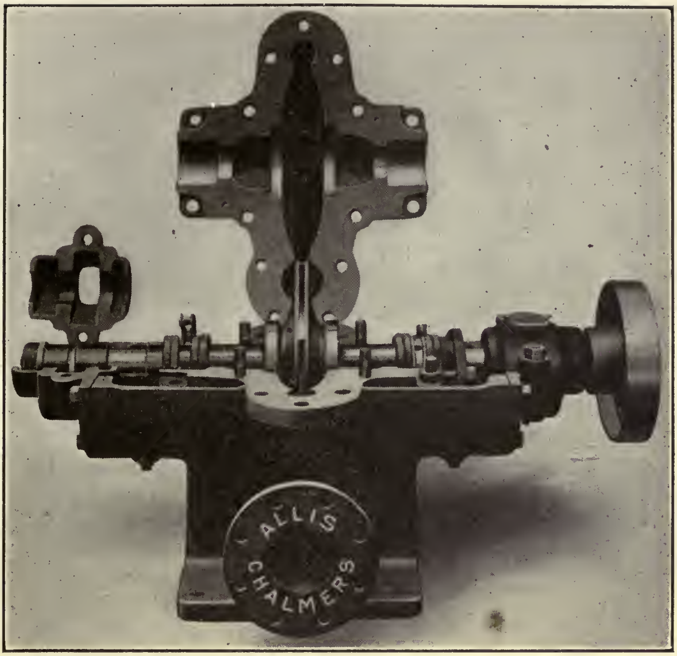
Courtesy of the Allis-Chalmers Mfg. Co.

Fig. 224.—Double-suction volute pump.

struction, as Fig. 229 will show. The water in a multi-stage turbine pump usually passes from one impeller to the next through passages which are like those shown in Fig. 230. There are other arrangements besides this, but they will not be described here.

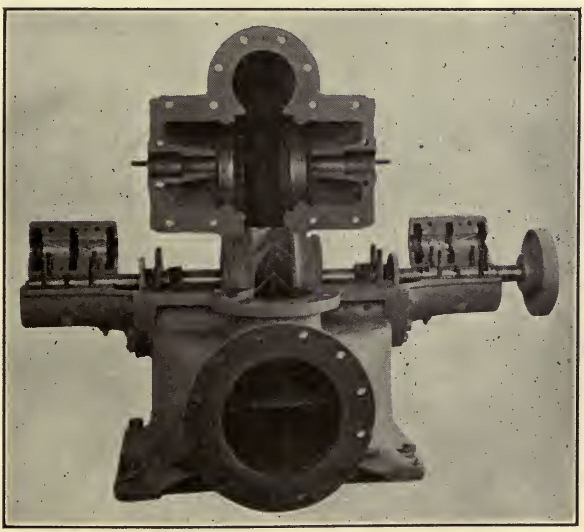
158. Conditions of Service.—Centrifugal pumps are used for lifting water a few feet only or as much as several thousand feet, if necessary. Several such pumps have been built for heads of 2,000 ft.

The capacities of centrifugal pumps range from very small quantities up to as high as 300 cu. ft. per sec. (134,500 G.P.M. or 194,000,000 gal. per 24 hr.). The I. P. Morris Co. has built several of the latter for a head of 16 ft. at 77.5 r.p.m.



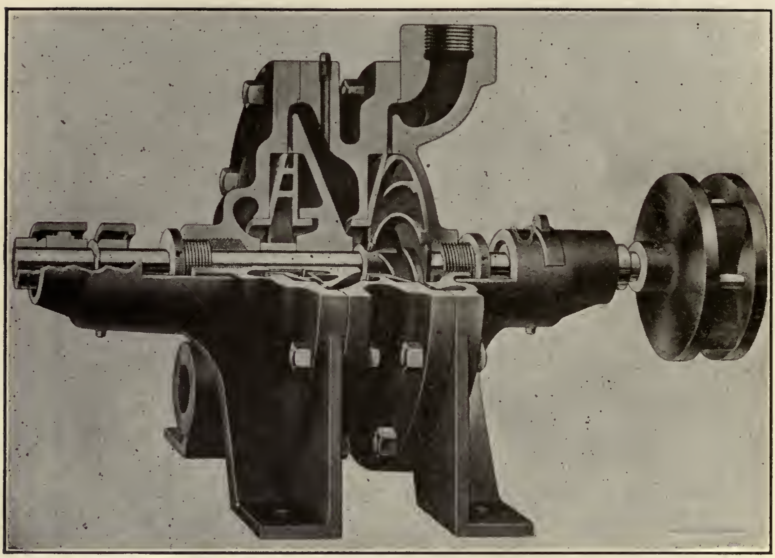
Courtesy of the Allis-Chalmers Mfg. Co.

FIG. 225.—Double-suction volute pump.



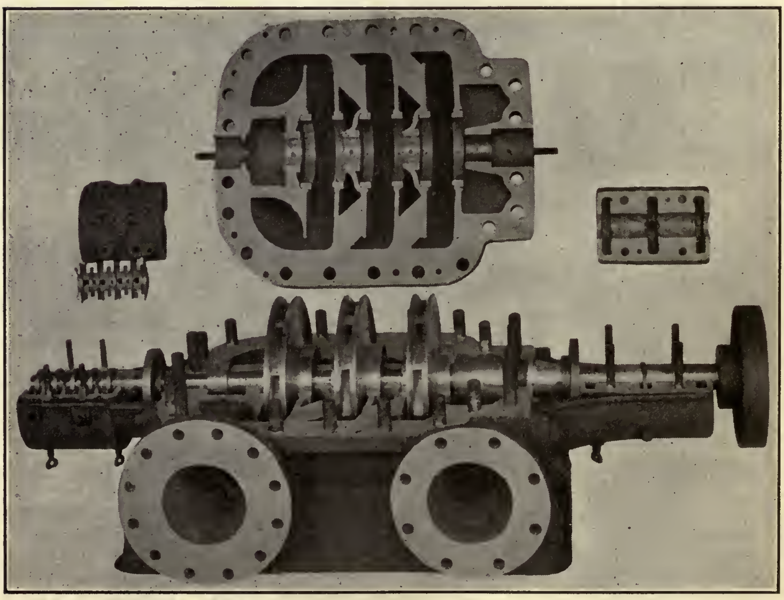
Courtesy of Platt Iron Wks.

FIG. 226.—Double-suction volute pump.



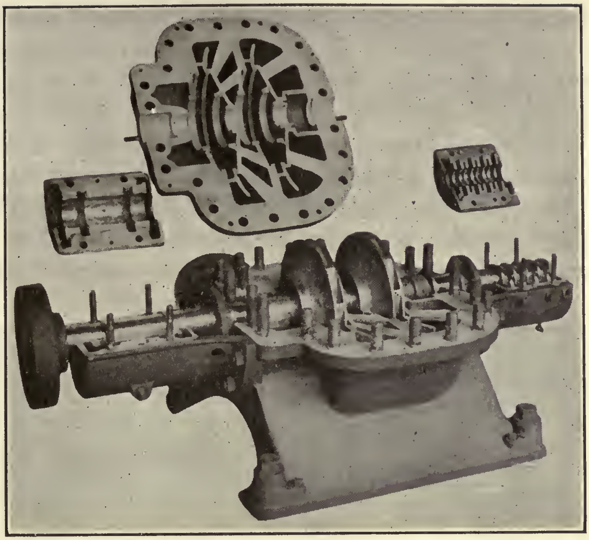
Courtesy of Chicago Pump Co.

FIG. 227.—Two-stage turbine pump.



Courtesy of Platt Iron Wks.

FIG. 228.—Three-stage centrifugal pump without diffusion vanes.



Courtesy of Platt Iron Wks.

Fig. 229.—Two-stage centrifugal pump with diffusion vanes.

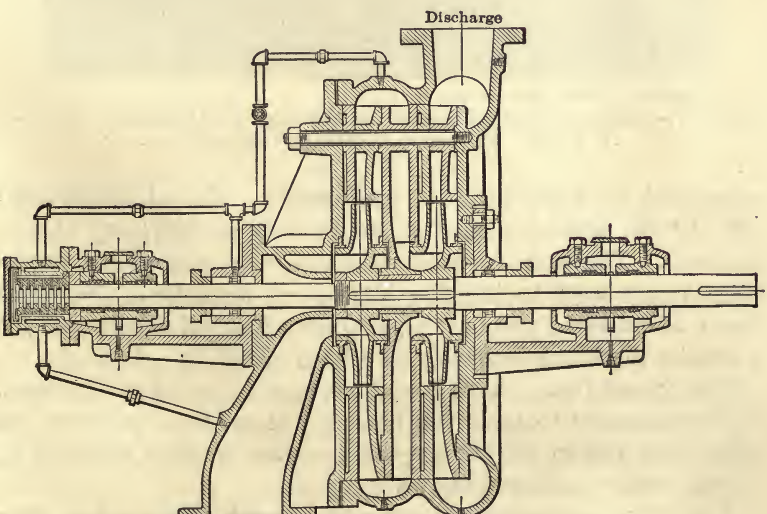


Fig. 230.—Worthington two-stage turbine pump.

The greatest power of any centrifugal pump is that of a pump installed by Sulzer Bros. in Italy. A single-stage pump running at 1,002 r.p.m. delivers 32,530 G.P.M. at a head of 498.6 ft. with an efficiency of 81.0 per cent. The water horsepower is 3,590 and the power required to run it is 4,430 hp. For most pumps the power required is less than 500 hp.

Rotative speeds may vary all the way from 30 to 3,000 r.p.m. in ordinary practice according to circumstances. The highest speed ever employed was 20,000 r.p.m. for a single-stage volute

Courtesy of Allis-Chalmers Mfg. Co.

FIG. 231.—72-in. centrifugal pump for drainage at Memphis. $h = 15'$; $N = 100$. Capacity, 194,000,000 gal. per day.

pump with an impeller 2.84 in. in diameter. The pump delivered 250 G.P.M. against a head of 700 ft. with an efficiency of 60.0 per cent. The highest peripheral speed used was with a single-stage pump with an impeller 3.15 in. in diameter. At 18,000 r.p.m. it delivered 189 G.P.M. against a head of 863 ft. and for a smaller discharge it developed a head of 995 ft.

Centrifugal pumps have been built with as many as 12 stages. It is customary to limit the head per stage to a value of not more than 100 to 200 ft., but this has been greatly exceeded in several cases mentioned above.

Water turbines are rated according to the diameters of their runners, but the size of a centrifugal pump is usually designated

by giving the diameter of the discharge pipe. The rated head and discharge for a centrifugal pump are the values for which the efficiency is a maximum under a given speed. This value of the rate of discharge is often designated as the *normal discharge.* These values will be different for different speeds.

159. Head Developed.—The head developed by a centrifugal pump when no flow occurs is called the "shut-off head" or the

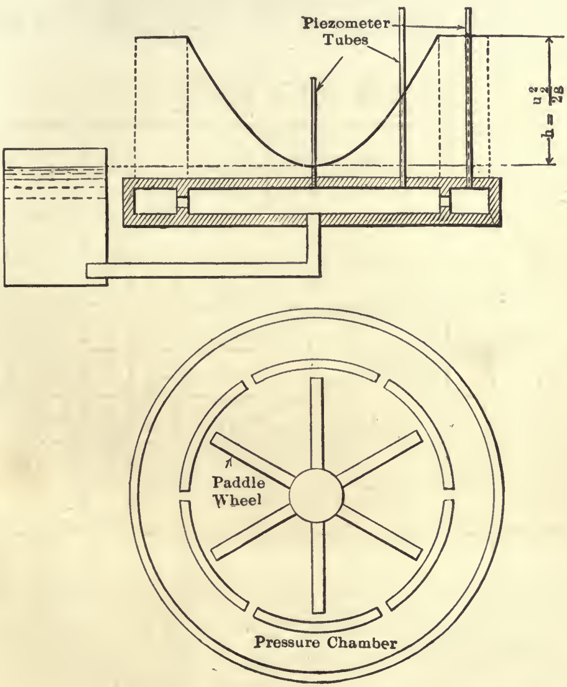


FIG. 232.—Crude centrifugal pump.

"head of impending delivery." Its value may be found by applying the principles of Art. 119.

If water in a closed chamber be set in motion by a paddle-wheel as in Fig. 232, there will be an increase in pressure from the center to the circumference. If the water is assumed to rotate at the same speed as the impeller, the peripheral velocity of which is u_2, it may be seen from equation (117) that $p_2 - p_1 = u_2^2/2g$, where p_1 denotes the pressure at the center. If this water is in communication with a pressure chamber to which a pie-

zometer tube is attached, as in Fig. 232, water will rise in the latter to such a height that

$$h = u_2^2/2g \tag{140}$$

If the height of the tube were less than this, water would flow out and we should have a crude centrifugal pump.

Actually there are certain influences at work in the real pump which affect this relation slightly. Some of these factors tend to increase the head and others to decrease it. The net effect is that for the usual type of centrifugal pump the head of impending delivery is

$$h = 0.85 \text{ to } 1.10\ u_2^2/2g. \tag{141}$$

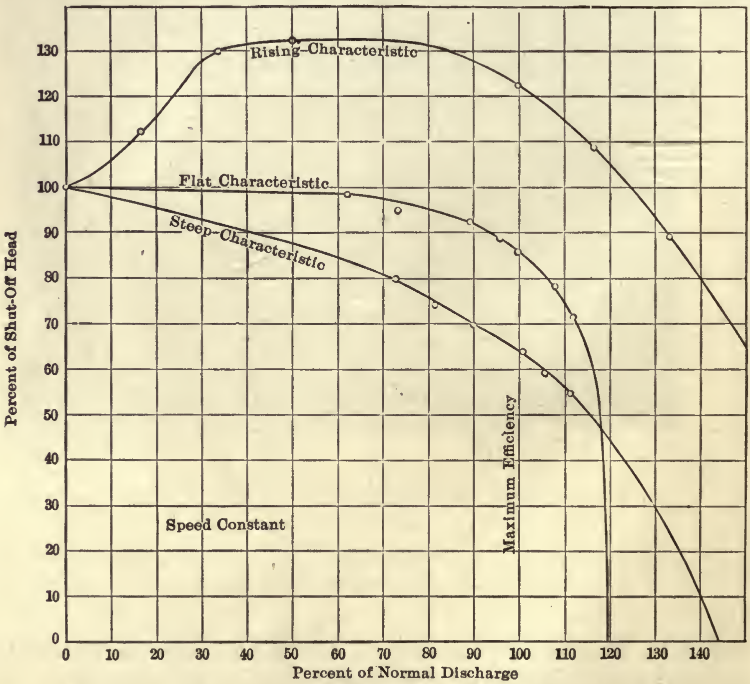


FIG. 233.—Head-discharge characteristics of different pumps.

But as soon as flow occurs the above relation is no longer true, as may be seen in Figs. 233 and 234. When water is being delivered, the head may be either greater or less than the shut-off head, according to the design of the pump. We shall now derive a general relation between head, impeller speed, and rate of discharge for all conditions of operation.

As the water in the suction pipe approaches the impeller it may have a rotary motion imparted to it before it ever reaches the latter, due to the viscosity of intervening particles of water. Hence, we shall write equations between points (2) and (s) in Fig. 221, the latter point being removed far enough from the impeller so that the water has no rotational flow imparted to it.

In Fig. 234a is shown the hydraulic gradient in the case of zero flow and, as in equation (140), $h_o = \frac{u_2^2}{2g}$.

When flow occurs there will be a drop in pressure at (2), which is just within the impeller, due to the velocity head at that point

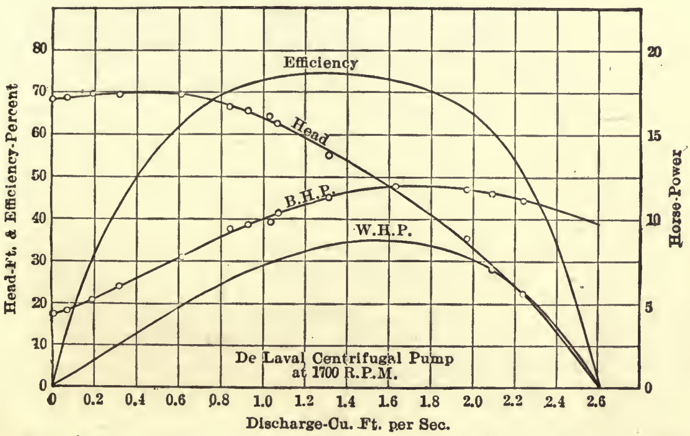


FIG. 234.—Characteristics of a 6-in. pump at a constant speed.

and also due to the loss of head within the impeller passages or from (s) to (2). If k'' is the coefficient of loss the drop in pressure at the outlet of the impeller will be $(1 + k'')v_2^2/2g$.[1] But the pressure at (s) likewise decreases by an amount equal to $V_s^2/2g$. Also as the water flows from (2) to (d) in Fig. 234a, there is a reduction in velocity head from $V_2^2/2g$ to $V_d^2/2g$. This means.

[1] As an illustration consider a hose with a nozzle on the end. When the nozzle is opened so that water may flow, the pressure at the base of the nozzle is decreased below the value obtained when it is closed, by an amount equal to the velocity head at that point and to the friction losses up to that point. If next the hose should be moved around, this pressure drop would not be affected in the least, for it is a function of the velocity of flow within the hose, which is the relative velocity, and does not depend upon the velocity of the water with respect to the earth.

a corresponding gain in pressure, though not without some loss. Thus $mV_2^2/2g$ is converted into pressure plus $V_d^2/2g$, the loss being $(1 - m)V_2^2/2g$.

From equation (145) the total head developed by the pump, including impeller and case, is (see Fig. 234a)

$$h = H_d - H_s = (p_d + z_d + V_d^2/2g) - (p_s + z_s + V_s^2/2g)$$
$$= h_0 - (1 + k'')v_2^2/2g + mV_2^2/2g$$
$$= \frac{u_2^2}{2g} - (1 + k'')\frac{v_2^2}{2g} + m\frac{V_2^2}{2g} \qquad (143)$$

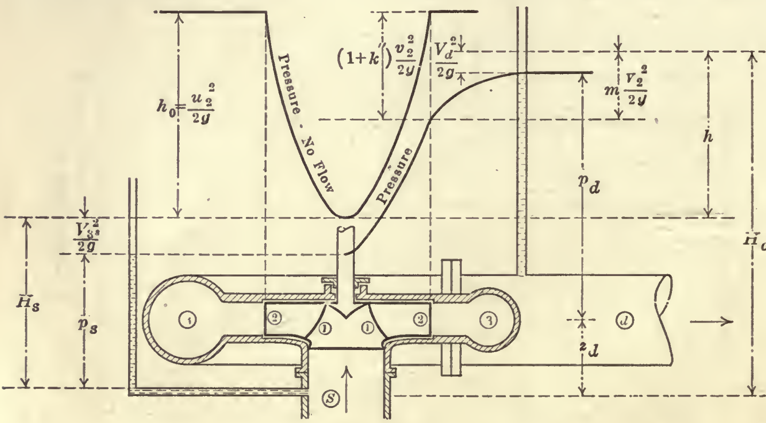


FIG. 234a.

In reality there will be a further drop in pressure at (s) when flow takes place, due to the loss of head in friction in the suction pipe. However, this would also have the effect of decreasing the pressures at (2) and (d) by the same amount. Hence the difference in pressure, with which we are here concerned, would be exactly the same.

It must be noted that the quantity m is a variable. When the discharge from the impeller is such that the angle A_2 (see Fig. 235) agrees with the angle of the diffusion vanes of a turbine pump, or the velocity V_2 is the proper value for a volute pump, the maximum proportion of the velocity head will be saved. For larger or smaller discharges than this there will be additional losses attending this conversion. For a turbine pump the maximum value of m is about 0.75 and for a volute pump it is somewhat less.

With a centrifugal pump the impeller areas are fixed and

constant in value and hence it is convenient to express the rate of discharge as

$$q = f_2 v_2. \tag{144}$$

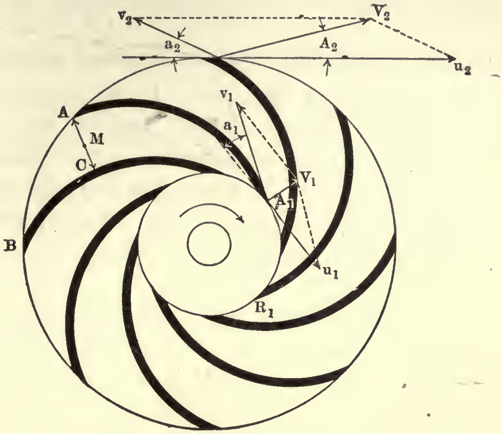


FIG. 235.—Velocity diagrams.

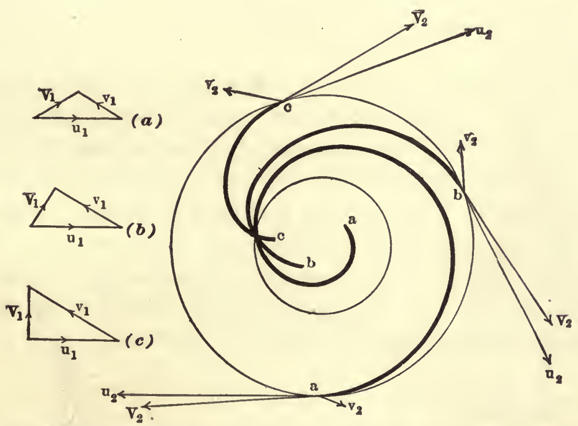


FIG. 236.—Stream lines for three different rates of discharge.

Now referring to the vector diagrams shown in Figs. 235 and 236 it may be noted that as the rate of discharge varies the values of V_2 and A_2 change. It may be seen that as q approaches zero, v_2 and A_2 approach zero, while V_2 approaches u_2. Hence for an infinitesimal discharge the value of V_2 may be regarded as equal to u_2, while the velocity of the water in the case surrounding the impeller is practically zero. Therefore, a particle of water leaving the impeller with a high velocity enters

a body of water at rest and loses all of its kinetic energy. Thus, as the rate of discharge approaches zero, the factor m approaches zero. Hence it may be seen that when we have zero discharge the value of h in equation (143) reduces to that given by equation (140).

An inspection of equation (143) serves to explain the rising or falling characteristics of Fig. 233. If the increase of pressure due to the conversion of the velocity head of discharge is more than enough to offset the decrease due to the velocity and the losses within the impeller, we have a rising characteristic. If they are about equal we have a flat characteristic, and if the quantity mV_2^2 is less than $(1 + k'')v_2^2$ we have a falling characteristic.

Because of the difficulty of efficiently transforming velocity head into pressure head it is desirable to keep V_2 as small as possible. It may be seen that, for a given value of v_2, the smaller the angle a_2 the less will be the magnitude of V_2. Therefore, in almost all centrifugal pumps the value of a_2 is from 20° to 30°, though occasionally this angle is as small as 10° or as large as 80°. It is rarely made larger than 90° because of the inefficiency of such designs.[1]

160. Measurement of Head.—We may compute the head which a pump is required to work against by equations (72) or (74). The head which the pump can develop may be estimated by equation (143), but when we desire to measure the head which the pump actually does develop we do so by taking certain readings on the discharge and suction sides of the pump. Thus in Fig. 237 the difference between the energy which the water has as it enters the pump at (s) and that with which it leaves at (d) is due solely to the pump. Hence we may write

$$h = H_d - H_s.$$

But $H_d = p_d + z_d + V_d^2/2g$, and $H_s = p_s + z_s + V_s^2/2g$. Therefore it follows that

$$h = (p_d - p_s) + (z_d - z_s) + (V_d^2 - V_s^2)/2g. \qquad (145)$$

[1] In the turbine theory the angle a has been defined as the angle between v and u. This is satisfactory for that purpose, but with ordinary centrifugal pumps this angle is always greater than 90°. Hence it is much more convenient to define it here as the angle between v and $-u$, as may be seen in Fig. 235.

As a usual thing the water enters the pump under a pressure less than that of the atmosphere, in which case the value of p_s will be negative. If the suction and discharge pipes at the points where the gages are attached are of the same diameter the velocity heads will cancel, in which event the value of h

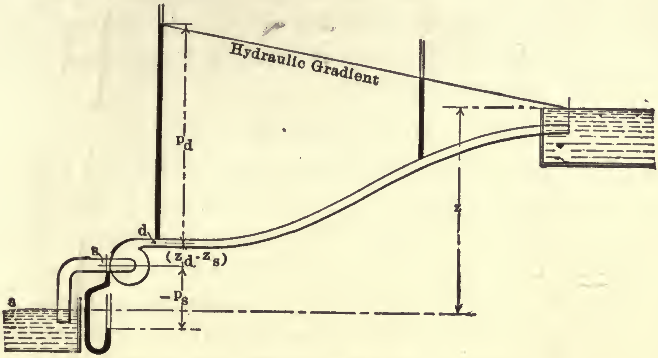


FIG. 237.—Head developed by pump.

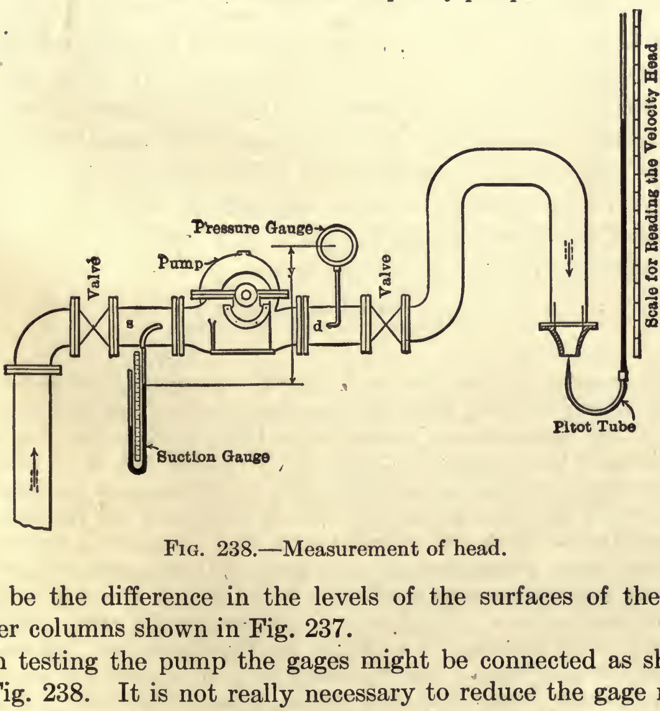


FIG. 238.—Measurement of head.

will be the difference in the levels of the surfaces of the two water columns shown in Fig. 237.

In testing the pump the gages might be connected as shown in Fig. 238. It is not really necessary to reduce the gage readings to the pressures which would be found at the center line

of the pipe. If the gage readings are used direct in equation (145) and the value of y represents $(z_d - z_s)$, it may be shown that the result is the same.[1]

161. Head Imparted by Impeller.—The amount of energy delivered to the water by the impeller is greater than that actually delivered in the water, the difference being due to hydraulic friction losses within the pump. If the head actually developed by the pump is represented by h, the head imparted to the water by the impeller is

$$h'' = h + h' \quad \text{or} \quad h'' = h/e_h.$$

In an ideal pump without hydraulic losses of any kind these two quantities would be equal, but in any real pump they represent two entirely different things. For a given pump under different conditions of operation, h'' and h neither differ from each other by a constant amount nor is one a constant proportion of the other. Hence the curves representing actual values of both h'' and h not only do not coincide but they are not even of the same shape. This may be seen in Fig. 239 in which the curve "*Actual Head Input*—h''" has been determined with a reasonable degree of accuracy from test data by the author. It may be seen that for rates of discharge from 0 to 0.6 cu. ft. per sec. the value of h increases while that of h'' decreases. In some other cases the difference is more marked than is here shown.

An expression for h'' may be derived by using the value of T given in equation (102). Since $Wh'' = T\omega$, $h'' = u_2 s_2/g$. The value of s_2 is $u_2 - v_2 \cos a_2$, hence

$$h'' = \frac{u_2(u_2 - v_2 \cos a_2)}{g} \tag{146}$$

[1] The question is often raised as to why it is necessary to deduct $V_s^2/2g$ in determining the head, since the pump has imparted that velocity to the water. The first answer is that equation (145) is the result of a direct application of the principles of energy, but the explanation of the matter is that we also have included p_s, whose value is a function of V_s. Suppose, for example, that the suction pipe were so large that the velocity in it were negligible. Then the measured value of p_s would give a higher pressure than when the suction pipe is smaller and, disregarding losses, the values of p_s in the two cases would differ by $V_s^2/2g$. If we are to omit the velocity head at (s) we should omit the pressure reading also. We might then obtain the total head by adding to the "discharge head" the value of z_s + *suction pipe losses.* But we should have to compute the latter and it may be shown that they are determined experimentally when p_s is measured and the velocity head $V_s^2/2g$ also employed.

This could also be obtained from equation (143) by eliminating the hydraulic losses. This would require values of $k'' = 0$ and $m = 1.0$. The next step would be to solve the vector triangle for V_2 in terms of u_2, v_2, and a_2. The result would agree with equation (146).

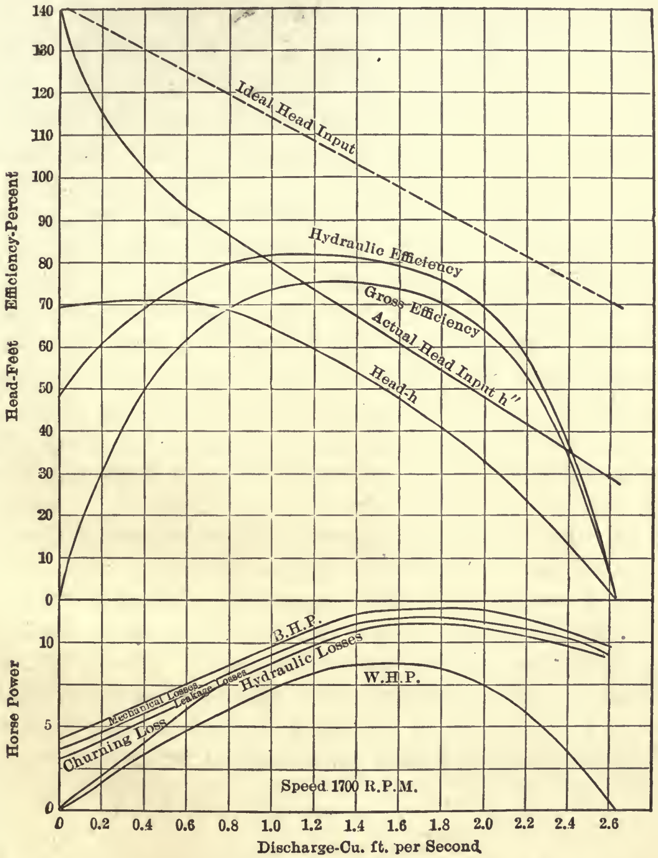


FIG. 239.—Analysis of centrifugal pump at a constant speed.

It may be seen that for a constant value of impeller speed the value of h'' given by equation (146) will increase with q (and v_2) for values of a_2 greater than 90°, will be independent of q for $a_2 = 90°$, and will decrease for values of a_2 less than 90°. It is sometimes argued from this that rising or falling characteristics are obtained by a suitable choice of a_2, but that is due to confusing

h'' and h. The value of a_2 does have some effect upon this, but it alone does not determine the matter. The author has found a decidedly rising characteristic in a pump he has tested with an angle of $a_2 = 26°$. And tests of other pumps with $a_2 = 90°$ have shown steep falling characteristics. The real explanation may be seen only in equation (143).

The hydraulic efficiency is the ratio h/h''. For the same reasons as are given in Art. 142, it is difficult to calculate true values of h'' and thus the *true* hydraulic efficiency can be determined only by test. The latter gives directly the value of the total efficiency only and it is necessary to allow for other losses or determine them by special methods in order to get the actual hydraulic efficiency. Applying equation (146) and using the actual impeller dimensions the computed values of h'' may be found to lie on a straight line, such as is labelled "*Ideal Head Input*" in Fig. 239. Thus the ratio between the actual h and the h'' computed in the ordinary manner is much less than the hydraulic efficiency in all cases. It is very often less than the gross efficiency thus proving that it is not a true value. But it is still useful for some purposes of design and is called "manometric coefficient." The value of this ratio is usually between 0.55 and 0.65.

162. Centrifugal-pump Factors.—Just as in the case of turbines, it is found that to obtain the best efficiency with a given centrifugal pump there must be a certain relation between head, speed, and discharge. Also the equations show that these three quantities are mutually interrelated. Hence it may be seen from equation (143) that for a velocity diagram of the same shape to be formed it is necessary that u_2, v_2, and V_2 vary as $\sqrt{h}$. Hence we shall find it convenient to write[1]

$$u_2 = \phi\sqrt{2gh} \qquad (147)$$

$$v_2 = c\sqrt{2gh} \qquad (148)$$

We find that a certain value of ϕ is required to obtain the maximum efficiency just as with the turbine. And a definite value of c is associated with every value of ϕ as may

[1] For the pump ϕ has the same meaning as in the inward flow reaction turbine since it gives the peripheral speed in both cases. But c has a different meaning, since we find it more convenient to deal with v_2 rather than with V_2.

be seen from equation (143). For ordinary types of pumps we find the following values of these factors:

For shut-off	$\phi = 0.95$ to 1.09
For normal discharge	$\phi_e = 0.90$ to 1.30
For normal discharge	$c_e = 0.10$ to 0.30

The value of ϕ_e will depend upon the design of the pump. Thus the smaller the angle a_2 and the fewer the number of impeller vanes, the larger the value of ϕ_e.

Just as in Art. 150, it may be shown that

$$N = \frac{1{,}840\,\phi\sqrt{h}}{D}. \tag{137}$$

163. Specific Speed.—The specific speed factor for turbines involves the developed horsepower, since that is the quantity with which we are concerned. But with centrifugal pumps we are primarily interested in their capacity and it will be more useful if we derive a similar expression giving N_s in terms of discharge. Since power and discharge are really proportional to each other it may be seen that we are merely expressing the specific speed for the pump in terms of different units.

Proceeding just as in Art. 151, except that we use equation (138) direct, we obtain for the centrifugal pump

$$N_s = \frac{N\sqrt{\text{G.P.M.}}}{h^{3/4}}* \tag{149}$$

The capacity of a centrifugal pump is generally expressed in gallons per minute rather than in cubic feet per second. Thus the expression will probably be the handiest in the above form. (1 cu. ft. = 7.48 U. S. gal.).

For an impeller, either single-suction or double-suction, values of specific speed may be found between the following limits:

$$N_s = 500 \text{ to } 8{,}000.$$

For special constructions even higher values may be attained. It must be noted that these apply only to single stages. For a multi-stage pump it is necessary to divide the total head by the number of stages to obtain the proper value of h for use in the equations in this chapter.

*Note that $h^{3/4} = h \div h^{1/4} = h \div \sqrt{\sqrt{h}}$. Some values of $h^{3/4}$ will be found on page 263.

Values of specific speeds are obtained from tests of actual pumps and they may then be applied to other pumps of the same type. For N_s is an index of the type of pump just as it is in the case of the turbine. Its great value is that it enables us to determine the combinations of speed, capacity, and head per stage that are possible or desirable. And if we desire to employ a certain type of pump with a definite value of N_s, we may then find the combinations of these factors that are required.

164. Operation at Different Speeds.—In this chapter we have shown characteristics of centrifugal pumps operating under variable heads at constant speeds. We may now desire to know how the pump is affected by a change in speed. This is shown by equations (147) and (148). To obtain similar conditions of operation it is necessary that the values of ϕ and c be maintained constant. If they are, it may be seen that both the speed and discharge of the pump will vary as the square root of the head. But if ϕ and c are not constant then we have no simple index to the variation of the equantities. We can then only resort to some second degree equation of the form shown in Art. 159. Hence if the head is varied due to a change in speed it must be understood that the rate of discharge varies also if the following simple ratios are to apply.

From equation (147) it may be seen that

$$h = \frac{1}{\phi^2} \frac{u_2^2}{2g}. \tag{150}$$

Which shows that if ϕ remains constant, the head developed varies as the square of the pump speed. From equation (148) we may obtain, after substituting the value of h given by equation (150),

$$v_2 = \frac{c}{\phi} u_2 \tag{151}$$

Hence it follows that if ϕ remains constant, c will also remain constant and the rate of discharge must vary directly as the speed. Since power is a function of the product of h and q it may be seen that it will vary as the cube of the speed. Just as in the case of the turbine, the hydraulic efficiency of a centrifugal pump is independent of the speed, within reasonable limits, as long as ϕ is constant. But the maximum gross efficiency of a given pump will increase slightly as higher speeds are attained.

165. Factors Affecting Efficiency.—The considerations of Art. 153 apply here also. The most important factor in determining the efficiency of a centrifugal pump is its capacity, as may be seen in Figs. 240 and 241. A pump of small capacity will have a low volumetric efficiency because of the relatively large per cent. of the water which will leak back into the

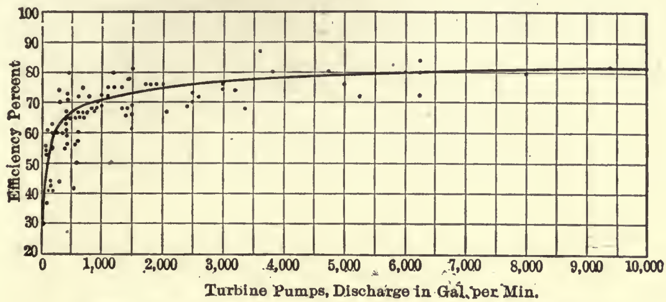


FIG. 240.—Efficiency as a function of capacity.

suction side through the clearance rings. Also the disk friction of such a pump is a greater percentage of the total power expended.

It may be shown that the head per stage has only a slight effect upon the efficiency of the pump, providing the design is carefully made.

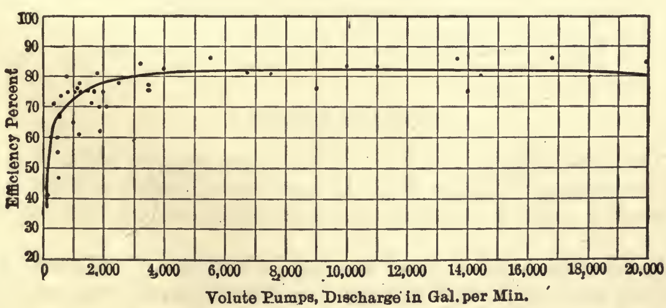


FIG. 241.—Efficiency as a function of capacity.

For a given capacity, however, the efficiency will be found to differ with different pumps, due not only to variations in workmanship and construction but also to the other factors such as speed and head. Since these are all involved in the specific speed, it would seem reasonable that efficiency may be expressed

as a function of the latter. Figs. 242 and 243 show the relation between efficiency and specific speed for a large number of turbine and volute pumps. But it should be borne in mind that for any given specific speed the larger the capacity the higher the efficiency. Hence we can have no single curve that will enable us to select definite values for any case.

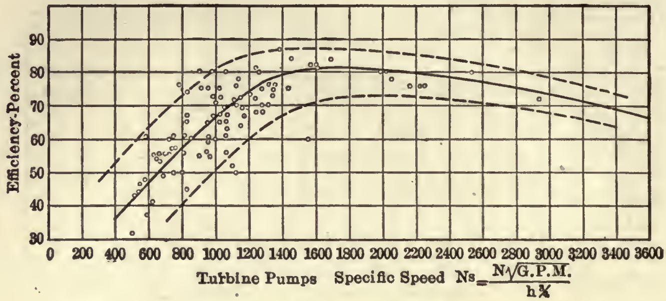


FIG. 242.—Efficiency as a function of specific speed.

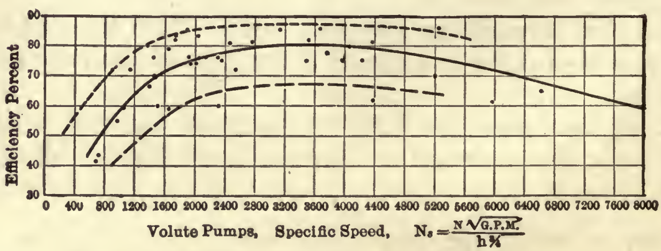


FIG. 243.—Efficiency as a function of specific speed.

166. PROBLEMS

1. The curves of Fig. 239 are for a single-stage pump in which $D = 9.12$ in., $f_2 = 0.0706$ sq. ft., $a_2 = 27°$. At 1,700 r.p.m. when $q = 1.315$ cu. ft. per second, $h = 55.7$ ft. If it be assumed that $m = 0.50$, find the value of k''.

2. A two-stage turbine pump running at 1,700 r.p.m. delivered 0.429 cu. ft. per sec. at a head of 225 ft. The essential dimensions were: $D = 12$ in., $f_2 = 0.0244$ sq. ft., $a_2 = 26°$. If it be assumed that $m = 0.70$, compute the value of k''.

3. The curves of Fig. 234 were obtained from the test of a single-stage pump having the following dimensions: $D = 9.12$ in., $f_2 = 0.0706$ sq. ft. Find the values of ϕ_o and c_e.

4. If it be assumed in Fig. 233 that the value of ϕ for shut-off is 1.0, what is the value of ϕ for the maximum lift of the pump with the rising characteristic? What is the value of ϕ for maximum efficiency in each case?

5. The diameter of a pump impeller is 10 in. The speed is to be 1,200 r.p.m. If $\phi = 1.20$, what is the value of h?

6. Compute the value of the specific speed for the pump shown in Fig. 231.

7. Compute the value of the specific speed for the pump whose dimensions are given in problem (2).

8. What would be the capacity, head, and power of the pump whose performance is shown in Fig. 234, if it were run at a speed of 1,000 r.p.m.?

9. What speed would be necessary to double the capacity of the pump whose curves are shown in Fig. 234? What speed would be required to double its lift?

10. If the speed of the pump of Fig. 234 were doubled, what would be the head for a discharge of 2.4 cu. ft. per sec.? What would be the efficiency for this rate of discharge at the higher speed?

11. It is desired to deliver 1,600 G.P.M. at a head of 900 ft. with a single-stage pump. What would be the minimum rotative speed that could be used?

12. If a speed of 600 r.p.m. is desired in problem (11), how many stages must the pump have at least?

13. It is desired to use a type of pump whose specific speed is 2,000 under a head of 16 ft. If the speed is to be 1,800 r.p.m., what will be the capacity?

14. Compute the specific speeds of the pumps for which data are given in Art. 158.

APPENDIX—TABLES

Table 9.—Areas of Circles

Diameter		Area		Diameter		Area	
Inches	Feet	Square inches	Square feet	Inches	Feet	Square inches	Square feet
¼	0.0021	0.0491	0.00034	30	2.500	706.9	4.90
½	0.0042	0.1963	0.00136	32	2.667	804.3	5.58
¾	0.0062	0.4417	0.00306	34	2.830	907.9	6.30
1	0.083	0.7854	0.00545	36	3.000	1,018.0	7.07
1¼	0.104	1.227	0.00853	38	3.17	1,134.0	7.88
1½	0.125	1.767	0.0123	40	3.44	1,257.0	8.72
1¾	0.146	2.405	0.0167	42	3.50	1,385.0	9.62
2	0.167	3.142	0.0218	44	3.67	1,521.0	10.57
2½	0.208	4.909	0.0341	46	3.83	1,662.0	11.53
3	0.250	7.069	0.0492	48	4.00	1,810.0	12.56
3½	0.292	9.621	0.0668	50	4.17	1,964.0	13.63
4	0.333	12.566	0.0872	52	4.33	2,124.0	14.75
4½	0.375	15.909	0.1105	54	4.50	2,290.0	15.90
5	0.417	19.635	0.1362	56	4.67	2,463.0	17.10
6	0.500	28.27	0.196	58	4.83	2,642.0	18.35
7	0.583	38.48	0.267	60	5.00	2,827.0	19.62
8	0.667	50.26	0.349	62	5.17	3,019.0	20.93
9	0.750	63.62	0.442	64	5.33	3,217.0	22.3
10	0.833	78.54	0.545	66	5.50	3,421.0	23.8
12	1.000	113.1	0.785	68	5.67	3,632.0	25.2
14	1.167	153.9	1.068	70	5.83	3,848.0	26.7
16	1.333	201.1	1.395	72	6.00	4,072.0	28.3
18	1.500	254.5	1.765	76	6.33	4,536.0	31.4
20	1.667	314.2	2.18	80	6.67	5,027.0	34.9
22	1.833	380.1	2.64	90	7.50	6,362.0	44.2
24	2.000	452.4	3.14	100	8.33	7,854.0	54.5
26	2.164	530.9	3.68	110	9.17	9,503.0	66.0
28	2.332	615.8	4.27	120	10.0	11,310.0	78.5

TABLE 10.—STANDARD WROUGHT-IRON PIPE SIZES

Diameter		Internal area		Diameter		Internal area	
Nominal, inches	Actual internal, inches	Square inches	Square feet	Nominal, inches	Actual internal, inches	Square inches	Square feet
⅛	0.27	0.0573	0.0004	3½	3.548	9.887	0.0687
¼	0.364	0.1041	0.0007	4	4.026	12.73	0.0884
⅜	0.494	0.1917	0.0013	4½	4.508	15.96	0.1108
½	0.623	0.3048	0.0021	5	5.045	19.99	0.1388
¾	0.824	0.5333	0.0037	6	6.065	28.89	0.2006
1	1.048	0.8626	0.0060	7	7.023	38.74	0.2690
1¼	1.380	1.496	0.0104	8	7.982	50.04	0.3474
1½	1.611	2.038	0.0141	9	8.937	62.73	0.4356
2	2.067	3.356	0.0233	10	10.019	78.84	0.5474
2½	2.468	4.784	0.0332	11	11.000	95.03	0.6600
3	3.067	7.388	0.0513	12	12.000	113.1	0.7854

TABLE 11.—VALUES OF $m^{2/3}$

m	$m^{2/3}$	m	$m^{2/3}$	m	$m^{2/3}$	m	$m^{2/3}$
0.2	.342	.2	1.69	4.0	2.52	8.0	4.00
0.4	.543	.4	1.79	.2	2.60	.5	4.17
0.6	.712	.6	1.89	.5	2.73	9.0	4.33
0.8	.863	.8	1.98	5.0	2.92	10.0	4.63
1.0	1.000	3.0	2.08	.5	3.12	11.0	4.93
.2	1.13	.2	2.17	6.0	3.29	12.0	5.22
.4	1.25	.4	2.26	.5	3.48	13.0	5.52
.6	1.37	.6	2.35	7.0	3.66	14.0	5.80
.8	1.48	.8	2.44	.5	3.83	15.0	6.10
2.0	1.58						

TABLE 12.—VALUES OF $h^{3/4}$

h	$h^{3/4}$	h	$h^{3/4}$	h	$h^{3/4}$	h	$h^{3/4}$
10	5.62	25	11.18	70	24.20	140	40.6
11	6.03	30	12.82	80	26.77	150	42.8
12	6.45	35	14.38	90	29.33	170	47.1
13	6.85	40	15.90	100	31.6	200	53.2
14	7.24	45	17.38	110	33.9	230	59.0
16	8.00	50	18.80	120	36.2	260	64.8
18	8.73	60	21.25	130	38.5	300	72.0
20	9.45						

FUNDAMENTAL TRIGONOMETRY

In a right angle triangle, such as Fig. 244:

$$\sin A = a/c \qquad \sec A = c/b$$
$$\cos A = b/c \qquad \csc A = c/a$$
$$\tan A = a/b \qquad \cot A = b/a$$

Any function of A is the same numerically as the co-function of any combination of A with an odd multiple of 90°. Thus:

$$\sin A = \cos (90° \pm A) = \cos (270° \pm A).$$

Any function of A is the same numerically as the function of any combination of A with an even multiple of 90°. Thus:

$$\sin A = \sin (180° \pm A).$$

The sign of the function depends in any case upon the quadrant in which the angle itself lies.

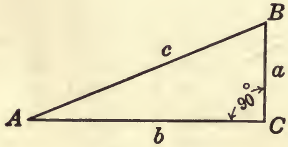


FIG. 244.

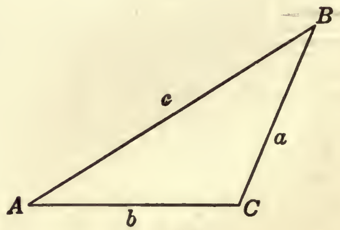


FIG. 245.

For the solution of an oblique triangle, such as that shown in Fig. 245, we have

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$a^2 = (b - c)^2 + 4bc \sin^2 \frac{A}{2}$$

$$a^2 = (b + c)^2 - 4bc \cos^2 \frac{A}{2}$$

$$a^2 = (b \sin A)^2 + (c \cos A - b)^2$$

This is as much as is required for the solution of the vector triangles that will be encountered with turbines and centrifugal pumps.

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